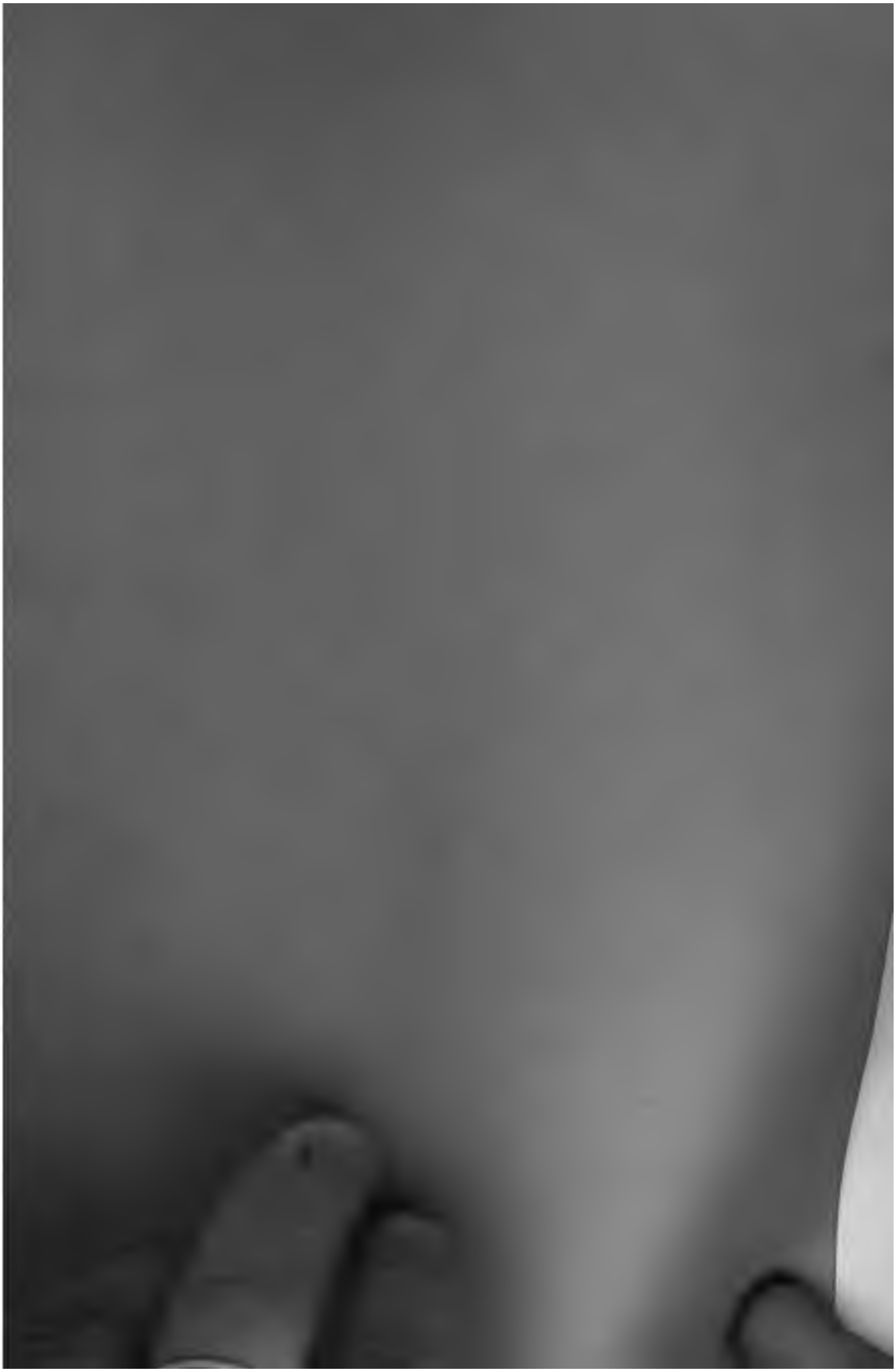

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THE

# ASTROPHYSICAL JOURNAL

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## ON THE PERIOD OF THE SOLAR SPOTS.

By SIMON NEWCOMB.

IN discussing periodic phenomena in which the times of recurrence of a given phase are subject to irregularities two hypotheses may be made. One is that underlying the periodic phenomena which we observe, there is a primary cause going through a perfectly uniform period; but that, on the action of this cause are superseded irregular actions which may delay or accelerate the occurrence of a phase without affecting the primary cause. When this is the case we shall have a series of perfectly equidistant normal epochs for the recurrences of the same phase, and the observed deviations from these epochs will be in the nature of separate and independent accidental errors. That is to say, if $P$ be the true value of the normal period then, at the end of $n$ periods, however great $n$ may be, the time of occurrence of the phase will differ from $n\ P$ only by a small quantity $\pm\epsilon$ indicating the irregularity in the general mean. This value of $\epsilon$ will be the same, no matter how great $n$ may be.

As an example of this we may take the hypothesis that the variations in the solar spots are due to the action of some planet, say *Jupiter*, which acts differently in different parts of its orbit,

but always produces the same action when it returns to the same point. Then the normal phase of the solar spots would always be that corresponding to the longitude of the planet, and the deviations from this phase would be in the nature of accidental irregularities.

The other hypothesis is that, while there is still a certain normal mean period, this period is nevertheless subject to change in such a way that, if a phase is once accelerated, the advance thus produced will go on indefinitely into all subsequent phases. For example, if the maximum of spot activity indicates a state of things leading in a regular way to a following minimum, then, if this maximum is accelerated, the occurrence of the minimum would be equally accelerated. These accidental accelerations or retardations being supposed purely accidental would follow the law of accidental error; that is to say, if $\pm\epsilon$ be the probable acceleration or retardation in a single period, then, at the end of $n$ periods the probable deviation from the normal would be $\pm\epsilon\times\sqrt{n}$. As $n$ increased there would be no limit to the possible deviation of the phase.

In deciding which of these two hypotheses applies to a given case we are met with the difficulty that we cannot determine the normal period except by the observations themselves. Consequently, although at the end of $n$ periods the phase should be accelerated or retarded by any amount, we should be obliged, in determining the mean period, to divide the excess or deficiency among all the periods so that, apparently, we should fail to get evidence of the accumulation. But probable evidence would still be obtainable if the number of consecutive periods observed was great. We might then divide the whole period of observation into a certain number, say two or three, of equal parts. If $n$ be the number of periods in each of these parts, then each part would probably differ from the normal length by the mean amount $\pm\epsilon\sqrt{n}$. These variations being quite independent of each other, the probable differences between the accumulated errors of any two parts would be $\pm\epsilon\sqrt{2n}$.

If, in any special case, a difference between the accumulated

lengths was found exceeding the probable deviations, we should have strong evidence that the period was of the second class. In the contrary case the evidence would be more or less probable according to the number of periods, but never quite conclusive.

Maxima and minima of solar spots have been derived by Wolf with more or less certainty through about twenty-five full periods. It seems possible to decide whether or not there is a normal period underlying the changes of activity which they exhibit.

The method of determining a period of a varying phenomenon from the observed phases is worthy of consideration. A plausible method frequently applied is this: some definable phase is seen to occur at the $n$ epochs $t_1, t_2, \ldots t_n$. Then, the observed periods will be $t_2 - t_1$; $t_3 - t_2$; .... $t_n - t_{n-1}$. It is common to take the mean of these periods as the normal period to be derived from the observations. But we readily see that the sum of all these periods is nothing but $t_n - t_1$; that is to say, the mean which we get is nothing more than the extreme time divided by the number of periods. Thus we completely ignore the periods which might be derived from all the intermediate observations. It is clear enough that these intermediate periods are worthy of consideration. If, for example, there were twenty recurrences, the result following from the interval between the second and the nineteenth would fall little short in accuracy from the comparison of the first and twentieth. In fact, the several values of the period,

$$\frac{t_{20} - t_1}{19}; \quad \frac{t_{19} - t_2}{17} \quad \ldots\ldots,$$

would be so many independent determinations of continually diminishing weight, of which all but the first are ignored in the method in question.

If the determinations of the epochs were all of equal weight the best result would be obtained by the weighted mean of the several determinations thus found. But the general method is to proceed by a least square solution which shall give an

equidistant set of times differing as little as possible from the observed times.

In applying this method to the determination of the period of the solar spots I shall depend mainly upon the epochs derived by Rudolf Wolf so far as they go. The results of his investigations up to 1872 are found in his *Handbuch der Mathematik*, etc., Zurich, 1872, Vol. II, page 296, etc. His researches in detail are found scattered through his *Astronomische Mittheilungen*. For more recent phases and for the correction of one or two of Dr. Wolf's last phases, I have depended upon the Greenwich observations. Since 1874 photographs of the Sun have been taken regularly at Greenwich and at some points in India and Mauritius. These are carefully measured on a uniform plan, and the results published in the annual volumes of Greenwich observations. In this way the mean daily spot-area of the Sun is given from month to month and year to year, with a multitude of details useful in investigating the question of their behavior. Thus we have in all a series of twenty-six observed maxima, extending from 1615 to 1893, and an equal number of minima, from 1610 to 1889.

I have not, however, depended entirely on these. In cases where a continuous series of observations are available, maxima and minima are not the phases which can be determined with the greatest precision. I have therefore included what we may call half-tide phases, increasing and decreasing. The epochs of these phases I have determined in the following way: We have at hand, from year to year, annual numbers expressive of the mean spot activity in each year. In the case of the Greenwich observations this is simply the mean daily spot area. In the case of the older observations it is something a little different, depending largely on the number of spots, but it is not to be expected that there will be any great differences between the two measures of activity. Half the sum of the numbers corresponding to the year of minimum and the following maximum, or *vice versa*, gives the number corresponding to a certain mid-phase. From the annual numbers, assumed to correspond to the middle of each

year, is found a time corresponding to this mid-phase. Such a time would, from the nature of the variation, be subject to less uncertainty than the time of a maximum or minimum. In forming these mid-phases I use the numbers of Dr. Wolf so far as they are found in his *Mittheilungen* and in the spot areas derived at Greenwich.

In the case of any one phase our method of proceeding is now this: We assume a certain epoch, as near as possible to one of the phases, and a certain provisional period. With this provisional epoch and period we form an arithmetical series expressing the provisional times at which the phase would occur were the assumed epoch and period correct. Each observed epoch gives rise to an equation of condition of the form

$$x + by = \Delta t$$

in which $x$ is the correction to the assumed zero phase, $y$ the correction to the length of the period, and $\Delta t$ the discrepancy between the observed and computed epochs. The least square solution of the equations thus formed gives the concluded results from each series of epochs.

The question of the relative weights to be assigned to the observed phases is a difficult one. In the observed time of a phase two classes of errors enter. One is that of the observations themselves, the other the irregularities of the actual phase. In the case of the Greenwich results the former error is almost evanescent. The magnitude of the latter or real irregularity is shown by the fact that the epoch of maximum or minimum may be a year different for the two solar hemispheres. As an example of this, extreme cases occurred in the years preceding and following the maximum of 1883 and the minimum of 1889 The actual mean irregularity in the best determined phase would seem to be at least $0^y.4$, perhaps $0^y.5$. On this actual irregularity is superposed the uncertainty rising from the irregularity of the observations. This last is such that Dr. Wolf sometimes assigns a probable error as great as two years to the times of a phase derived from the observations. In assigning weights,

however, I do not deem it advisable to use so great a difference as would be indicated by these probable errors.

A preliminary solution was first made of each of the four phases, from which it would be inferred that the normal period was quite near to 11.13 years. This period, however, and the epochs derived in connection with it were considered only as provisional values to be corrected by a definitive solution. The deviations are, however, of such a nature that they may be regarded as accidental errors. In order to save space I omit this preliminary solution as well as the numbers not necessary to judge the results. The actual numbers on which my conclusions are based are presented as follows, in order that the method of derivation may be most easily seen.

In the first column is given an equidistant series of computed epochs for each of the four phases, maximum, minimum, mid-phase rising and mid-phase falling.

In the column following is given the observed epoch of the phase.

Next follows the weight assigned to this determination. In the case of the older observations this weight was generally the same as that of the preliminary solution. The comparison of the latter with observation showed, however, that the modern observations were entitled to a relative weight nearly double that assigned to them in the first solution. Their weights were, therefore, increased.

The next column gives the observed corrections to the computed phase, which is, in fact, merely the difference of the first two columns. Next we have in column *b* the number of the period from an epoch quite near the weighted mean of all the epochs, divided by ten.

A few remarkable deviations demand attention. It would seem that during the decade 1670–1680 there was a considerable retardation in the phases. This might be taken to indicate that the second hypothesis was the correct one, and that the actual period was subject to acceleration and retardation. But we find that, during the two following decades, this seeming retardation

| Maxima | | | | | Minima | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Comp. | Obs. | Wt. | *n* | *b* | Comp. | Obs. | Wt. | *n* | *b* |
| 1615.36 | 15.5 | 3 | +0.1 | −1.8 | 1610.79 | 10.8 | 5 | 0.0 | −1.8 |
| 26.49 | 26.0 | 5 | −0.5 | −1.7 | 21.92 | 19.0 | 2 | −2.9 | −1.7 |
| 37.62 | 39.5 | 5 | +1.9 | −1.6 | 33.05 | 34.0 | 3 | +1.0 | −1.6 |
| 48.75 | 49.0 | 3 | +0.2 | −1.5 | 44.18 | 45.0 | 3 | +0.8 | −1.5 |
| 59.88 | 60.0 | 1 | +0.1 | −1.4 | 55.31 | 55.0 | 1 | −0.3 | −1.4 |
| 71.01 | 75.0 | 1 | +4.0 | −1.3 | 66.44 | 66.0 | 1 | −0.4 | −1.3 |
| 82.14 | 85.0 | 3 | +2.9 | −1.2 | 77.57 | 79.5 | 1 | +1.9 | −1.2 |
| 93.27 | 93.0 | 1 | −0.3 | −1.1 | 88.70 | 89.5 | 1 | +0.8 | −1.1 |
| 1704.40 | 05.5 | 5 | +1.1 | −1.0 | 99.83 | 98.0 | 1 | −1.8 | −1.0 |
| 15.53 | 18.2 | 5 | +2.7 | −0.9 | 1710.96 | 12.0 | 3 | +1.0 | −0.9 |
| 26.66 | 27.5 | 5 | +0.8 | −0.8 | 22.09 | 23.5 | 3 | +1.4 | −0.8 |
| 37.79 | 38.7 | 5 | +0.9 | −0.7 | 33.22 | 34.0 | 3 | +0.8 | −0.7 |
| 48.92 | 50.0 | 5 | +1.1 | −0.6 | 44.35 | 45.0 | 3 | +0.6 | −0.6 |
| 60.05 | 61.5 | 10 | +1.4 | −0.5 | 55.48 | 55.5 | 10 | 0.0 | −0.5 |
| 71.18 | 69.9 | 15 | −1.3 | −0.4 | 66.61 | 66.5 | 5 | −0.1 | −0.4 |
| 82.31 | 79.5 | 10 | −2.8 | −0.3 | 77.74 | 75.8 | 5 | −1.9 | −0.3 |
| 93.44 | 89.0 | 10 | −4.4 | −0.2 | 88.87 | 84.8 | 5 | −4.1 | −0.2 |
| 1804.57 | 04.0 | 5 | −0.6 | −0.1 | 1800.00 | 98.5 | 5 | −1.5 | −0.1 |
| 15.70 | 16.8 | 10 | +1.1 | 0.0 | 11.13 | 10.5 | 5 | −0.6 | 0.0 |
| 26.83 | 28.7 | 10 | +1.9 | +0.1 | 22.26 | 23.2 | 5 | +0.9 | .1 |
| 37.96 | 37.2 | 20 | −0.8 | 0.2 | 33.39 | 33.8 | 15 | +0.4 | .2 |
| 49.09 | 48.6 | 20 | −0.5 | 0.3 | 44.52 | 44.0 | 20 | −0.5 | .3 |
| 60.22 | 60.2 | 20 | 0.0 | 0.4 | 55.65 | 56.2 | 20 | +0.6 | .4 |
| 71.35 | 70.9 | 20 | −0.4 | 0.5 | 66.78 | 67.2 | 20 | +0.4 | .5 |
| 82.48 | 83.7 | 20 | +1.2 | 0.6 | 77.91 | 78.8 | 20 | +0.9 | .6 |
| 93.61 | 93.6 | 20 | 0.0 | 0.7 | 89.04 | 89.4 | 20 | +0.4 | .7 |

| Mid-phase rising | | | | | Mid-phase falling | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Comp. | Obs. | Wt. | *n* | *b* | Comp. | Obs. | Wt. | *n* | *b* |
| 1757.79 | 58.7 | 1 | +0.9 | −.8 | 1751.69 | 52.0 | 2 | +0.3 | − .9 |
| 68.92 | 68.3 | 2 | −0.6 | −.7 | 62.82 | 62.8 | 2 | 0.0 | − .8 |
| 80.05 | 77.5 | 1 | −2.5 | −.6 | 73.95 | 72.2 | 1 | −1.8 | − .7 |
| 91.18 | 86.2 | 1 | −5.0 | −.5 | 85.08 | 81.8 | 2 | −3.3 | − .6 |
| 1802.31 | .... | .. | ..... | .... | 96.21 | .... | .. | ..... | .. .. |
| 13.44 | 15.0 | 5 | +1.6 | −.3 | 1807.34 | .... | .. | ..... | .. .. |
| 24.57 | 26.5 | 5 | +1.9 | −.2 | 18.47 | 19.5 | 3 | +1.0 | − .3 |
| 35.70 | 35.8 | 10 | +0.1 | −.1 | 29.60 | 31.8 | 6 | +2.2 | − .2 |
| 46.83 | 46.7 | 20 | −0.1 | 0 | 40.73 | 40.0 | 10 | −0.7 | − .1 |
| 57 96 | 58.4 | 20 | +0.4 | +.1 | 51.86 | 52.5 | 10 | +0.6 | .0 |
| 69.09 | 69.3 | 20 | +0.2 | .2 | 62.99 | 62.9 | 15 | −0.1 | + .1 |
| 80.22 | 80.7 | 20 | +0.5 | .3 | 74.12 | 73.2 | 15 | −0.9 | .2 |
| 91.35 | 91.8 | 20 | +0.4 | .4 | 85.25 | 86.1 | 15 | +0.8 | .3 |
| | | | | | 96.37 | 96.0 | 15 | −0.4 | .4 |

was lost. More remarkable yet is the acceleration about 1790, which, it will be seen, seems to affect all the phases. But, again, during the following two or three decades, this acceleration is changed into a retardation. I was at first disposed to think that these perturbations of the period might be real, but, on more mature consideration, I think they are to be regarded as errors rising from the imperfection of the record. The derivation of any exact epoch requires a fairly continuous series of observations made on a uniform plan. If we compare and combine the results of observations made in any irregular or sporadic way it may well be that the actual changes are masked by the apparent changes due only to these imperfections.

A curious case of this is afforded by a comparison of Carrington's results with the contemporaneous observations of Schwabe. During the years 1856–1859, which immediately followed a minimum, the number of new groups noted by each observer was nearly the same as we should expect, but, as the maximum approached, Carrington had decidedly more groups than Schwabe.

On this consideration was based the great disparity of weights which were finally assigned to the several observed phases.

A curious anomaly is the preponderance of positive corrections given by nearly all the maxima from the beginning to 1760. For this reason the period as it comes out from the maxima is smaller than that from the other phases. The fact appears to be that, while modern observations show that the maximum follows the minimum by less than five years, and between six and seven years are required to again fall to the minimum, the older observations seem to place the two phases nearly equidistant. I regard this only as resulting from the accidental errors of the observations, as we can scarcely suppose a change in the law of variation to have occurred.

In the first of the definitive solutions I include the remarkably discordant epochs about 1780–1790, on the ground that previous observations might well have been affected with the same kind of an error arising from the imperfect continuity of the record. With the weights as assigned we have the following

normal equations and solutions for $y$. Here $\epsilon_1$ is the mean error for weight 1 as derived from the residual values of $n$, and $\epsilon$ is the mean error of the value of $y$.

MAXIMA.

$$\begin{aligned} 227x - 15y &= -7^y.1 \\ -15 + 106 &= -30.7 \end{aligned} \qquad \epsilon_1 = \pm 4^y.6.$$

$$\text{Solution: } y = -0^y.297 \pm 0^y.43.$$

MINIMA.

$$\begin{aligned} 185x + 7y &= +16^y.7 \\ 7 + 83 &= +23.7 \end{aligned} \qquad \epsilon_1 = \pm 2^y.8.$$

$$\text{Solution: } y = +0^y.220 \pm 0^y.31.$$

MID-PHASE RISING.

$$\begin{aligned} 125x + 13y &= +40^y.0 \\ 13 + 9 &= +7.4 \end{aligned} \qquad \epsilon_1 = \pm 2^y.5.$$

$$\text{Solution: } y = +0^y.42 \pm 0^y.91.$$

MID-PHASE FALLING.

$$\begin{aligned} 96.0x + 6.6^y &= -1^y.8 \\ 6.6 + 9.3 &= +0.3 \end{aligned} \qquad \epsilon_1 = \pm 2^y.7.$$

$$\text{Solution: } y = +0^y.05 \pm 0^y.92.$$

Applying $y \div 10$ as a correction to the provisional value $11^y.13$ of $P$ we have the following four-results and combination:

| | | | |
|---|---|---|---|
| Maxima | $P = 11^y.100 \pm 0^y.043$ | | |
| Minima | $11.152 \pm$ | .031 | |
| Mid-phase R. | $11.172 \pm$ | .091 | |
| Mid-phase F. | $11.135 \pm$ | .092 | |
| Mean period | $11.136 \pm$ | .023 | (A) |

The striking abnormality of the phases between 1781 and 1792 is shown by the fact that they contribute about one third of the whole sum of the squares of the errors. We are therefore justified in at least undertaking a solution in which these discordant epochs, one for each phase, are dropped out. When this is done the equations of condition and solution will be:

$$\text{Maxima} \quad \begin{aligned} 217x - 13.3y &= +37.0 \\ 13 + 105.3 &= -39.5 \end{aligned} \qquad \epsilon_1 = \pm 3.8.$$

$$\text{Result: } y = -0.354 \pm 0.37.$$

$$\begin{array}{llr} \text{Minima} & 180.0x + \phantom{0}7.8y = +\ 29.2 & \\ & \phantom{00}7.8\phantom{x} + 83.0\phantom{y} = +\ 19.8 & \epsilon_1 = \pm\ 2.2. \end{array}$$

Result: $y = +\ 0.223 \pm 0.24.$

$$\begin{array}{llr} \text{Mid-phase R.} & 124.0x + 13.7y = +\ 45.0 & \\ & \phantom{0}13.7\phantom{x} + \phantom{0}8.8\phantom{y} = +\ \phantom{0}4.9 & \epsilon_1 = \pm\ 2.1. \end{array}$$

Result: $y = -\ 0.013 \pm 0.80.$

$$\begin{array}{llr} \text{Mid-phase F.} & 94.0x + 7.8y = +\ 4.8 & \\ & \phantom{0}7.8\phantom{x} + 8.6\phantom{y} = -\ 3.7 & \epsilon_1 = \pm\ 2.5. \end{array}$$

Result: $y = -\ 0.512 \pm 0.89.$

We therefore have

| | | | |
|---|---|---|---|
| From Maxima - - - - | $P = 11^y.095 \pm 0^y.037$ | | |
| From Minima - - - - | $11\ .152 \pm$ | $.024$ | |
| From Mid-phase R. - - - | $11\ .129 \pm$ | $.080$ | |
| From Mid-phase F. - - - | $11\ .079 \pm$ | $.089$ | |
| Mean - - - - - | $11\ .132 \pm$ | $.018$ | (B) |

The difference between the two results A and B is much less than the probable error of either. It is not therefore necessary, in order to fix the period, that we should decide between them. Omitting useless decimals we conclude that the normal period of the solar spots is

| | |
|---|---|
| | $P = 11^y.13 \pm 0^y.02$ |
| Rise from minimum to maximum | $4\ .62$ |
| Fall from maximum to minimum | $6\ .51$ |

We may now consider the important question whether there is any evidence of an accumulation of accidental irregularities in the course of the successive cycles. One way of doing this would be to find the mean period separately for the first half of the series, and the second half. But then the result would depend almost entirely on the retention or omission of the abnormal residuals. I therefore divide the whole period of observation into three parts, the first extending from 1610 to 1720, the second from 1720 to 1820, and the third from 1820 to the present time. Taking the weighted mean residuals for each of these periods, and including all the epochs, we find the mean deviations to be

| | From Maxima | From Minima |
|---|---|---|
| 1st period - - - - - | $+1^y.2$ | $+0^y.1$ |
| 2d period - - - - - | $-0\ .9$ | $-0\ .8$ |
| 3d period - - - - - | $+0\ .1$ | $+0\ .2$ |

It would thus appear that near the mid-epoch there was an apparent systematic acceleration amounting to somewhat more than a year. But this conclusion rests on the hypothesis that there is nothing abnormal in the great residuals. If we exclude those between 1775 and 1790, in which the evidence of abnormality is so strong, the deviations of the middle period will be:

$$\text{Maxima} + 0^y.3\,;\ \text{Minima} - 0^y.1.$$

The deviation from uniformity is now reduced to $0^y.3$, a quantity markedly less than the probable variation of a single period. If each period were subject independently to an accidental error liable to accumulate, the deviation at the mid-epoch would be between 1.5 and 2 years. Were this the case a more systematic character would be seen in the residuals for the middle period. The contrast between the sudden deviations in the residuals of the doubtful period and the small ones of the recent well observed epochs make it almost certain that the errors between 1770 and 1800 are due to imperfections of the record. I therefore consider the most probable conclusion to be that there is no accumulation of accidental errors in the course of successive cycles, and that the first of the two hypotheses set forth in the beginning is the correct one. If during each short period, say one year, the progress of a cycle was measured by the apparent spot activity of the time, there would be an accumulation of the kind we have been looking for. Our final conclusion is therefore this:

*Underlying the periodic variations of spot activity there is a uniform cycle, unchanging from time to time and determining the general mean of the activity.*

Whether the cause of this cycle is to be sought in something external to the Sun, or within it; whether, in fact, it is in the nature of a cycle of variations within the Sun, we have, at present, no way of deciding.

It would seem from what precedes that a revision of the conclusions to be drawn from the observations of Sun-spots during the interval of 1775–1790 is very desirable.

The preceding conclusions rest upon a discussion of four

separate phases, expressing the entire general degree of spot activity. The question may arise whether the singular change which takes place in the distribution of the spots in latitude at the time of minimum may not give a more definite series of epochs than those of the phases we have considered. The best defined of these additional epochs seems to be that at which, about the time of a minimum, spots begin to show themselves in very high latitudes. I find that we can regard this phenomenon as a fairly well-defined one, the mean epoch of, we may say, the first four spots being taken. A comparison of these phases shows, however, that they are even less equidistant than the epochs of minimum. I find the following dates to be thus defined:

| First spot | Mean of first four |
|---|---|
| 1856.4 | 1856.6 |
| 1867.2 | 1867.3 |
| 1879.3 | 1879.5 |
| 1889.0 | 1889.4 |

The conclusion seems to be that here the phenomenal phase which we observe is not definitely and invariably connected with the exact cycle of change, the existence of which we have shown to be so probable. It may, indeed, well happen that the minimum which is derived from a series of phenomena extending over two or three years will be better connected with the cycle than with a single phenomenon like that in question.

I have remarked that the epochs of a phase derived independently from the spot activity in the northern and southern solar hemispheres separately may differ by a year from each other. It was remarked by Spörer that the phases in the southern hemisphere preceded those in the northern. A mere glance at the Greenwich numbers, however, will show that this is not a general rule for at least the maximum and minimum. I have endeavored to settle this question by deriving the epochs of mid-phase raising and falling, in a rough way, from the two hemispheres separately. The results are as follows:

| | North | South | N—S |
|---|---|---|---|
| Rising phase - - - | 1859.2 | 1858.1 | +1.1 |
| Falling " - - | 62.5 | 62.7 | —0.2 |
| Rising " - - - | 69.4 | 69.2 | +0.2 |
| Falling " - - | 73.2 | 73.5 | —0.3 |
| Rising " - - - | 80.5 | 82.1 | —1.6 |
| Falling " - - | 85.7 | 85.9 | —0.2 |
| Rising " - - - | 91.2 | 91.3 | —0.1 |
| Mean - - - - - - - - - | | | —0.16 |

It will be seen that this very small mean difference arises from the large difference in the rising phase 1880–1882. There seems to have been an abnormal delay in the increase of the spottedness of the southern hemisphere during the years 1880–1881. Our general conclusion is, therefore, that there is no systematic difference between the phases on the two hemispheres, but that those of each hemisphere are separately subject to considerable irregularities.

Spörer has also pointed out that the spottedness of the southern hemisphere was, from observations of Carrington and himself, greater than that of the northern. This is shown to be the case in a yet greater degree by the more recent Greenwich observations. We now have four cycles through which a comparison may be made. For the two cycles from 1856 to 1877 I have used Spörer's numbers expressing the frequency of spots; from 1878 onward I use the Greenwich mean daily areas. The sum total for the two hemispheres is as follows:

| | North | South | S—N | Authority |
|---|---|---|---|---|
| 1856 to 66 - - - | 3442 | 3680 | 238 | Spörer |
| 67 to 77 - - | 3247 | 3698 | 451 | Spörer |
| 78 to 88 - - - | 2479 | 3434 | 955 | Greenwich |
| 89 to 98 - - | 3198 | 3908 | 710 | Greenwich |

The difference is several times larger than the probable accumulation of accidental irregularities and shows with fair conclusiveness that, for at least four cycles, the spottedness of the southern hemisphere has been one fifth greater than that of the northern. Whether this is a permanent feature of solar activity is an interesting question which only the future can decide.

For convenient reference we give the epochs of some maxima and minima derived from the concluded theory:

| Maxima | Minima |
|---|---|
| 1871.52 | 1878.03 |
| 1882.65 | 1889.16 |
| 1893.78 | 1900.29 |
| 1904.91 | 1911.42 |
| 1916.04 | 1922.55 |
| 1927.17 | 1933.68 |

# ON AN APPARATUS FOR THE LABORATORY DEMONSTRATION OF THE DOPPLER-FIZEAU PRINCIPLE.

By A. BÉLOPOLSKY.

AN apparatus for this purpose was suggested by me in the year 1894,[1] and since then I have made numerous attempts to construct it. Thanks to a grant of three hundred dollars which I received early in 1898 from the "Elizabeth Thompson Science Fund," I have succeeded in my attempts this year. The other necessities, such as the spectroscopic apparatus, the electric current, etc., were supplied me by the Pulkowa Observatory. I express here my thanks to both of these scientific institutions.

The principle of the apparatus is as follows: If a source of light is reflected in two nearly parallel mirrors, the distance $S$ of the $n$th reflection from the source itself may be expressed as

$$S = h + 2nx + l\,,$$

where $h$ is the distance from the source to a plane midway between the two mirrors, $l$ is the distance of the image from the same plane after $n$ reflections, and $x$ is the distance between the two mirrors. If we differentiate this expression according to $t$ we obtain

$$\frac{dS}{dt} = 2n\frac{dx}{dt}\,.$$

$\frac{dx}{dt}$ is the velocity of the mirror. We see that although $\frac{dx}{dt}$ may itself attain no very large value, $\frac{dS}{dt}$ will be $2n$ times larger. If the mirror had, for instance, a velocity of 50 meters per second, its image after ten reflections would move with a velocity of $2 \times 10 \times 50 = 1000$ meters per second.

We can also show that the wave-length of a homogeneous beam of light after $n$ reflections from plane, moving mirrors

[1] *Mem. Spettr. Ital.*, 23; *A. N.*, No. 3267.

alters according to the direction of the motion.[1] We obtain the following expression for the wave-length $\lambda$ after one reflection from a moving mirror:

$$\lambda_1 = \lambda_0 \left(1 \pm \frac{2v}{V} \cos \psi\right),$$

where $\lambda_0$ is the normal wave-length, $v$ the velocity of the mirror, $V$ the velocity of light, $\psi$ the angle between the direction of motion of the mirror and the normal to its surface. If the beam is successively reflected from several mirrors, we shall obtain the following wave-lengths, provided that all the mirrors have the same velocity and that $\psi$ is constant:

After the 1st reflection, $\lambda_1 = \lambda_0 \left(1 \pm \frac{2v}{V} \cos \psi\right)$.

After the 2d reflection, $\lambda_2 = \lambda_1 \left(1 \pm \frac{2v}{V} \cos \psi\right)$.

After the $n$th reflection, $\lambda_n = \lambda_{n-1} \left(1 \pm \frac{2v}{V} \cos \psi\right)$.

Hence we obtain, with sufficient approximation,

$$\lambda_n = \lambda_0 \left(1 \pm \frac{2nv}{V} \cos \psi\right).$$

The sign depends upon the direction of $v$. With a large enough value of $n$, $\lambda_n - \lambda_0$ will have an appreciable value, even if $v$ remains comparatively small.

The apparatus must therefore move at least two mirrors as rapidly as possible in opposite directions. In its simplest construction it would consist of two wheels, like those of a water wheel, each rotating rapidly and carrying several mirrors. The axes of the wheels are so connected by gears that each pair of mirrors will come into a position near to parallelism at the same time. The wheels are of aluminum, of 250 mm diameter, and each carries eight silvered mirrors of size 20 × 105 × 3 mm. The mirrors are so regulated by five adjusting screws that a beam falls upon the slit of a powerful spectrograph after $n$ reflections by all of the eight pairs. The support of the wheels is of cast iron and weighs 175 pounds. Each wheel is placed on the

[1] KETTELER, *Astronomische Undulationstheorie.*

common shaft between two electric motors, of which there are four in all. With 50 volts and from 1.5 to 2 amperes per motor, they should make 6000 revolutions per minute. Two rheostats are used to introduce gradually the current from the storage batteries, and two switches, one for each pair of motors, permit a change in the direction of rotation as desired.

The shafts of the motors are nearly parallel, but displaced somewhat sidewise in order to make room for the beam before incidence and after repeated reflections. The apparatus is mounted upon a very solid wooden table A rather poor heliostat reflects the sunlight upon a slit 10 × 20 mm in front of the apparatus.

It soon appeared that the brightness of a beam undergoing repeated reflections falls off very much, and a spectrograph of large light-power is necessary for the production of spectrograms when moving mirrors are used. For this I employed three compound prisms, set at minimum deviation for $\lambda$ 433, a collimator of 1.5 meters focus, and a camera of 1.75 meters, both in wooden mountings, the prism-box being of steel, however. The whole spectrograph rests upon four long wooden screws with lock-nuts. The stability of the apparatus is increased by weights, in all amounting to some 280 pounds, which are placed at different points upon the spectrograph.

The slit is at a distance of about a meter from the mirror which reflects the light last; and either half of the slit may be employed in turn by means of a device placed directly in front of the slit. A cylindrical condenser is introduced between this device and the apparatus. The slit is observed by the light reflected from the first surface of the first prism. A diaphragm is placed closely over the collimator objective in order to see that the rays pass through the objective centrally.

Directly in front of the plate are two slides, movable from outside, which permit the exposure of any desired portion of the plate, so that the central part of the plate can be covered up and the edges exposed to the light, or the reverse. The plate-holder can be pressed into position by a screw, and has a strong spring.

With this arrangement the experiment may be made as follows:

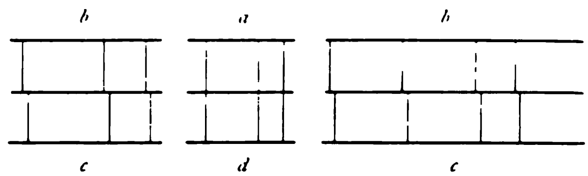
I. (*a*) One half of the slit open, say the upper half; mirror at rest; central part of plate exposed.

(*b*) Central part of plate covered, edges uncovered; mirror moving in one direction. Expose.

II. (*c*) The other half of the slit is now opened, while the first is covered; central part of plate covered, edges free; exposure made with mirrors rotating in the opposite direction.

(*d*) Mirrors at rest; central part of plate free; edges covered. Expose.

In this way we obtain upon a single plate the following spectra: Two spectra at the center of the plate, one above the




other, from mirrors at rest, to check the stability of all parts of the apparatus during the experiment; four spectra, two on each side of the central part, which should exhibit the displacements of the lines, two giving a displacement toward red, and two that toward violet.

The figure gives a schematic representation of the spectra obtained, the letters indicating the order of the spectra. *a* and *d* are the spectra for control of the stability; *b* and *c* the displaced spectra; hence the plate shows a double displacement of the lines.

The region of spectrum from $\lambda 438$ to $450\mu\mu$ was employed, as the violet region was very faint in the reflected light (since the silvering of the mirrors transmits the violet rays). The lines found on the plate serve to compute the coefficient $K$ for converting the measured displacements into kilometers per second.

To show that the dispersion alters very little on different plates, I give the measured distance in revolutions of the micrometer between the two lines $\lambda 4456.24$ and $\lambda 4425.63$, viz., January 27, 33.849 rev.; July 5, 33.877 rev.; August 9, 33.867 rev.

For these plates the following values of the coefficient $K$ for $\lambda 4444.18$ were computed by the method of least squares:

| | | |
|---|---|---|
| 1900 | June 27, | $\log K = 1.7878$ |
| | July 6, | 1.7826 |
| | Aug. 9, | 1.7871 |

If the magnitude of the displacement of the lines was 0.010 rev., a difference of one unit in the second place of $\log K$ would make a difference of only 14 meters in the calculated velocity. It would therefore be permissible to use the mean value of $\log K = 1.7858$ for all my plates.

The stellar spectrograph of this Observatory gives a precision seven times smaller in the determination of velocities in the line of sight; and since it has been shown that the velocity for stars of the second type can be determined with a probable error of $\pm 2$ km per second for each plate, we might expect that this spectrograph could give a probable error of a few hundred meters.

The preliminary experiments, which have been in progress since the completion of the apparatus in April, have shown that a very long exposure is required to obtain a measurable spectrogram when the light is repeatedly reflected by moving mirrors. Thus sunlight reflected eight times required more than an hour, and more than two hours were necessary for two spectra. So long an exposure as this is hardly possible, for the summer sky is seldom free from clouds for so long a time, and variations in temperature and other causes may produce large changes in the parts of the spectrograph. I convinced myself that under favorable conditions no displacements of the spectral lines occurred, as was shown by two adjacent spectrograms taken two and one half hours apart.

To indicate the large loss by reflections from moving mirrors,

I may say that with the mirrors at rest, quite strong spectrograms could be obtained with 2 seconds' exposure.

Thereafter I accordingly employed only the sixth reflection, which cut the exposure time down to 30 minutes for each spectrum.

I made use of a speed indicator for determining the velocity of rotation of the apparatus; and also employed the acoustical method of estimating the pitch when a piece of paper was held against the gears. These determinations yielded the following average results:

With a current of 4½ amperes (rheostat cut out) there were 2016 revolutions in 63 seconds, or 32 per second. With the same circuit, at another time, the pitch was estimated as *La* of the third octave, corresponding to 1740 vibrations per second. The wheel had 49 teeth, so that there were 35 revolutions per second. With 7¼ amperes there were 1512 revolutions in 34 seconds, or 44 per second. At another time 505 revolutions were counted in every 11½ seconds, giving the same number of revolutions per second as above. During the whole time of the experiment the ammeter showed no variations of over ¼ ampere, whence we may infer that the rotation of the dynamos was very constant.

The breadth of the mirrors being 20 mm, the largest diameter between the edges of two mirrors standing 180° from each other was 230 mm, and the least was 190 mm, whence it follows that with the sixth reflection and 32 revolutions per second the limits of the linear velocity were 276 and 230 meters per second. With 44 revolutions per second they were 389 and 318 meters per second. It is perhaps for this reason that the lines upon the spectrogams taken with rotating mirrors have a broader appearance than those taken with mirrors at rest.

The spectrograms were measured with a micrometer screw 65 mm long attached provisionally to the microscope, as the regular screw of 35 mm was too short for the purpose; 199.4 divisions of the head corresponded to one millimeter.

The plates were always so placed under the microscope that

the readings of the head increased from red to violet. Only that half of the plate toward the violet was measured, as the length of the spectrograms was 100 mm. Settings (usually four or five on each line) were first made on the lines of the upper spectrum, then on those of the lower. The direction of the rotation of the mirrors, whether receding (−) or approaching (+) each other, is marked on the plate. I have obtained measurable plates only since June 27. The small number secured since that time is explained in part by the unfavorable weather and in part by the fact that I was also making spectrograms of the Sun with the 30-inch refractor. The measures of the plates and explanatory remarks follow.

JUNE 27, 1900.

Sixth reflection; exposure $30^m$; exposures with mirrors at rest at beginning and end; the first motion was +; current of 4½ amperes; under microscope the upper spectrogram corresponded to a negative, the lower to a positive direction of rotation. The difference of the readings is always expressed as lines of upper − lines of lower spectrum.

| Comparison spectrum (mirrors at rest) | | Spectrum with rotating mirrors | |
|---|---|---|---|
| Line 1 | −0.008 rev. | λ 4461.9 | +0.011 rev. |
| 2 | −0.006 | 4462.1 | +0.007 |
| 3 | −0.022 | 4457.6 | −0.008 |
| 4 | −0.016 | 4456.0 | −0.005 |
| | ———— | 4454.7 | −0.010 |
| Mean | −0.013 | 4451.7 | +0.003 |
| | | 4448.0 | +0.003 |
| | | 4444.0 | −0.005 |
| | | 4425.6 | 0.000 |

The absolute displacement (toward red in lower spectrum) equals 0.012 rev., corresponding to a velocity of 0.75 km per sec. The mirrors had a maximum velocity of 0.55 km per sec.

JULY 1.

The comparison spectra on this plate show an unaccountably large displacement, so that at first I rejected it altogether.

If these spectra are neglected, however, and those obtained from moving mirrors are treated independently, the micrometer

thread being oriented by the dividing line between the spectra, a displacement is obtained which corresponds with sufficient accuracy to the velocity of the mirrors. The large displacement might be explained by a jar received by the spectrograph just after the first or before the last exposure.

The pitch indicated a velocity of rotation of the motor of 35 revolutions per second. Under the microscope the upper spectrum corresponded to a +, the lower to a − motion.

| λ | Upper—lower spectrum |
|---|---|
| 4459.4 | −0.011 rev. |
| 4457.9 | −0.003 |
| 4456.0 | −0.009 |
| 4451.8 | −0.010 |
| 4448.0 | −0.020 |
| 4444.0 | +0.003 |
| 4442.5 | −0.006 |
| 4425.6 | −0.017 |
| 4415.7 | −0.022 |
| 4408.0 | −0.015 |
| Mean | −0.011 |

The displacement corresponds to a velocity of 0.67, the maximum motion of the mirrors to one of 0.60 km per sec.

JULY 6.

Current = 7¼ amperes; motion at first in the negative direction; exposure $30^m$. The temperature changed 0°.4 C. during the experiment. Under microscope the upper spectrum was due to −, the lower to + motion.

| Comparison spectrum | | Spectrum from rotating mirrors | |
|---|---|---|---|
| λ 4482.5 | −0.074 rev. | λ 4462.0 | −0.058 rev. |
| 4482.5 | −0.075 | 4456.1 | −0.052 |
| 4476.2 | −0.084 | 4451.8 | −0.057 |
| 4468.7 | −0.080 | 4436.0 | −0.060 |
| | | 4425.6 | −0.053 |
| Mean | −0.078 rev. | 4418.0 | −0.058 |
| | | 4415.8 | −0.064 |
| | | 4407.9 | −0.055 |
| | | Mean | −0.057 |

Absolute displacement (toward red in lower spectrum) = 0.021 rev.; velocity = 1.28; maximum velocity of mirrors = 0.78 km per sec.

JULY 9.

Current = 7¼ amperes; exposure $30^m$. Under microscope the upper spectrum was due to +, the lower to — motion.

| | Comparison spectrum | | Spectrum from rotating mirrors |
|---|---|---|---|
| 1 | +0.002 rev. | 1 | −0.007 rev. |
| 2 | +0.009 | 2 | −0.028 |
| 3 | +0.003 | 3 | −0.009 |
| 4 | +0.006 | 4 | −0.011 |
| 5 | −0.006 | 5 | +0.009 |
| Mean | +0.003 rev. | 6 | +0.002 |
| | | 7 | +0.006 |
| | | 8 | −0.0017 |
| | | 9 | −0.010 |
| | | 10 | −0.014 |
| | | Mean | −0.008 rev. |

Absolute displacement (toward violet for lower spectrum) = 0.011 rev.; corresponding velocity = 0.67; maximum velocity of mirrors = 0.78 km per second.

AUGUST 7.

Current 7¼ amperes; first direction of motion —; under microscope upper spectrum corresponds to — motion.

| | Comparison spectrum | | Spectrum from rotating mirrors |
|---|---|---|---|
| 1 | +0.045 rev. | 1 | +0.080 rev. |
| 2 | +0.062 | 2 | +0.080 |
| 3 | +0.068 | 3 | +0.060 |
| 4 | +0.064 | 4 | +0.062 |
| 5 | +0.068 | 5 | +0.074 |
| Mean | +0.059 rev. | 6 | +0.068 |
| | | 7 | +0.063 |
| | | Mean | +0.070 rev. |

Absolute displacement (toward red for lower spectrum) = 0.011 rev.; corresponding velocity = 0.67; maximum velocity of mirrors = 0.78 km per second.

AUGUST 9.

Current $7\frac{1}{4}$ amperes; exposure $30^m$; first direction of motion —; under microscope the upper spectrum corresponds to — motion.

| | Comparison spectrum | Spectrum from rotating mirrors | |
|---|---|---|---|
| 1 | +0.082 rev. | 4461.8 | +0.079 rev. |
| 2 | +0.071 | 4456.0 | +0.100 |
| 3 | +0.072 | 4451.8 | +0.083 |
| 4 | +0.077 | 4448.0 | +0.086 |
| 5 | +0.08 | 4444.0 | +0.095 |
| 6 | +0.075 | 4442.5 | +0.079 |
| 7 | +0.079 | 4437.1 | +0.091 |
| | | 4435.9 | +0.088 |
| Mean | +0.077 rev. | 4425.6 | +0.088 |
| | | 4418.6 | +0.087 |
| | | 4417.9 | +0.096 |
| | | Mean | +0.088 rev. |

Absolute displacement (toward red for lower spectrum) = 0.011 rev.; velocity = 0.67; maximum velocity of mirrors = 0.78 km per second.

The velocities measured may be summarized as follows:

| 1900 | From displacements | From rotation |
|---|---|---|
| June 27 | 0.73 km per sec. | 0.46—0.55 km per sec. |
| July 1 | 0.67 | 0.50—0.60 |
| July 6 | 1.28 | 0.64—0.78 |
| July 9 | 0.67 | 0.64—0.78 |
| Aug. 7 | 0.67 | 0.64—0.78 |
| Aug. 9 | 0.67 | 0.64—0.78 |

Probable error of each velocity = ±0.17 km

These results are to be regarded as only the first experiments with the apparatus above described. Much remains in the way of its improvement, and it is especially desirable to put the wheels into a vacuum in order to avoid the resistance of the air. It is hoped that in time better results will be attained.

PULKOWA,
October 1900.

# THE RADIATION OF A BLACK BODY.

By C. E. MENDENHALL and F. A. SAUNDERS.

IN the present article it is proposed to give, first, a brief review of recent work in connection with the radiation of an absolutely black body, and, second, an account of some experiments of this character carried on in the Physical Laboratory of the Johns Hopkins University. The results of the latter were largely negative; but a statement of methods and difficulties may be of service to others engaged in the same line of work.

In most cases the method of producing the "black body" has been based on Kirchhoff's discussion of the problem of radiation in a uniformly heated enclosure; a hollow body, preferably of good conducting material and having an aperture, being heated as uniformly as possible, the radiation emerging from the aperture has been taken as that of a "black body," and examined by appropriate means. Another method has been suggested and used by Paschen,[1] but it seems to be, on the whole, less satisfactory; in this case a radiating strip is put near the center of a reflecting enclosure having an aperture through which passes the radiation to be examined. In order to consider the subject to the best advantage it will be well to group the materials around the most important of the so-called "laws"—which have been obtained for the most part theoretically, and which have in turn been put to test by recent experiment. For brevity these laws will sometimes be referred to by number, corresponding to those given below, the following symbols being used:

$S$ = total radiant energy at any absolute temperature.
$T$ = this absolute temperature.
$Ed\lambda$ = energy radiated in waves of length $< \lambda + d\lambda$ and $> \lambda$.
$\lambda$ = any wave-length expressed in thousandths of a millimeter — $\mu$.

[1] PASCHEN, *Wied. Ann.*, 60, 1897; PASCHEN and WANNER, ASTROPHYSICAL JOURNAL, 9, 40, 1899; 11, 297, 1900.

$\lambda_m$ = wave-length of maximum energy at temperature $T$.
$E_m d\lambda$ = amount of (maximum) energy at temperature $T$, between the limits,

$$\lambda_m \pm \frac{d\lambda}{2}.$$

$A$, $B$, $C$, $c$, $\alpha$, are constants.

The following "laws" will be considered:

| | | |
|---|---|---|
| (I) | $S = \text{const. } T^4$. | A relation between total radiation and temperature. |
| (II) | $\lambda T = \text{const.}$ | A relation between wave-length and temperature. |
| (III) | $\lambda_m T = A$. | A relation between wave-length of maximum of energy-curve and the corresponding temperature. |
| (IV) | $E_m T^{-5} = B$. | A relation between maximum ordinate of energy-curve and the corresponding temperature. |
| (V) | $E = C\lambda^{-5} e^{-\frac{c}{\lambda T}}$. | Gives distribution of energy in spectrum at any temperature; *i. e.*, is equation of energy-curve. |

Equation (I), $S = \text{const. } T,^4$ expressing the well-known law of Stefan, has been subjected to experimental test recently by Lummer and Pringsheim,[1] and by Paschen.[2] The first named attacked the problem most directly and found, as can be seen from Table I, a fairly satisfactory agreement with theory.

TABLE I.

| *T.* | *373°* | *492°* | *733°* | *755°* | *799°* | *820°* |
|---|---|---|---|---|---|---|
| Obs. S. | 156 | 638 | 3320 | 3810 | 4440 | 5150 |
| Calc. S. | 143 | 600 | 3270 | 3700 | 4660 | 5170 |

| *T.* | *877°* | *1106°* | *1125°* | *1403°* | *1492°* | *1522°* | *1561°* |
|---|---|---|---|---|---|---|---|
| Obs. S. | 6190 | 16400 | 17700 | 44700 | 57400 | 60600 | 67800 |
| Calc. S. | 6180 | 17200 | 18500 | 45000 | 57600 | 62400 | 69100 |

It is to be noted here that no correction (apparently) was made for the absorption of $CO_2$ and $H_2O$ vapor in the atmosphere, nor were any precautions taken to diminish this absorption. This work has since been extended to 1700° abs. in one direction and about 100° abs. in the other, and Stefan's law found to be satisfied to within a few per cent. The Stefan relation has also been deduced by Planck,[3] from the basis of the electromagnetic theory of light.

[1] LUMMER and PRINGSHEIM, *Wied. Ann.*, 63, 395, 1897.

[2] PASCHEN, *Wied. Ann.*, 58, 60, 1896, 1897.

[3] M. PLANCK, *Drude's Ann.*, 1, No. 1, 1900.

The expressions (III), (IV), $\lambda_m T = A$ and $E_m T^{-5} = B$, follow at once from Wien's[1] so-called "Verschiebungsgesetz," $\lambda T =$ const. (II), and this, as originally developed, assumed the truth of Stefan's law. This "Verschiebungsgesetz" of Wien states nothing as to the distribution of energy at any one temperature, but states that this distribution must change with the temperature in such a manner that if there is any definite amount of energy corresponding to a given wave-length $\lambda$, at a temperature $T$, this same amount of energy will at any other temperature $T_1$ be emitted in waves whose length is determined by the relation $\lambda T = \lambda_1 T_1$. The expressions (III) and (IV) also are necessary consequences of (V), which, in turn, has been theoretically developed by Planck (*loc. cit.*). Thiesen[2] has objected to part of Wien's reasoning, and has deduced the relation $\lambda T =$ const. by another process.

The expressions (III) and (IV) have been very elaborately tested by experiment. Not to mention earlier work, Lummer and Pringsheim[3] obtained the following series of values for $A$ and $B$:

| $A$ | $B$ |
|---|---|
| 2928 | $2246 \times 10^{17}$ |
| 2974 | 2184 |
| 2959 | 2176 |
| 2966 | 2164 |
| 2956 | 2166 |
| 2980 | 2208 |
| 2950 | 2166 |
| 2814 | 2190 |
| 2940 | 2188 |

Two other series of observations, with different arrangement of apparatus in each case, gave values for $A$ of 2940 and 2930, and equally satisfactory constancy for $B$.

Paschen[4] finds, as a mean for a number of independent series

[1] W. WIEN, *Ber. d. Berl. Akad.*, **6**, 1893.

[2] THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, No. 5, 1900.

[3] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 12.

[4] PASCHEN, ASTROPHYSICAL JOURNAL, **10**, 40, 1899; **11**, 288, 1900; Paschen and Wanner, *ibid.*, **9**, 300, 1899.

of observations, extending from about 150° C. to 1300° C. and over a range of wave-lengths from $0.5\,\mu$ to $9.2\,\mu$, the value of $A$ to be 2907; while the maximum variation of $B$ is 4 per cent.

In the later work of Lummer and Pringsheim the apparatus was so arranged as largely to exclude $CO_2$ and $H_2O$ vapor from the atmosphere between the radiator and bolometer strip, so that the gaps due to the selective absorption of these two substances were very greatly reduced in extent and depth. On the other hand, in Paschen's later work, the bolometer strip was placed at the center of a reflecting hemisphere, in order to approach more closely to the condition of a perfect absorber. According to Kurlbaum[1] a thinly-coated bolometer strip, which appears, however, black to the eye, may absorb but 40 per cent. of the incident total radiation, so that the difference between the results of Lummer and Pringsheim and of Paschen may be partly due to the imperfect absorption of Lummer and Pringsheim's bolometer strip.

So far, then, as the expressions (III) and (IV) are concerned, which tell us nothing as to the distribution of energy in the spectrum, the predictions of theory seem to be verified to within the outstanding experimental errors, and these are being steadily reduced. Equation (V), $E = C\lambda^{-5}e^{-\frac{c}{\lambda T}}$ which concerns this distribution of energy, remains to be considered. As regards its theoretical foundation, Wien's original method is not rigorous, as has been pointed out by Lummer and Pringsheim.[2]

The most careful attempts at experimental verification have been made by Paschen[3] and by Lummer and Pringsheim.[2] Full accounts of the work of the former have been published in this JOURNAL He finds $c$ (see Table II for meaning of these constants) to be 14531, with a possible error of 80 from a series of experiments, while $C$ is more variable. No systematic variation

[1] KURLBAUM, *Wied. Ann.*, **67**, 1899.

[2] LUMMER and PRINGSHEIM, *Verh. Deutsch. Phys. Ges.*, **1**, 1, 1900.

[3] PASCHEN, ASTROPHYSICAL JOURNAL, **10**, 40, 1899; **11**, 288, 1900.

of either $C$ or $c$ is evident; he further tests (V) by plotting it logarithmically.

The theoretical and observed curves agree to within the errors of experiment, except in the region of very short wave-lengths, where the observed energy is greater than the theoretical; and precisely here, as Paschen points out, it is very difficult to avoid stray light.

On the other hand, Lummer and Pringsheim find that while the computed and observed curves (graphs of V) agree quite well in the region of wave-lengths near the maximum, particularly at lower temperatures, this agreement is not so good for the long wave-lengths, and that this disagreement increases as the temperature increases. This can be stated in another way. If the expression $E = C\lambda^{-5}\, e^{-\frac{c}{\lambda T}}$ is transformed by the introduction of logarithms we have $\log E = \log C - \frac{c}{\lambda T}\log e - 5 \log \lambda$, and if $\log E$ is plotted against $\frac{1}{T}$ we have the so-called "isochromatic" curve, evidently a straight line. This may be compared with the curve plotted for corresponding values of $E$ and $T$, observed at a fixed point in the spectrum ($\lambda$ constant). Evidently the slope of this line is proportional to $c$, while from the constant term can be obtained the value of $C$. According to Lummer and Pringsheim's observations the observed isochromatic is convex toward the $\frac{1}{T}$ axis, and the values for $C$ and $c$ obtained from these isochromatic curves increase systematically as the temperature rises. These investigations of Lummer and Pringsheim are still in progress. Thiesen has, however, found, from a recalculation of the results of Lummer and Pringsheim, that a modification of $V$, by changing the coefficient of $\lambda$ from $-5$ to $-4.5$, would completely satisfy their observations. The law thus modified would be satisfied by the observations of Beckmann,[1] and is further strengthened by some recent work of Lummer and Pringsheim.[2]

[1] BECKMANN, *Inaug. Diss.*, Tübingen, 1898.

[2] Referred to by THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, 5, 1900.

As regards the variation of these constants, Rubens[1] has discussed the results of Beckmann, who used a hollow "black body" as source of radiation, and produced a more or less perfect isolation of certain wave-lengths by repeated reflection from fluor spar, and thus determined an approximate "isochromatic" curve in a spectral region not heretofore studied in this connection, viz., for a mean wave-length of about 28 $\mu$. In order that the Wien formula (V) should represent Beckmann's work (as recalculated by Rubens) it is necessary that $c$ should have the value 26000. On account of the method used, this result ought not, perhaps, to be considered conclusive. However, Rubens points out that the change in the ordinate of the energy curve at $\lambda = 25\ \mu$ produced by a change in $c$ from 26000 to 14500 would be (at 2000° C.) $\frac{2}{10000}$ of the maximum ordinate; so that it would be difficult to detect such a change in $c$ by study of the energy curves.

The present knowledge respecting these various laws of radiation can perhaps best be summed up in the following table:

TABLE II.

| | |
|---|---|
| (I) $S = \int E d\lambda = \text{const. } T^4$. | First given by Stefan.[2] Thermodynamically deduced by Boltzmann[3] with certain assumptions. Experimentally tested by Paschen[4] and more especially by Lummer and Pringsheim.[5] Deduced by Planck from electro-magnetic theory, involving electro-magnetic definition of entropy and temperature. |

[1] RUBENS, *Wied. Ann.*, **69**, 1899.

[2] STEFAN, *Sitzber. d. k. Gesellsch. zu Wien*, **79**, 1879.

[3] BOLTZMANN, *Wied. Ann.*, **22**, 1884.

[4] PASCHEN, *Wied. Ann.*, **58**, 1896; **60**, 1897.

[5] LUMMER and PRINGSHEIM, *Wied. Ann.*, **63**, 1897.

(II) $\lambda T =$ const.
from which follow:
(III) $\lambda_m T = A$.
(IV) $E_m T^{-5} = B$.

Developed theoretically by Wien,[1] assuming Stefan's law (I). Tested experimentally by Paschen,[2] Lummer and Pringsheim,[3] Paschen and Wanner,[4] and found to hold with increasing accuracy as experimental methods are improved. Outstanding difference between Paschen and Lummer and Pringsheim of about 1 per cent. in value of $A$; $B$ not so good. (II) Theoretically developed by Thiesen,[5] who questions the rigor of Wien's original method, and by Planck[6] from an electro-magnetic basis.

(V) $E = C\lambda^{-5}e^{-\frac{c}{\lambda T}}$.

(III) and (IV) followed by differentiation from (V).

(VI) $E = C\lambda^{-a}e^{-\frac{c}{\lambda T}}$.

Theoretically deduced by Wien[7] —but with rather arbitrary assumptions and not altogether rigorous reasoning.[8] An expression of the same form (VI) was given by Paschen as best representing the energy curves of various radiating surfaces. Tested by Lummer and Pringsheim,[3] who found systematic variations in $C$ and $c$ with temperature; also by Paschen and Wanner[4] and Paschen,[2] who finds no systematic variation of the constants and a quite satisfactory agreement of the observed and computed curves. Also deduced theoretically by Planck.

(III) and (IV) are necessary, but not sufficient conditions for the truth of (V).

[1] WIEN, *Ber. d. Berl. Akad.*, **6**, 1893; *Wied. Ann.*, **52**, 1894.

[2] PASCHEN (*loc. cit.*) and ASTROPHYSICAL JOURNAL, June 1899, May 1900.

[3] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 1 and 12, 1899.

[4] PASCHEN and WANNER, *Ber. d. Berl. Akad.*, January 1899.

[5] THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, 5, 1900.

[6] PLANCK, *Drude's Annalen*, **1**, 1900; **4**, 1900.

[7] WIEN, *Wied. Ann.*, **58**, 662, 1896.

[8] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 1, 1899.

[9] PASCHEN, *Wied. Ann.*, **60**, 1897.

Here

$S$ = total energy of radiation at any absolute temperature.

$T$ = total energy of radiation at this absolute temperature.

$E d\lambda$ = energy radiated in waves of length $< \lambda + d\lambda$ and $> \lambda$.

$\lambda$ = any wave-length in $\mu$, (0.001 mm.)

$\lambda_m$ = wave-length of maximum energy at temperature $T$.

$E_m d\lambda$ = amount of maximum energy at temperature $T$, between the limits

$$\lambda_m \pm \frac{d\lambda}{2} .$$

$A$, $B$, $C$, $c$, $\alpha$, are constants.

The following work on the radiation of an "absolutely black body" was begun by Mr. C. E. Mendenhall in conjunction with Dr. H. F. Reid, the part relating to temperatures above 500° C. being carried out by him, while that relating to temperatures below 500° C. was carried out by Mr. F. A. Saunders. The work was done in the Physical Laboratory of the Johns Hopkins University, Baltimore.

We shall first consider the work at temperatures above 500° C., then point out the differences in procedure adopted for the low temperature curves, and give the corresponding results.

The method of realizing the "black body," based on Kirchhoff's theoretical investigation of the radiation inside a uniformly heated inclosure, had been suggested by Dr. Reid in this JOURNAL,[1] and independently by Wien and Lummer.[2]

Our black body was either a cast-iron or copper cylinder, about 8 cm in diameter and 12 to 18 cm high, with a slit in the side through which passed the radiation to be examined; a furnace, or, for the lower temperatures, appropriate baths, served for the heating.

The spectrometer was practically a reproduction of Langley's early one, used at Allegheny.

The available rock salt consisted of a 60° prism, having faces about 5 cm × 7 cm, and a lens, about 10 cm diameter and 40 cm focus, with Brashear surfaces — the material for which had been kindly loaned by Professor Langley.

[1] H. F. REID, ASTROPHYSICAL JOURNAL, 2, 160, 1895.

[2] W. WIEN and O. LUMMER, *Wied. Ann.*, 56, 1895.

In order to increase the sensitiveness of the bolometer, two diagonal arms of the Wheatstone bridge quadrilateral were exposed to radiation. Theoretically this should be better than a single strip of the same width in the ratio $\sqrt{2}$ to 1. Considerations of freedom from "drift" with general changes in temperature made it desirable that the balancing arms should be as nearly like the exposed strips as possible, and similarly situated. All four arms were accordingly made of similar strips of annealed platinum foil, and mounted in the same cell. Balancing was accomplished by moving the galvanometer terminals independently along two copper wires; one, being comparatively fine (No. 24 about), gave rough adjustment, while the other (No. 12), gave fine adjustment.

The spectrometer, balancing bridge, etc., were inclosed in a double-walled box — for the purpose not only of protecting the bolometer and its appendages from temperature changes, but also of protecting the rock salt from moisture. No attempt was made to exclude or remove $CO_2$, nor water vapor, except in so far as was needed for the protection of the rock salt. The battery consisted of a number of Edison-Lalande cells, connected in multiple, and, on the advice of Mr. C. G. Abbot, of the Smithsonian Institution, carefully protected, as were the main leads, also, from temperature changes. The galvanometer was of the Thomson 4-coil pattern, of low resistance (about 4 ohms), and with the needle used in the first part of the work gave 1 mm deflection at 1 m with about $2.5 \times 10^{-10}$ amp., with a complete period of 10 sec., though only rarely was this maximum sensibility needed.

The bolometer showed a very satisfactory freedom from "drift" and from disturbance in general. The galvanometer, however, located, as it was, in the midst of a city, was, with the best needle-system which we were able to produce, and with a quadruple iron magnetic shield, quite unusable in the day time, so that all observations had to be carried on at night. For temperatures above 500° the black body was heated in a furnace of fire-clay and the temperatures were determined by the use of

several platinum, platinum-iridium thermo-couples, according to the potentiometer method, much as outlined by Barus. These were calibrated by the use of a number of standard melting and boiling temperatures, viz., water, napthalin, mercury, potassium chloride and gold. The higher temperature determinations were perhaps in error by 5° or 10°. Temperatures were measured at four points, two at the top of the cylinder (black body), and two at the bottom. With the furnace method of heating the black body, as we used it, differences of temperature of from 10° to 20° were usually found between some of these four points.

## PART I.

With the above apparatus observations of the distribution of energy in the black body spectrum were taken at many temperatures between 500° C. and 1100° C.; and a few sets of observations of energy at various (fixed) points in the spectrum while the temperature varied — giving data for the so-called isochromatic curves. When the observations were used with the corresponding observation of minimum deviation to plot energy curves, the characteristic absorption bands of $CO_2$ and $H_2O$ vapor were very marked. These curves were then put in the normal form by changing from minimum deviation to wavelength, using the dispersion curve of rock salt found by Rubens,[1] and by Rubens and Trowbridge.[2] The corresponding change in the ordinates of these curves, viz., multiplication by $\frac{d\lambda}{d\delta}$, was made — also corrections for impurity of spectrum, according to Runge,[3] and for variation in sensibility of apparatus.

It was at first attempted to allow for the absorption bands of $H_2O$ and $CO_2$ in the usual way by "bridging over" the gaps with a free-hand curve. Upon comparing these curves, however, it was concluded that the amount of absorption had, over part

[1] RUBENS, *Wied. Ann.*, **54**, 436, 1895.

[2] RUBENS and TROWBRIDGE, *Am. Jour. Sci.*, January 1898.

[3] RUNGE (Paschen) *Wied. Ann.*, **60**, 1897, and Schlömilch's *Zeit. fur Math. u. Phys.*, **43**, 1897.

of the curve, been greatly underestimated. This made the entire middle portion of the curves uncertain; especially it made the wave-length of maximum energy very difficult to determine, and hence made it impossible to test accurately equation (III). In fact, by properly bridging over the absorption gaps, the curves can be made to satisfy (III) as exactly as may be desired. Paschen[1] has stated that the expression

$$\lambda_m = \frac{(\log \lambda_2 - \log \lambda_1)\, \lambda_2\, \lambda_1}{(\lambda_2 - \lambda_1) \log \epsilon},$$

where $(\lambda_1, \lambda_2)$ are any two wave-lengths on opposite sides of the maximum corresponding to equal energies of radiation, serves to give consistent values for $\lambda_m$, and it has accordingly been used in connection with these curves. The wave-lengths $(\lambda_1, \lambda_2)$ were taken at points where the absorption was as small as possible, and for each of seven curves several values of $\lambda_m$ were calculated; these values agreed usually to about 0.1 $\mu$. The resulting values of $\lambda_m T$ are as follows:

| $T$, C° | $\lambda_m$ | $\lambda_m T$ |
|---|---|---|
| 570 | 3.34 | 2815 |
| 704 | 2.72 | 2657 |
| 771 | 2.53 | 2641 |
| 837 | 3.36 | 2619 |
| 896 | 2.20 | 2571 |
| 944 | 2.20 | 2611 |
| 1030 | 2.00 | 2586 |

With the exception of the first one, the numbers in the last column are as nearly constant as could be expected considering the possible errors of measurement—but the mean value differs by nearly 300 from Paschen's mean value 2907, or Lummer and Pringsheim's 2930. This could be accounted for by imperfect "blackness" of our radiator, but this seems a rather improbable explanation considering the size and form of our enclosure. It is perhaps more probable that the heavy absorption on the descending side of our curves has led to an apparent shifting of all the $\lambda_m$ toward the short wave-lengths.

[1] PASCHEN, *Wied. Ann.*, 50, 409.

As far as (I) is concerned our method is at best a poor one — analyzing the radiation only to integrate the energy-curve afterwards; with the absorption as large a part of the total energy as the curves would indicate, an attempt to confirm (I) becomes still less fruitful.

As for (IV) $E_m$ is rendered uncertain by the bands above referred to — but not to the same extent as $\lambda_m$; for the entire change in $\lambda_m$ through the temperature range used in the high temperature work is but about 1.2 $\mu$; and an examination of the curves shows that the uncertainties are a large part of this.

The causes of this extremely strong absorption undoubtedly lay in the use of a furnace to heat the black body, which became filled with the products of combustion, notably $CO_2$ and $H_2O$. That no more elaborate means to prevent this were taken was due to the conclusion, drawn from a comparison of some of the final curves roughly plotted, with some curves previously taken with slightly different arrangements — that the amount of furnace gas in the black body was not sufficient to produce extraordinary absorption. This conclusion was evidently in error.

The following table gives the values of (B) for seven curves:

| °C | $T$ | $E_m\, T^{-5} \times$ const. |
|---|---|---|
| 1020 | 1293 | 59 |
| 914 | 1187 | 65 |
| 896 | 1169 | 61 |
| 837 | 1110 | 50 |
| 771 | 1044 | 55 |
| 704 | 977 | 51 |
| 570 | 843 | 54 |

As to Stefan's law (I), $S$ can be approximately determined from the area of the various curves as finally corrected. From these we obtain the following table of values of $\frac{S}{[T^4 - T_1^4]}$, where $T_1$ is the absolute temperature of the shutter used to exclude radiation.

| $T$ | $\frac{S}{[T^4 - T_1^4]} \times$ const. |
|---|---|
| 1293 | 493 |
| 1187 | 528 |
| 1169 | 457 |
| 1110 | 436 |
| 1044 | 470 |
| 977 | 458 |
| 843 | 433 |

Here also there is very unsatisfactory constancy. The five lower temperature curves agree fairly well among themselves, but we think it probable that the absorption has not been completely allowed for in these curves. The error in the 1187° curve seems to be rather larger than could be accounted for by an error in the absorption correction alone; unless, as suggested above, the other curves have been undercorrected. If this is the case, then the coincidence of an error of 8° in the estimation of temperature (for the 1187° curve) with an easily allowable overestimation of the absorption correction would account for the discrepancy.

## PART II.

On account of the extremely small amount of radiation with which one has to deal in measuring the energy in the spectrum of a radiating body at comparatively low temperatures, it was absolutely necessary to have a more sensitive needle system in the galvanometer for this part of the work, and accordingly a series of experiments was undertaken to determine what form of system would be most efficient. The vertical needle system of Weiss having proved inadequate when the highest sensibility was required, the ordinary form of system was used and a number of modifications in it were tried, with results which it may be of interest to give. It seemed obvious from the work of Paschen and others that the lighter the system was, other things being equal, the more sensitive it was, and also that the greater the proportion of the weights of the magnets to the total weight, the greater the sensitiveness. Starting from these facts, fourteen different systems were made and tested. The magnets used throughout were made from watch and clock-spring material and were

tempered, magnetized and boiled before being mounted. A great many magnets were made at the beginning and from these the best were selected by observing their activity when laid upon a glass plate and tapped in a vertical magnetic field; only the best were used. The sensitiveness of each system was found by mounting it on a fine quartz fiber in the galvanometer and observing the deflection produced by a measured current. The figures given for this result are the currents in amperes which produced a deflection of 1 mm on a scale 1 meter distant when the complete period of the system was ten seconds.

Each system was built on a very fine glass rod and furnished with a minute copper wire loop at its upper end, by means of which it could be hung on a corresponding hook on the lower end of the fiber. The mirrors used were fragments of the finest microscope cover-glasses obtainable, silvered, and cut into pieces roughly circular in shape with an area of about 1.5 sq. mm. Their weights varied from 0.4 to 0.7 mg.

A study of the results obtained brought out the following conclusions, which, of course, apply exactly only to a galvanometer whose "free space" is circular and of the same diameter as ours (3 mm):

1. It is unwise to make the magnets shorter than 1 mm.
2. It is unwise to make the system as deep (measured along the stem) as the diameter of the free space.
3. It is somewhat disadvantageous to make the magnets themselves as long as the free space is wide; such systems are also very troublesome to use.
4. There is a slight disadvantage in making the magnets of material thinner than 0.05 mm.

One system was finally chosen as the best of all, and the systems used in the subsequent work were made after the same pattern. Each group in the system consisted of three magnets, two of which were 1.6 mm long while the third was 2.3 mm long; the width of each magnet was 0.2 mm and its thickness 0.05 mm. Each group was spaced along the stem so as to cover about 1.5 mm. The total weight was 2.5 mg; of which 1.3 mg was of steel, 0.6 mg of mirror and 0.6 mg of glass and shellac.

An effort was next made to bring a number of these systems to a highly astatic condition. This, of course, requires that the magnets shall lie in the same plane, or in parallel planes, and that the upper and lower sets shall be equal in magnetic strength but opposite. As this occupied an unexpectedly long time, it may perhaps be well to give an account of the difficulties encountered in this apparently simple operation. In the first systems the magnets were all fastened on the same side of the glass rod and the mirror was on the opposite one. The magnets of the systems being fastened in place while lying on a piece of plate glass they were nearly all in the same plane, and the fine adjustment was made by loosening one of the magnets a little by heating and turning it through a small angle. For a considerable time no consistent results were obtained, owing to the proximity of a magnetized steam-pipe which had an unexpectedly great effect on the uniformity of the Earth's magnetic field in the place in which the systems were kept. Having moved the systems to a place where the field was uniform, we once more adjusted them by turning one magnet of one set so that the plane of the equivalent magnet was parallel to that of the other set. This process, since it involved heating, usually resulted in a slight weakening of the magnet turned, which was corrected by bringing near it a powerful permanent magnet. Any system the planes of whose magnets are not parallel should, when the magnetic strength of the two sets are made equal, stand with its magnets pointing east and west, and its period should be longer the nearer its magnet sets are to being in the same plane. This was found to be the case in about one system only out of ten. The others as they approached an astatic condition reached a condition when they would oscillate about *two* positions of equilibrium, usually in the two directions northwest to southeast and northeast to southwest. When this was the case, there was no means of finding, by its positions of equilibrium or by its period of oscillation, which of the two magnet sets was the stronger, nor in what direction one of them must be turned in order to bring the sets more nearly into the

same plane. It was therefore impossible to proceed with the astaticizing at all.

This anomalous condition was in no way due to torsion in the supporting fiber, for a complete revolution of this produced no more than five degrees change in the natural position of the system, and six whole turns were necessary to force the system to revolve. The fibers were from 15 to 25 cm in length and not more than 0.0025 mm in diameter, and usually somewhat less. They were made by the blowing process invented by Boys,[1] in which the oxy-hydrogen flame melts and at the same time blows out a minute fragment of quartz into a long fiber, a process which proved extremely efficient and simple after a little practice had been obtained with it.

Each fiber was hung in a glass tube so that its lower end projected beyond the tube into a small wooden box whose front was closed by a glass plate fastened by a bit of wax. It was found that the act of detaching this glass plate charged it quite highly with electricity, and it was hoped that the action between this charge and the charge induced on the magnets might account for the existence of two positions of equilibrium. That it did not, however, was amply shown by the use of a brass plate instead of the glass one, which was carefully discharged whenever handled, by passing it through a flame. The systems behaved just as before.

A lack of symmetry in the system itself was next suggested as a possible explanation, though by no means, at first sight at least, a very rational one. Accordingly, several systems were constructed and astaticized in which the following precautions were taken: the weights of the two sets of magnets were made equal to within 0.1 mg; the weight of the magnets fastened on one side of the glass stem was made equal to the weight of the magnets and mirror on the other side of the stem, to the same degree of approximation (it was not possible to do this more accurately, as the total weight of the parts was not often more than 1 mg and the weighings were not certain to less than

[1] V. C. Boys, *London Electrician*, December 11, 1896.

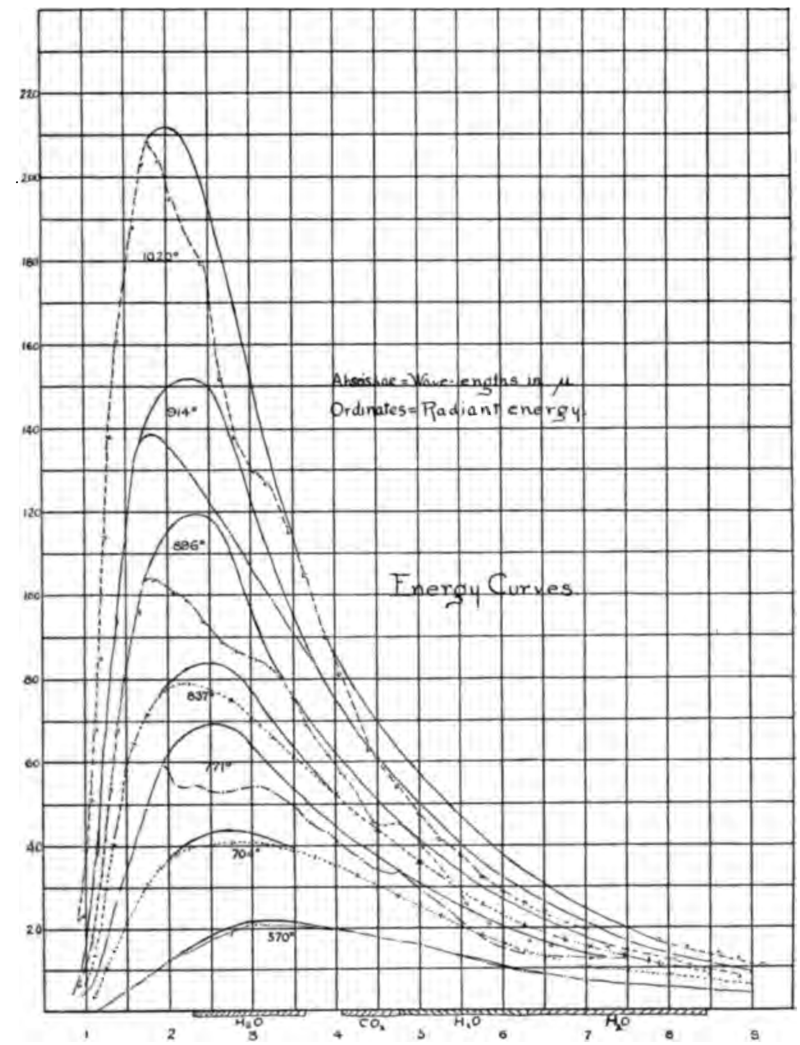



FIG. 1.

Broken lines, uncorrected or partially corrected curves.
Full lines, corrected curves, drawn to fit relation $\lambda_m T = \text{const.}$
Absorption bands indicated at bottom.

0.1 mg or to 10 per cent.); the greatest care was taken to have the glass stem fastened at the middle points of the magnets, and the mirror was so adjusted as to make the oscillations of the whole system aperiodic when it was supported horizontally so as to be free to turn about the stem as axis; and, finally, the loop of wire at the upper end of the system was dispensed with and the quartz fiber was fastened directly to the stem. The systems that were produced after all these precautions were taken were in no way better than those made before, and as no further changes could be thought of which would prevent this anomalous behavior all the systems that exhibited it were discarded.

It would be quite possible to account for this difficulty by remembering that an absolutely astatic system, *i. e.*, one whose magnets were all in the same plane and whose sets were exactly equal and opposite in strength when they were in an east and west plane, would be thrown out of astaticism when the plane of the magnets became north and south by the magnetism induced in the magnets by the Earth's field, and would hence oscillate about two positions of equilibrium. It seemed very unlikely that these systems were so nearly perfect that this explanation could apply to them, particularly as the induced magnetism must be very feeble in hardened watch-spring steel. No other explanation has, however, been thought of.

The best systems finally chosen, three or four in all, were then brought to a fairly astatic condition, as shown by their period of oscillation (complete period six or seven seconds), and they were then examined from day to day, being as far as possible undisturbed in the intervals, to see how their condition altered with the time. This alteration proved to be very great, and was apparently as great in the case of systems composed of boiled magnets as it was with some constructed of "raw" material. All of them at first lost their astaticism rapidly, but, after readjusting them once or twice a day, at the end of two weeks we obtained one which held a period of about five seconds for the ensuing two weeks without appreciable alteration. This system was therefore mounted in the galvanometer and was used

in the subsequent work. It had a sensitiveness of about $1 \times 10^{-10}$ amp. At the close of the investigation, however, after three months use, it was found to have a period of about three seconds, indicating a considerable fall in astaticism.

Seven curves in all were obtained, all of them duplicated in important regions, namely, at the temperatures 100°, 175°, 243°, 313°, 399°, 503°, and 578° C. Of these curves the four taken at the lowest temperatures are drawn in Fig. 2, and reveal at once the presence of the absorption bands due to the presence of carbon dioxide and of water vapor in the air, which are to be found in all the curves taken. The full lines with which the observed curves in part coincide indicate the curves filled in according to a method explained below. It was hoped that five or six vessels containing concentrated sulphuric acid, which were put inside the spectrometer box, would keep the air inside reasonably dry. They did keep the rock-salt surfaces from being fogged, but did not prevent the water-vapor bands from being prominent in every curve. With the spectrum as impure as it was, these bands in some places overlap and affect the curves continuously for a considerable distance. A few points in the spectrum seemed, however, to be free from absorption, and the observations at these points only were used.

Paschen[1] has found that if Wien's law be true, and if the radiation curves be plotted, not as is usually done, with wave-length and intensity as coördinates, but with the logarithms of these quantities instead, then the curves have the property of congruency. By this is meant that any one curve is an exact copy of any other, but shifted, unaltered in shape, to another part of the diagram. Now, since the maximum energy occurs at different wave-lengths as the temperature is changed, it is evident that any region of absorption will fall upon a different part of each curve, and hence if the curves are congruent as above explained, the points cut out of any curve by absorption can be supplied from another curve by merely laying one above the other. In this way, by the use of curves taken at different temperatures,

[1] F. PASCHEN, *Wied. Ann.*, **60**, 1897.

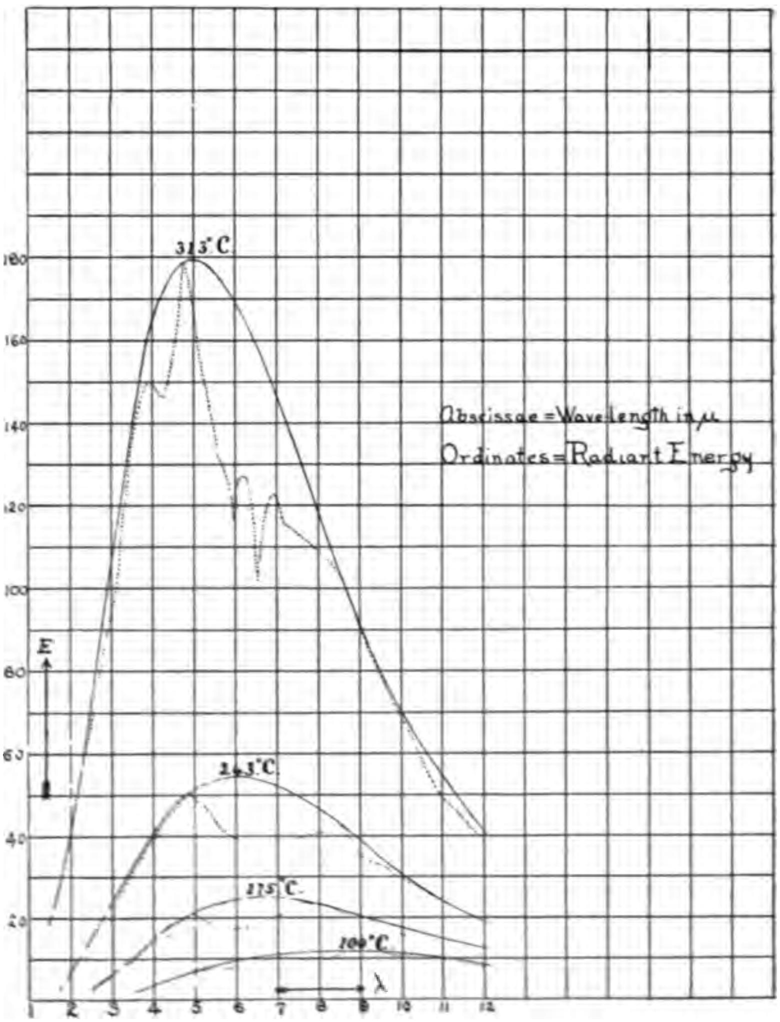



FIG. 2.

the entire curve can be constructed from a few points. This has been the method used. Our curves when plotted logarithmically are roughly congruent, though not accurately so, as there seems to be a slight change in form as the temperature changes. The curve is roughly of an inverted U shape, and our curves show a

slight tendency towards a widening of the U as the temperature increases. The mean curve was, however, taken, as the deviations did not seem too large, and by means of this the missing points in the observed curves were supplied.

Four tests were applied to this set of curves. Law I was first tested, with the results shown by the following tables:

| $T$ | $\lambda_m$ | $\lambda_m T$ |
|---|---|---|
| 373° | 8.30 | 3090 |
| 448 | 6.50 | 2910 |
| 516 | 5.90 | 3040 |
| 586 | 4.90 | 2870 |
| 672 | 4.37 | 2940 |
| 776 | 3.81 | 2950 |
| 851 | 3.24 | 2760 |
| | Average | 2940 |

These figures are somewhat irregular, but the determination of the maximum of the energy curve is an extremely uncertain thing when the curves are as much cut up by absorption bands as these were. The average value of this constant, 2940, is somewhat higher than that found by Paschen, and there is a slight tendency shown in the series of values towards an increase in the value of the constant with decrease of temperature. The marked difference between this value and that found in the earlier part of this work may perhaps point to the same result. The expression (IV) was next tested. This set of observations fails entirely to conform with this law. The maximum energy varies nearly as the sixth power of the absolute temperature.

The following relation should also hold:

$$\frac{E}{E_m}=\left\{\frac{\lambda_m}{\lambda}\cdot e^{\frac{\lambda-\lambda_m}{\lambda}}\right\}^a,$$

where $E$ is the intensity at wave-length $\lambda$, and $E_m$ is the maximum intensity. This relation was found experimentally by Paschen, and according to Wien's law the constant $a$ should be 5. This constant was calculated from a great number of points on all the curves, and the average value obtained was slightly less than 5, but very near it.

Finally, Wien's expression (V) for the energy at any wave-length, gives, on integration with respect to $\lambda$ from $O$ to $\lambda$,

$$\int_0^\lambda C\lambda^{-5}\epsilon^{-\frac{c}{\lambda T}}\cdot d\lambda = C\left(\frac{T}{c}\right)^4\cdot\lambda^{-3}\cdot\epsilon^{-\frac{c}{\lambda T}}\cdot$$

$$\left\{\left(\frac{c}{T}\right)^3 + 3\left(\frac{c}{T}\right)^2\lambda + 6\left(\frac{c}{T}\right)\lambda^2 + 6\lambda^3\right\}.$$

This enables us to compare the areas of the curves up to a certain wave-length with those required by the formula. If we knew the entire curve we could simply integrate it and then its area, representing as it does the total radiation, should follow Stefan's law, *i. e.*, should be proportional to the fourth power of the absolute temperature. It is not possible, however, to do this on account of the large part of the curve lying in the region of the longer wave-lengths, which is influenced by the absorption of the prism, etc. This circuitous method was tried on our curves, and since Wien's law is based on Stefan's, should lead to the same result. This was not found to be the case here, our areas being nearly proportional to the seventh power of the absolute temperature; but this method of obtaining the total radiation is too indirect to lead to accurate results.

The conclusions to be drawn from our results are, as before stated, rather negative in character. It is evident that some of the deductions from Wien's law are satisfied, while others are not; but no results of sufficient accuracy can be obtained without taking excessive precautions in regard to the "blackness" of both radiator and bolometer strip, and in excluding from the air about the apparatus all traces of carbon dioxide and water vapor.

It is interesting to note that if law III holds, the maximum of the radiation curve of a body at the temperature of the boiling of liquid air under atmospheric pressure should lie at about $30\,\mu$, and would therefore be beyond the reach of any rock-salt dispersion apparatus. An effort was made to test this by cooling the black body with liquid air, but at the last it proved impossible to obtain enough for our purposes. With the small

quantity that we had, however, the body was cooled to about —90° C., and at this point caused the greatest deflection at about $10\mu$, which is roughly where the maximum should lie, according to law III. The deflection was also of the order to be expected, though the working conditions at the time were not good enough to allow any accurate measurements to be taken.

The writers wish to express their sincere indebtedness to Dr. H. F. Reid for his continued interest and valuable advice, and to Professors Rowland and Ames, not only for their kind supervision, but for the generosity with which they placed all the facilities of the laboratory at our disposal.

NOTE.—Mr. H. C. Dickinson of Williams College has suggested that the two positions of equilibrium of the delicate galvanometer needles, above referred to, may be due to the disturbance of the co-planarity of the two systems of magnets, by the couples between these systems and the magnetic field, opposed by the torsional rigidity of the connecting glass rod. This assumes that the two groups of magnets are initially very closely in the same plane—and of very nearly equal moments. Some experiments have been performed which indicated that a change of several minutes might be expected in the angular relation of the two groups, and this seems sufficient to account for the observed phenomena. C. E. M.

November 1900.

# EXAMINATION OF *PLEIADES* AND *EROS* PLATES TAKEN WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.[1]

In reply to a letter received September 21, 1900, from the Acting Director of the Lick Observatory, Professor W. W. Campbell, requesting the aid of the Columbia University Observatory in determining the accuracy of the photographs of *Eros* to be taken with the Crossley reflector,[2] Professor Rees stated that the Columbia Observatory staff would make very gladly the measurements required.

Professor Campbell was asked to send three plates of the *Pleiades* taken on the same night or different nights, having the center of each plate near the greatest number of stars. In the meantime a plate of *Eros* was taken on September 19 with the Crossley reflector and forwarded to the Columbia Observatory. The *Pleiades* plates were obtained on September 27 and received October 8. The plates were placed at once in charge of Professor Jacoby.

Dr. S. A. Mitchell and Miss F. E. Harpham made the measurements and in the reductions were assisted by the Observatory computing force. The results of the measurements and reductions are given below in the report from Professor Jacoby.

### REPORT ON THE MEASUREMENT AND REDUCTION OF THREE PLEIADES PHOTOGRAPHS MADE WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

The following pages contain a detailed account of the measurement and reduction of three photographs of the *Pleiades*, made with the Crossley reflector of the Lick Observatory.

[1] Communicated by Professor J. K. Rees, Director of Columbia University Observatory, New York City.

[2] *Note by W. W. C.*—The Crossley reflector is in charge of Assistant Astronomer C. D. Perrine. He is assisted by Mr. H. K. Palmer, Fellow in Astronomy. The plates were secured by them.

The data relative to the exposure of the negatives are contained in Table I.

TABLE I.

| Plate | Date | Exposure | Mt. H. Sid. Time (Middle of exposure) | Barometer | Att. Therm. | Ext. Therm. |
|---|---|---|---|---|---|---|
| | 1900 | | | | | |
| 1..... | Sept. 27 | 30 sec. | $4^h$ $2^m$ $50^s.0$ | 26.127 | 57°.0 | 56°.1 |
| 2..... | " 28 | 10 sec. | 4 17 39.0 | 26.137 | 61.0 | 58.3 |
| 3..... | " 27 | 5 sec. | 0 47 5.5 | 26.127 | 57.0 | 57.0 |

The plates were measured in rectangular coördinates by two observers and reduced according to Jacoby's method as given in Dr. Schlesinger's paper on the *Praesepe* Group.[1]

Table II contains the results of the individual measures in millimeters, and the differences between the two observers reduced to seconds of arc. Ten stars were measured on each plate, the central star numbered 17 being observed twice. The numbers in the first column are from Jacoby's catalogue of the *Pleiades*.[2]

TABLE II.

PLATE I. $x$ MEASUREMENTS.

| Star | $x$ direct | | | | $x$ reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | ½ *m* or Scale *minus* Star: Harpham | Mitchell *minus* Harpham | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | ½ *m* or Scale *minus* Star: Harpham | Mitchell *minus* Harpham |
| 1 | 111 | 0.0470 | 0.0592 | −0″.47 | 8 | 0.5750 | 0.5840 | −0″.35 |
| 5 | 96 | 0.0548 | 0.0495 | + .20 | 23 | 0.5808 | 0.5858 | − .19 |
| 16 | 71 | 0.7915 | 0.7975 | − .23 | 47 | 0.8405 | 0.8410 | − .02 |
| 17 | 70 | 0.5398 | 0.5455 | − .22 | 49 | 0.0855 | 0.0908 | − .20 |
| 17 | 70 | 0.5455 | 0.5370 | + .33 | 49 | 0.0890 | 0.0905 | − .06 |
| 19 | 69 | 0.2488 | 0.2460 | + .11 | 50 | 0.3892 | 0.3902 | − .04 |
| 20 | 66 | 0.3750 | 0.3755 | − .02 | 53 | 0.2598 | 0.2655 | − .22 |
| 22 | 65 | 0.4765 | 0.4702 | + .24 | 54 | 0.1530 | 0.1628 | − .38 |
| 24 | 61 | 0.7955 | 0.7970 | − .06 | 57 | 0.8368 | 0.8410 | − .16 |
| 25 | 57 | 0.7328 | 0.7350 | − .08 | 61 | 0.9005 | 0.9005 | .00 |
| 34 | 37 | 0.8740 | 0.8935 | − .75 | 81 | 0.7498 | 0.7390 | + .42 |

[1] "The *Praesepe* Group: Measurement and Reduction of the Rutherfurd Photographs," by FRANK SCHLESINGER, *Annals N. Y. Acad. Sci.*, **10**, 189–286, 1898, or *Contrib. Obsy. Columbia Univ.*, No. 15.

[2] *Annals N. Y. Acad. Sci.*, **6**, 323, or *Contrib. Obsy. Col. Univ.*, No. 3.

PLATE II. $x$ MEASUREMENTS.

| Star | $x$ direct | | | | $x$ reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | Harpham | Mitchell *minus* Harpham | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | Harpham | Mitchell *minus* Harpham |
| 1 | 115 | 0.3722 | 0.3815 | $-0^{r}.36$ | 4 | 0.2488 | 0.2618 | $-0^{r}.50$ |
| 5 | 100 | 0.3765 | 0.3852 | − .34 | 19 | 0.2520 | 0.2560 | − .15 |
| 16 | 76 | 0.1235 | 0.1165 | + .27 | 43 | 0.5070 | 0.5035 | + .14 |
| 17 | 74 | 0.8748 | 0.8865 | − .45 | 44 | 0.7465 | 0.7498 | − .13 |
| 17 | 74 | 0.8800 | 0.8855 | − .21 | 44 | 0.7462 | 0.7480 | − .07 |
| 19 | 73 | 0.5895 | 0.5895 | .00 | 46 | 0.0402 | 0.0390 | + .05 |
| 20 | 70 | 0.7115 | 0.7225 | − .42 | 48 | 0.9132 | 0.9088 | + .17 |
| 22 | 69 | 0.8332 | 0.8438 | − .41 | 49 | 0.7935 | 0.7878 | + .22 |
| 24 | 66 | 0.1075 | 0.1120 | − .17 | 53 | 0.5090 | 0.5118 | − .11 |
| 25 | 62 | 0.0915 | 0.0990 | − .29 | 57 | 0.5365 | 0.5320 | + .17 |
| 34 | 42 | 0.2602 | 0.2700 | − .38 | 77 | 0.3670 | 0.3698 | − .11 |

PLATE III. $x$ MEASUREMENTS.

| Star | $x$ direct | | | | $x$ reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | Harpham | Mitchell *minus* Harpham | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | Harpham | Mitchell *minus* Harpham |
| 1 | 119 | 0.1038 | 0.0975 | $+0^{r}.21$ | 0 | 0.5135 | 0.5272 | $-0^{r}.53$ |
| 5 | 104 | 0.1158 | 0.1090 | + .26 | 15 | 0.5110 | 0.5170 | − .23 |
| 16 | 79 | 0.8710 | 0.8702 | + .03 | 39 | 0.7638 | 0.7620 | + .07 |
| 17 | 78 | 0.6218 | 0.6158 | + .23 | 41 | 0.0098 | 0.0145 | − .18 |
| 17 | 78 | 0.6208 | 0.6220 | − .05 | 41 | 0.0088 | 0.0060 | + .11 |
| 19 | 77 | 0.3275 | 0.3258 | + .07 | 42 | 0.3155 | 0.3078 | + .30 |
| 20 | 74 | 0.4670 | 0.4610 | + .23 | 45 | 0.1670 | 0.1788 | − .46 |
| 22 | 73 | 0.5668 | 0.5495 | + .67 | 46 | 0.0765 | 0.0790 | − .10 |
| 24 | 69 | 0.8615 | 0.8680 | − .25 | 49 | 0.7658 | 0.7702 | − .17 |
| 25 | 65 | 0.8248 | 0.8300 | − .20 | 53 | 0.8062 | 0.8038 | + .09 |
| 34 | 46 | 0.0098 | 0.0060 | + .15 | 73 | 0.6262 | 0.6345 | − .32 |

PLATE I. $y$ MEASUREMENTS

| Star | $y$ direct | | | | $y$ reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | Harpham | Mitchell *minus* Harpham | Line of Scale | ½ *m* or Scale *minus* Star: Mitchell | Harpham | Mitchell *minus* Harpham |
| 1 | 63 | 0.7568 | 0.7670 | $-0^{r}.39$ | 55 | 0.8630 | 0.8588 | $+0^{r}.16$ |
| 5 | 65 | 0.7032 | 0.7028 | + .02 | 53 | 0.9278 | 0.9235 | + .17 |
| 16 | 72 | 0.2210 | 0.2228 | − .07 | 47 | 0.4070 | 0.4080 | − .04 |
| 17 | 60 | 0.3382 | 0.3442 | − .23 | 59 | 0.2932 | 0.2948 | − .06 |
| 17 | 60 | 0.3368 | 0.3412 | − .17 | 59 | 0.2912 | 0.2920 | − .03 |
| 19 | 55 | 0.9748 | 0.9752 | − .02 | 63 | 0.6495 | 0.6585 | − .35 |
| 20 | 63 | 0.3548 | 0.3613 | − .25 | 56 | 0.2670 | 0.2730 | − .23 |
| 22 | 32 | 0.7345 | 0.7372 | − .10 | 86 | 0.8952 | 0.8875 | + .30 |
| 24 | 78 | 0.2908 | 0.3028 | − .46 | 41 | 0.3322 | 0.3270 | + .20 |
| 25 | 47 | 0.3902 | 0.3932 | − .12 | 72 | 0.2398 | 0.2425 | − .10 |
| 34 | 55 | 0.1102 | 0.1050 | + .20 | 64 | 0.5210 | 0.5260 | − .19 |

PLATE II. *y* MEASUREMENTS

| Star | *y* direct | | | | *y* reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ *m* or Scale *minus* Star | | Mitchell *minus* Harpham | Line of Scale | ½ *m* or Scale *minus* Star | | Mitchell *minus* Harpham |
| | | Mitchell | Harpham | | | Mitchell | Harpham | |
| 1 | 59 | 0.0062 | 0.0120 | −0″.22 | 60 | 0.6192 | 0.6200 | −0″.03 |
| 5 | 60 | 0.9275 | 0.9268 | + .03 | 58 | 0.6995 | 0.7030 | − .14 |
| 16 | 67 | 0.4052 | 0.4055 | − .01 | 52 | 0.2218 | 0.2268 | − .19 |
| 17 | 55 | 0.5232 | 0.5250 | − .07 | 64 | 0.1052 | 0.1118 | − .25 |
| 17 | 55 | 0.5162 | 0.5215 | − .20 | 64 | 0.1025 | 0.1090 | − .25 |
| 19 | 51 | 0.1590 | 0.1600 | − .04 | 68 | 0.4688 | 0.4615 | + .28 |
| 20 | 58 | 0.5300 | 0.5392 | − .36 | 61 | 0.0895 | 0.0970 | − .29 |
| 22 | 27 | 0.9185 | 0.9162 | + .09 | 91 | 0.6895 | 0.6948 | − .20 |
| 24 | 73 | 0.4708 | 0.4755 | − .18 | 46 | 0.1520 | 0.1550 | − .12 |
| 25 | 42 | 0.5688 | 0.5692 | − .02 | 77 | 0.0548 | 0.0610 | − .24 |
| 34 | 50 | 0.2678 | 0.2505 | + .67 | 69 | 0.3752 | 0.3702 | + .19 |

PLATE III. *y* MEASUREMENTS

| Star | *y* direct | | | | *y* reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ *m* or Scale *minus* Star | | Mitchell *minus* Harpham | Line of Scale | ½ *m* or Scale *minus* Star | | Mitchell *minus* Harpham |
| | | Mitchell | Harpham | | | Mitchell | Harpham | |
| 1 | 58 | 0.7968 | 0.7942 | +0″.10 | 60 | 0.8435 | 0.8385 | +0″.19 |
| 5 | 60 | 0.7118 | 0.7138 | − .08 | 58 | 0.9208 | 0.9152 | + .22 |
| 16 | 67 | 0.2162 | 0.2175 | − .05 | 52 | 0.4125 | 0.4122 | + .01 |
| 17 | 55 | 0.3252 | 0.3215 | + .14 | 64 | 0.3112 | 0.3112 | .00 |
| 17 | 55 | 0.3228 | 0.3270 | − .16 | 64 | 0.3070 | 0.3082 | − .05 |
| 19 | 50 | 0.9778 | 0.9730 | + .19 | 68 | 0.6572 | 0.6620 | − .19 |
| 20 | 58 | 0.3450 | 0.3482 | − .12 | 61 | 0.2865 | 0.2898 | − .13 |
| 22 | 27 | 0.7512 | 0.7482 | + .12 | 91 | 0.8820 | 0.8810 | + .04 |
| 24 | 73 | 0.3015 | 0.2998 | + .07 | 46 | 0.3308 | 0.3302 | + .02 |
| 25 | 42 | 0.3985 | 0.4020 | − .14 | 77 | 0.2378 | 0.2362 | + .06 |
| 34 | 50 | 0.1028 | 0.1015 | + .05 | 69 | 0.5342 | 0.5402 | − .23 |

These measures were corrected for division errors of the millimeter scale and errors of the micrometer screw, using the correction tables employed by Dr. Schlesinger in the computation of the *Praesepe* measures. The run of the micrometer screw was practically negligible. The corrected coördinates are given in Table III, where the quantities are in millimeters.

TABLE III.

PLATE I. CORRECTED COÖRDINATES.

| Star | x | | | y | | |
|---|---|---|---|---|---|---|
| | Direct | Reversed | Mean | Direct | Reversed | Mean |
| 1 | −40.5066 | .5126 | −40.5096 | + 3.4236 | .4334 | + 3.4285 |
| 5 | −25.5074 | .5066 | −25.5070 | + 5.3643 | .3692 | + 5.3668 |
| 16 | − 1.2525 | .2504 | − 1.2514 | +11.8832 | .8883 | +11.8858 |
| 17 | 0.0000 | .0000 | 0.0000 | 0.0000 | .0000 | 0.0000 |
| 19 | + 1.2938 | .2994 | + 1.2966 | − 4.3666 | .3630 | − 4.3648 |
| 20 | + 4.1660 | .1742 | + 4.1701 | + 3.0202 | .0232 | + 3.0217 |
| 22 | + 5.0686 | .0705 | + 5.0696 | −27.6060 | .5956 | −27.6008 |
| 24 | + 8.7476 | .7506 | + 8.7491 | +17.9574 | .9647 | +17.9610 |
| 25 | +12.8103 | .8125 | +12.8114 | −12.9513 | .9496 | −12.9504 |
| 34 | +32.6615 | .6572 | +32.6594 | − 5.2328 | .2319 | − 5.2324 |

PLATE II. CORRECTED COÖRDINATES.

| Star | x | | | y | | |
|---|---|---|---|---|---|---|
| | Direct | Reversed | Mean | Direct | Reversed | Mean |
| 1 | −40.4908 | .4943 | −40.4926 | + 3.4889 | .4898 | + 3.4894 |
| 5 | −25.4970 | .4946 | −25.4958 | + 5.4058 | .4078 | + 5.4068 |
| 16 | − 1.2398 | .2426 | − 1.2412 | +11.8862 | .8850 | +11.8856 |
| 17 | 0.0000 | .0000 | 0.0000 | 0.0000 | 0000 | 0.0000 |
| 19 | + 1.2919 | .2921 | + 1.2920 | − 4.3625 | .3578 | − 4.3602 |
| 20 | + 4.1637 | .1616 | + 4.1626 | + 3.0141 | .0150 | + 3.0146 |
| 22 | + 5.0421 | .0416 | + 5.0418 | −27.6042 | .5820 | −27.5931 |
| 24 | + 8.7696 | .7626 | + 8.7661 | +17.9536 | .9566 | +17.9551 |
| 25 | +12.7856 | .7874 | +12.7865 | −12.9528 | .9503 | −12.9516 |
| 34 | +32.6180 | .6229 | +32.6204 | − 5.2640 | .2659 | − 5.2650 |

PLATE III. CORRECTED COÖRDINATES.

| Star | x | | | y | | |
|---|---|---|---|---|---|---|
| | Direct | Reversed | Mean | Direct | Reversed | Mean |
| 1 | −40.4800 | .4922 | −40.4861 | + 3.4716 | .4707 | + 3.4712 |
| 5 | −25.4905 | .4970 | −25.4938 | + 5.3886 | .3934 | + 5.3910 |
| 16 | − 1.2500 | .2474 | − 1.2487 | +11.8950 | .8992 | +11.8971 |
| 17 | 0.0000 | .0000 | 0.0000 | 0.0000 | 0000 | 0.0000 |
| 19 | + 1.2926 | .3020 | + 1.2973 | − 4.3520 | .3499 | − 4.3510 |
| 20 | + 4.1552 | .1629 | + 4.1590 | + 3.0234 | .0224 | + 3.0229 |
| 22 | + 5.0613 | .0680 | + 5.0646 | −27.5747 | .5691 | −27.5719 |
| 24 | + 8.7542 | .7567 | + 8.7554 | +17.9784 | .9818 | +17.9801 |
| 25 | +12.7918 | .7946 | +12.7932 | −12.9244 | .9272 | −12.9258 |
| 34 | +32.6131 | .6224 | +32.6178 | − 5.2238 | .2282 | − 5.2260 |

The focal length of the Crossley reflector[1] is 17 feet, 6.1 inches, and the approximate scale value of the photographs is therefore 1 mm=38."65.

Multiplying the corrected $x$ and $y$ coördinates in Table III by 38."65 sec $\delta_0$ and 38."65 respectively (where $\delta_0$ is the declination of the central star), we obtain the quantities X sec $\delta_0$ and Y.

Coefficients for computing refraction are given in the following table:

TABLE IV.

REFRACTION COEFFICIENTS.

| Plate | $M_x$ | $N_x$ | $M_y$ | $N_y$ |
|---|---|---|---|---|
| 1 | + 0.000249 | − 0.000005 | + 0.000013 | + 0.000261 |
| 2 | + 250 | − 7 | + 21 | + 260 |
| 3 | + 367 | + 7 | − 135 | + 287 |

Four constants are necessary for the reduction of each plate. These constants are the corrections for scale value, orientation, and the two errors of the plate center. Put:

$p$ = the correction to the scale value, so that 1 mm at the center of the plate corresponds to 38."65 $(1+p)$;

$r$ = the orientation correction, or the sine of the angle through which the axes must be rotated, measured in the direction of decreasing position angles;

$k$ and $c$ = the number of seconds of arc through which the coördinate axes must be moved in the direction of decreasing right ascensions and declinations respectively.

We then have from each known star a pair of equations of the form:

$$pX + rY + k + n_x = 0$$
$$pY - rX + c + n_y = 0$$

In these equations $n_x$ and $n_y$ are computed as follows, putting:

$\Delta\alpha$, $\Delta\delta$ = excess of the right ascension and declination of any star over the corresponding coördinates of the central star.

Then

$n_x \sec \delta_0 = X \sec \delta_0$ *plus* corrections for transformation and refraction *minus* $\Delta\alpha$;

$n_y = Y$ *plus* corrections for transformation and refraction *minus* $\Delta\delta$.

The right ascensions and declinations for computing $\Delta\alpha$ and $\Delta\delta$ were taken from Jacoby's *second* reduction of the Rutherfurd *Pleiades* photographs, the results of which are now in course of publication. The adopted positions are given in Table V.

[1] KEELER, "The Crossley Reflector," ASTROPHYSICAL JOURNAL, **11**, 325, 1900.

TABLE V.

ADOPTED POSITIONS OF THE STARS.

| Star | α [illegible] | δ [illegible] |
|---|---|---|
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] |

It is not necessary to bring these positions up to the date of the plates, as precession, nutation, and aberration are taken into account by the constants $p$ and $r$. The right ascension and declination of the central star, $\alpha$ and $\delta$, used in determining the refraction were, however, brought up to [illegible] by precession.

These ten known stars furnish the following equations, in which the coefficients of $p$ and $r$ have been divided by [illegible] for convenience in making the solution.

RIGHT ASCENSIONS. DECLINATIONS.

Plate I

| Star | R.A. equation | Residual | Decl. equation | Residual |
|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | $-0''.13$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.48$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.25$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.12$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.01$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.15$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.57$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.50$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.54$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.31$ |

Plate II

| Star | R.A. equation | Residual | Decl. equation | Residual |
|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | $+0''.24$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.22$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.33$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.10$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.06$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.12$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.35$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.23$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $+.88$ |
| [illegible] | [illegible] | [illegible] | [illegible] | $-.45$ |

| | RIGHT ASCENSIONS. Plate III | | DECLINATIONS. Plate III | |
|---|---|---|---|---|
| 1 | $-15.6p+1.3r+k-0'.47=0$ | $-0'.78$ | $+1.3p+15.6r+c+8.86=0$ | $+0'.36$ |
| 5 | $-9.9p+2.1r+k+0.49=0$ | $-.29$ | $+2.1p+9.9r+c+5.13=0$ | $-.47$ |
| 16 | $-0.5p+4.6r+k+2.26=0$ | $+.11$ | $+4.6p+0.5r+c+0.59=0$ | $-.25$ |
| 17 | $0p+0r+k+0=0$ | $+.19$ | $0p\quad 0r+c+0=0$ | $-.53$ |
| 19 | $+0.5p-1.7r+k-0.96=0$ | $+.09$ | $-1.7p-0.5r+c+0.06=0$ | $-.19$ |
| 20 | $+1.6p+1.2r+k+0.89=0$ | $+.45$ | $+1.2p-1.6r+c-0.60=0$ | $-.32$ |
| 22 | $+2.0p-10.7r+k-5.47=0$ | $+.16$ | $-10.7p-2.0r+c-0.06=0$ | $+.54$ |
| 24 | $+3.4p+6.9r+k+3.83=0$ | $+.46$ | $+6.9p-3.4r+c-1.02=0$ | $+.12$ |
| 25 | $+5.0p-5.0r+k-2.56=0$ | $+.13$ | $-5.0p-4.9r+c-0.99=0$ | $+1.03$ |
| 34 | $+12.6p-2.0r+k-1.64=0$ | $-.56$ | $-2.0p-12.6r+c-6.24=0$ | $-.32$ |

Solving these equations by least squares the following results are obtained:

TABLE VI.

| Plate | $p$ | $r$ | Probable error of $p$ and $r$ | $k$ | $c$ | Probable error of $k$ and $c$ | $[vv]$ | Probable error of one equation of weight unity |
|---|---|---|---|---|---|---|---|---|
| 1 | −0.000619 | −0.004537 | ±0.000110 | +0′.0941 | +0′.1199 | ±0′.0963 | 3′.2647 | ±0′.3046 |
| 2 | −0.000175 | −0.005692 | ±0.000103 | +0.2451 | −0.0965 | ±0.0911 | 2.9222 | ±0.2882 |
| 3 | −0.000104 | −0.005104 | ±0.000119 | +0.1935 | −0.5254 | ±0.1043 | 3.8207 | ±0.3299 |

Collecting the residuals for comparison we have the second, third, and fourth columns of Table VII for the right ascensions and corresponding columns for the declinations.

TABLE VII.

| Star | Right ascensions: Plate I | II | III | Mean | Plate *minus* mean I | II | III | Declinations: Plate I | II | III | Mean | Plate *minus* mean I | II | III |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | −.74 | −.77 | −.78 | −.76 | +.02 | −.01 | −.02 | −.13 | +.24 | + .36 | +.16 | −.29 | +.08 | +.20 |
| 5 | −.15 | −.24 | −.29 | −.23 | +.08 | −.01 | −.06 | −.48 | −.22 | − .47 | −.39 | −.09 | +.17 | −.08 |
| 16 | +.19 | +.19 | +.11 | +.16 | +.03 | +.03 | −.05 | −.25 | −.33 | − .25 | −.28 | +.03 | −.05 | +.03 |
| 17 | +.09 | +.25 | +.19 | +.18 | −.09 | +.07 | +.01 | +.12 | −.10 | − .53 | −.17 | +.29 | +.07 | −.36 |
| 19 | −.17 | +.04 | +.09 | −.01 | −.16 | +.05 | +.10 | +.01 | −.06 | − .19 | −.08 | +.09 | +.02 | −.11 |
| 20 | +.76 | +.54 | +.45 | +.58 | +.18 | −.04 | −.13 | +.15 | −.12 | − .32 | −.10 | +.25 | −.02 | −.22 |
| 22 | −.47 | −.05 | +.16 | −.12 | −.35 | +.07 | +.28 | +.57 | +.35 | + .54 | +.49 | +.08 | −.14 | +.05 |
| 24 | +.28 | +.45 | +.46 | +.40 | −.12 | +.05 | +.06 | −.50 | −.23 | + .12 | −.20 | −.30 | −.03 | +.32 |
| 25 | +.14 | +.14 | +.13 | +.14 | .00 | .00 | −.01 | +.84 | +.88 | +1.03 | +.92 | −.08 | −.04 | +.11 |
| 34 | +.04 | −.51 | −.56 | −.34 | +.38 | −.17 | −.22 | −.31 | −.45 | − .32 | −.36 | +.05 | −.11 | +.04 |

These residuals are to be regarded as errors of the Lick plates on the assumption that the Rutherfurd results are absolutely correct, and that there has been no proper motion between [illegible], the date of the Rutherfurd photographs, and [illegible], the date of the Lick plates. Now, whatever may be the errors of the adopted Rutherfurd positions, and whatever may have been the effects of proper motion, the residuals ought to come out the same from each of the three Lick plates. Therefore the "mean residual" from the three plates has been computed for each star, and the divergences of the residuals (individual plate *minus* mean) set down in the last three columns of the table.

These latter residual numbers now give us a measure of the precision with which the Lick plates reproduce their own errors. Computing the sums of the squares of these numbers we find for:

| | |
|---|---|
| Plate 1 - - - - | [illegible] |
| 2 - - - - | [illegible] |
| 3 - - - - | [illegible] |

whereas the original residuals gave, according to Table VI:

| | |
|---|---|
| Plate 1 - - - - | [illegible] |
| 2 - - - - | [illegible] |
| 3 - - - - | [illegible] |

We see, therefore, that the agreement *inter se* of the Lick plates is much better than their accord with the Rutherfurd results; and this is favorable to the Crossley reflector. For the desideratum in an instrument is the ability to reproduce its own errors every time it is used, rather than that these errors should be extremely small. Nor should we ascribe the differences, "Lick *minus* Rutherfurd," to errors of the Lick instrument alone; since we have already pointed out that they are due in part to proper motion, and errors of the Rutherfurd catalogue.

Extended calculations have been made to ascertain whether the agreement between the Lick and Rutherfurd photographs can be improved by correcting the latter with proper motions derived from comparisons of the Königsberg and Yale heliometer measures. But these calculations have resulted unsuccessfully. Similarly, new least square solutions omitting two stars having large residuals failed to improve the result.

A computation of the probable error of the Lick plate measures from the data of Table II was also made. The method used was the same as that employed by Dr. Schlesinger.[1] It was found that the probable error of a final coördinate was ±0".0[illegible] for *X* and ±0".0[illegible] for *Y*. Dr. Schlesinger's corresponding values for the Rutherfurd *Praesepe* plates were respectively ±0".030 and ±0".0[illegible].

[1] *Annals N. Y. Acad. Sci.*, 10:272, or *Contrib. Observ. Columbia Univ.*, No. 15, p. [illegible].

Our final conclusion from all the evidence is therefore as follows: The Crossley plates have star images inferior to the Rutherfurd plates; but the distortion of the field is certainly small throughout a radius of about half a degree. Moreover, if there is a small distortion, it is very nearly constant in the same part of the field of different plates, even when taken at widely different hour angles.

MEASUREMENT AND REDUCTION OF A PHOTOGRAPH OF EROS MADE WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

This plate was made September 19, 1900, with six successive exposures, two of five seconds duration, two of ten seconds and two of thirty seconds. The measures and reductions, however, include only the first and third exposures, of five and ten seconds, respectively. A microscopic examination of the plate showed that the ten-second images of *Eros* are quite sufficiently well defined for measurement. Even the five-second images are measurable easily; while the thirty-second ones are somewhat over-exposed.

The following are the particulars relating to the two exposures measured:

| Exposures | | Middle of exposure | Duration |
|---|---|---|---|
| 1st | Pacific standard time | $14^h\ 45^m\ 17.5^s$ | $5^s$ |
| 3d | " " " | 14 45 45.0 | 10 |

Barometer and thermometer readings were not furnished, but the refraction will not be influenced appreciably, as the plate was taken very near the zenith, and values could be assumed from those employed in the *Pleiades* reductions.

Ten stars and *Eros* were measured. The following tables contain the measures and the corrected coördinates in a form similar to that used for the *Pleiades* plates:

FIRST IMAGE. $x$ MEASUREMENTS

| Star | $x$ direct | | | | $x$ reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ m or Scale *minus* Star: Mitchell | ½ m or Scale *minus* Star: Harpham | Mitchell *minus* Harpham | Line of Scale | ½ m or Scale *minus* Star: Mitchell | ½ m or Scale *minus* Star: Harpham | Mitchell *minus* Harpham |
| 1 | 113 | +0.3465 | +0.3618 | −0.″59 | 6 | +0.2835 | +0.2822 | +0.″05 |
| 2 | 111 | 0.2798 | 0.2840 | −0.16 | 8 | 0.3615 | 0.3570 | +0.17 |
| 3 | 97 | 0.4208 | 0.4285 | −0.30 | 22 | 0.2185 | 0.2110 | +0.29 |
| 4 | 96 | 0.3025 | 0.3050 | −0.10 | 23 | 0.3398 | 0.3315 | +0.32 |
| 5 | 69 | 0.0705 | 0.0650 | +0.21 | 50 | 0.5632 | 0.5700 | −0.26 |
| 5 | 69 | 0.0692 | 0.0590 | +0.39 | 50 | 0.5700 | 0.5700 | 0.00 |
| 6 | 66 | 0.4415 | 0.4418 | −0.01 | 53 | 0.1968 | 0.1985 | −0.07 |
| 7 | 52 | 0.0838 | 0.0568 | +1.04 | 67 | 0.5540 | 0.5628 | −0.34 |
| 8 | 41 | 0.8710 | 0.8548 | +0.63 | 77 | 0.7728 | 0.7690 | +0.15 |
| 9 | 39 | 0.6745 | 0.6592 | +0.59 | 80 | −0.0270 | −0.0182 | −0.34 |
| 10 | 33 | 0.3765 | 0.3792 | −0.10 | 86 | +0.2638 | +0.2595 | +0.17 |
| *Eros* | 72 | 0.6775 | 0.6708 | +0.26 | 47 | −0.0280 | −0.0280 | 00 |

FIRST IMAGE. *y* MEASUREMENTS

| Star | *y* direct | | | | *y* reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ m or Scale *minus* Star | | Mitchell *minus* Harpham | Line of Scale | ½ m or Scale *minus* Star | | Mitchell *minus* Harpham |
| | | Mitchell | Harpham | | | Mitchell | Harpham | |
| 1 | 35 | −0.0155 | −0.0200 | +0″.17 | 84 | +0.6540 | +0.6545 | −0″.02 |
| 2 | 78 | +0.0100 | +0.0175 | − .29 | 41 | 0.6258 | 0.6202 | + .22 |
| 3 | 46 | 0.8300 | 0.8352 | − .20 | 72 | 0.8102 | 0.8018 | + .32 |
| 4 | 76 | 0.8170 | 0.8315 | − .56 | 42 | 0.8082 | 0.8005 | + .30 |
| 5 | 58 | 0.7908 | 0.7968 | − .23 | 60 | 0.8478 | 0.8447 | + .12 |
| 5 | 58 | 0.8192 | 0.7955 | + .91 | 60 | 0.8498 | 0.8408 | + .35 |
| 6 | 31 | −0.0088 | −0.0085 | − .01 | 88 | 0.6478 | 0.6485 | − .03 |
| 7 | 31 | +0.4295 | +0.4222 | + .28 | 88 | 0.2412 | 0.2165 | + .95 |
| 8 | 57 | −0.0450 | −0.0455 | + .02 | 62 | 0.6768 | 0.6745 | + .09 |
| 9 | 67 | +0.2288 | +0.2350 | − .24 | 52 | 0.3928 | 0.3902 | + .10 |
| 10 | 61 | 0.1215 | 0.1045 | + .66 | 58 | 0.5158 | 0.5168 | − .04 |
| *Eros* | 67 | 0.6232 | 0.6178 | + .21 | 51 | 0.0100 | 0.0118 | − .07 |

THIRD IMAGE. *x* MEASUREMENTS

| Star | *x* direct | | | | *x* reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of Scale | ½ m or Scale *minus* Star | | Mitchell *minus* Harpham | Line of Scale | ½ m or Scale *minus* Star | | Mitchell *minus* Harpham |
| | | Mitchell | Harpham | | | Mitchell | Harpham | |
| 1 | 113 | +0.2832 | +0.2845 | −0″.05 | 6 | +0.3480 | +0.3572 | −0″.36 |
| 2 | 111 | 0.1950 | 0.2050 | − .39 | 8 | 0.4302 | 0.4295 | + .03 |
| 3 | 97 | 0.3418 | 0.3632 | − .83 | 22 | 0.2922 | 0.2852 | + .27 |
| 4 | 96 | 0.2198 | 0.2362 | − .63 | 23 | 0.4108 | 0.4118 | − .04 |
| 5 | 69 | −0.0075 | −0.0080 | + .02 | 50 | 0.6508 | 0.6490 | + .07 |
| 5 | 69 | −0.0055 | −0.0125 | + .27 | 50 | 0.6468 | 0.6495 | − .10 |
| 6 | 66 | +0.3645 | +0.3752 | − .41 | 53 | 9.2735 | 0.2735 | .00 |
| 7 | 52 | 0.0180 | 0.0070 | + .42 | 67 | 0.6620 | 0.6675 | − .21 |
| 8 | 41 | 0.7985 | 0.7750 | + .91 | 77 | 0.8410 | 0.8610 | − .77 |
| 9 | 39 | 0.6070 | 0.6240 | − .66 | 80 | 0.0532 | 0.0715 | − .71 |
| 10 | 33 | 0.3310 | 0.3080 | + .89 | 86 | 0.3470 | 0.3400 | + .27 |
| *Eros* | 72 | 0.5928 | 0.5900 | + .11 | 47 | 0.0498 | 0.0540 | − .16 |

THIRD IMAGE. *y* MEASUREMENTS.

| Star | *y* direct | | | | *y* reversed | | | |
|---|---|---|---|---|---|---|---|---|
| | Line of scale | ½ or Scale *minus* Star | | Mitchell *minus* Harpham | Line of scale | ½ m or Scale *minus* Star | | Mitchell *minus* Harpham |
| | | Mitchell | Harpham | | | Mitchell | Harpham | |
| 1 | 35 | +0.4665 | +0.4592 | +0″.28 | 84 | +0.1778 | +0.1730 | +0″.19 |
| 2 | 78 | 0.4940 | 0.4978 | − .15 | 41 | 0.1430 | 0.1405 | + .10 |
| 3 | 47 | 0.3092 | 0.3090 | + .01 | 72 | 0.3330 | 0.3232 | + .38 |
| 4 | 77 | 9.3010 | 0.3172 | − .63 | 42 | 0.3285 | 0.3225 | + .23 |
| 5 | 59 | 0.2602 | 0.2675 | − .28 | 60 | 0.3690 | 0.3692 | − .01 |
| 5 | 59 | 0.26 0 | 0.2690 | − .04 | 60 | 0.3692 | 0.3710 | − .07 |
| 6 | 31 | 0.4640 | 0.4705 | − .25 | 88 | 0.1658 | 0.1728 | − .27 |
| 7 | 31 | 0.9108 | 0.8895 | + .82 | 87 | 0.7958 | 0.7720 | + .92 |
| 8 | 57 | 0.4328 | 0.4318 | + .04 | 62 | 0.2052 | 0.2050 | + .01 |
| 9 | 67 | 0.7008 | 0.7105 | − .37 | 51 | 0.9165 | 0.9198 | − .13 |
| 10 | 61 | 0.6000 | 0.5878 | + .47 | 58 | 0.0495 | 0.0442 | + .20 |
| *Eros* | 68 | 0.1125 | 0.1125 | .00 | 51 | 0.5225 | 0.5265 | − .15 |

CORRECTED COÖRDINATES. FIRST IMAGE.

| Star | *x* | | | *y* | | |
|---|---|---|---|---|---|---|
| | Direct | Reversed | Mean | Direct | Reversed | Mean |
| 1 | −44.2809 | .2859 | −44.2833 | −23 8179 | .8086 | −23.8133 |
| 2 | −42.2102 | .2104 | −42.2103 | +19.2144 | .2239 | +19.2192 |
| 3 | −28.3534 | .3521 | −28.3528 | −11.9698 | .9616 | −11.9657 |
| 4 | −27.2337 | .2316 | −27.2327 | +18.0250 | .0422 | +18.0336 |
| 5 | 0.0000 | .0000 | 0.0000 | 0.0000 | .0000 | 0.0000 |
| 6 | + 2.6245 | .6314 | + 2.6280 | −27.8090 | .8017 | −27.8054 |
| 7 | +16.9983 | .9944 | + 16.9964 | −27.3751 | .3830 | −27.3790 |
| 8 | +27.2077 | .2061 | +27.2069 | − 1.8460 | .8310 | − 1.8385 |
| 9 | +29.4030 | .4124 | +29.4077 | + 8.4332 | .4541 | + 8.4436 |
| 10 | +35.6917 | .6944 | +35.6931 | + 2.3133 | .3287 | + 2.3210 |
| *Eros* | − 3.6064 | .5957 | − 3.6010 | + 8.8213 | .8342 | + 8.8278 |

CORRECTED COÖRDINATES. THIRD IMAGE.

| Star | *x* | | | *y* | | |
|---|---|---|---|---|---|---|
| | Direct | Reversed | Mean | Direct | Reversed | Mean |
| 1 | −44.2848 | .2967 | −44.2908 | −23.8038 | .8061 | −23.8050 |
| 2 | −42.2026 | .2104 | −42.2065 | +19.2301 | .2290 | +19.2296 |
| 3 | −28.3555 | .3580 | −28.3573 | −11.9600 | .9599 | −11.9600 |
| 4 | −27.2323 | .2367 | −27.2345 | +18.0439 | .0449 | +18.0444 |
| 5 | 0.0000 | .0000 | 0.0000 | 0.0000 | .0000 | 0.0000 |
| 6 | + 2.6170 | .6266 | + 2.6243 | −27.7996 | .7991 | −27.7994 |
| 7 | +16.9818 | 17.0200 | +17.0009 | −27.3673 | .4130 | −27.3902 |
| 8 | +27.2096 | .2055 | +27.2076 | − 1.8342 | .8367 | − 1.8354 |
| 9 | +29.3801 | .4166 | +29.3984 | + 8.4404 | .4537 | + 8.4471 |
| 10 | +35.6758 | .6955 | +35.6857 | + 2.3276 | .3220 | + 2.3248 |
| *Eros* | − 3.5980 | .5965 | − 3.5972 | + 8.8483 | .8469 | + 8.8476 |

The refraction coefficients were found to be as follows:

| $M_x$ | $N_x$ | $M_y$ | $N_y$ |
|---|---|---|---|
| +0.000244 | −0.000009 | +0.000004 | +0.000244 |

The positions of the ten stars were taken from the *A. G. C. Catalogue*, and are:

| No. | A. G. C. No. | $\alpha$ 1875.0 | | | $\delta$ 1875.0 | | |
|---|---|---|---|---|---|---|---|
| 1 | 2318 | $2^h$ | $35^m$ | $24^s.50$ | 40° | 7′ | 23″.4 |
| 2 | 2323 | | 35 | 30.24 | | 35 | 10.1 |
| 3 | 2333 | | 36 | 18.27 | | 15 | 8.7 |
| 4 | 2334 | | 36 | 21.30 | | 34 | 29.4 |
| 5 | 2351 | | 37 | 53.75 | | 22 | 55.3 |
| 6 | 2355 | | 38 | 3.06 | | 5 | 2.1 |
| 7 | 2366 | | 38 | 51.40 | | 5 | 17.5 |
| 8 | 2379 | | 39 | 25.88 | | 21 | 46.5 |
| 9 | 2380 | | 39 | 33.42 | | 28 | 24.2 |
| 10 | 2386 | | 39 | 54.66 | | 24 | 27.5 |

The following equations were formed from the above star-places and plate measures, using No. 3 as the central star:[1]

| Star | First Image. Right Ascensions | Residual | First Image. Declinations | Residual |
|---|---|---|---|---|
| 1 | [illegible] | [illegible] | [illegible] + 5″.31 = 0 | + 0″.91 |
| 2 | [illegible] | [illegible] | [illegible] + 2.04 = 0 | − 0.42 |
| 3 | [illegible] | [illegible] | [illegible] + 1.50 = 0 | − 0.69 |
| 4 | [illegible] | [illegible] | [illegible] 0.75 = 0 | − 0.59 |
| 5 | [illegible] | [illegible] | [illegible] 0 = 0 | + 1.37 |
| 6 | [illegible] | [illegible] | [illegible] − 1.75 = 0 | − 0.79 |
| 7 | [illegible] | [illegible] | [illegible] − 1.54 = 0 | + 1.12 |
| 8 | [illegible] | [illegible] | [illegible] − 4.55 = 0 | − 0.06 |
| 9 | [illegible] | [illegible] | [illegible] − 5.14 = 0 | − 0.11 |
| 10 | [illegible] | [illegible] | [illegible] − 0.30 = 0 | − 0.79 |

[1] [illegible] "central star" [illegible] necessary; in fact, theory [illegible] rather than to a star [illegible] introduced in the present [illegible] "central star."

| | Third Image. Right Ascensions | | Third Image. Declinations | |
|---|---|---|---|---|
| 1 | $-17.1p - 9.2r + k - 1''.05 = 0$ | $+1''.58$ | $-9.2p + 17.1r + c + 5''.63 = 0$ | $+0''.96$ |
| 2 | $-16.3p + 7.4r + k + 3.10 = 0$ | $+0.62$ | $+7.4p + 16.3r + c + 3.04 = 0$ | $-0.33$ |
| 3 | $-11.0p - 4.6r + k - 3.23 = 0$ | $-1.64$ | $-4.6p + 11.0r + c + 1.78 = 0$ | $-0.70$ |
| 4 | $-10.5p + 7.0r + k + 0.43 = 0$ | $-1.56$ | $+7.0p + 10.5r + c + 1.17 = 0$ | $-0.43$ |
| 5 | $0p \quad 0r + k \quad 0 = 0$ | $+0.85$ | $0p \quad 0r + c \quad 0 = 0$ | $+1.23$ |
| 6 | $+1.0p - 10.7r + k - 5.39 = 0$ | $-1.15$ | $-10.7p - 1.0r + c - 1.52 = 0$ | $-0.66$ |
| 7 | $+6.6p - 10.6r + k - 4.32 = 0$ | $+0.25$ | $-10.6p - 6.6r + c - 1.97 = 0$ | $+0.64$ |
| 8 | $+10.5p - 0.7r + k - 1.14 = 0$ | $+0.59$ | $-0.7p - 10.5r + c - 4.43 = 0$ | $+0.03$ |
| 9 | $+11.4p + 3.3r + k - 0.76 = 0$ | $-0.21$ | $+3.3p - 11.4r + c - 5.01 = 0$ | $-0.02$ |
| 10 | $+13.8p + 0.9r + k - 0.72 = 0$ | $+0.72$ | $+0.9p - 13.8r + c - 6.25 = 0$ | $-0.66$ |

A solution of these equations gave the following results for the unknowns :–

| Image | $p$ | $r$ | Probable error of $p$ and $r$ | $k$ | $c$ | Probable error of $k$ and $c$ | $[v\,v]$ | Probable error of 1 equation of weight unity |
|---|---|---|---|---|---|---|---|---|
| First.. | +0.000660 | −0.003020 | ±0.000172 | $+0''.7651$ | $+1''.3748$ | $\pm 0''.2224$ | 17.3947 | $\pm 1''.0427$ |
| Third. | +0.000631 | −0.003113 | ±0.000163 | +0.8458 | +1.2257 | ±0.2101 | 15.5225 | ±0.7033 |

The large probable errors, as compared with the *Pleiades* plates, are due in part to the smaller precision of the adopted star places. The *A. G. C. Catalogue* must, of course, be expected to furnish much less accurate relative positions than are obtained from a purely differential micrometric catalogue, such as that derived from the Rutherfurd *Pleiades* photographs. But some of the increase in the residuals is due to the star-images on the *Eros* plate. These are not so good as the Lick *Pleiades* images,[1] the probable errors of a final coördinate $x$ and $y$ being $\pm 0''.11$ and $\pm 0''.07$, whereas we found for the *Pleiades* only $\pm 0''.07$ and $\pm 0''.04$.

With the values of $p$, $r$, $k$, and $c$ obtained above, the place of *Eros* was computed from the measured coördinates.[2]

The results are, for the equinox of 1875.0:

| Place of *Eros*, 1875.0 | | | | | | |
|---|---|---|---|---|---|---|
| Image | $\alpha$ | | | $\delta$ | | |
| First ....... | 39° | 25′ | $22''.74$ | 40° | 28′ | $37''.71$ |
| Third ...... | | | 23.02 | | | 38.32 |
| Mean .... | 39 | 25 | 22.88 | 40 | 28 | 38.02 |

[1] *Eros* plates taken October 6–12 and recently received, appear to show images considerably better than the plates discussed in this paper.

[2] See Dr. Schlesinger's paper already quoted. *Annals N. Y. Acad. Sci.*, **10**, 246, or *Contrib. Obsy. Col. Univ.*, No. 15.

This position of *Eros* was brought up to 1900.0 by precession, and corrected with the usual "reduction to apparent place." A further correction for parallax was then applied, using for the planet's distance a value interpolated from the Ephemeris of *Eros* published in the Berlin *Jahrbuch* for 1902. Parallax corrections were also computed for Barnard's observations, made with the 40-inch refractor of the Yerkes Observatory[1] on September 19, which was also the date of the Lick plate. These observations were further corrected by the addition of $+0^s.07$ and $+1''.30$ to Barnard's right ascension and declination, which should reduce the place of Barnard's comparison star to the mean system of the ten stars upon which the Lick position is based. The numerical values of these corrections are simply $k$ sec. $\delta$ and $c$ from the least squares computation of the Lick plate.

The observed positions, thus corrected for parallax, were compared with places interpolated from the Berlin *Jahrbuch* Ephemeris of *Eros*, taking account of the aberration time. The Berlin *Jahrbuch* Ephemeris was also compared with the Ephemeris published by the Paris *Eros* committee in their Circular No. 3, dated August 17, 1900. This Paris Ephemeris is corrected with the results of observations made at Paris August 4 and 7, 1900. The following are, then, the corrections required by the Berlin *Jahrbuch* Ephemeris for the date September 19.

| Authority | $\alpha$ | $\delta$ |
|---|---|---|
| Lick Plate | $+13^s.24$ | $+31''.8$ |
| Barnard | $+13.23$ | $+32.1$ |
| Paris Ephemeris | $+13.64$ | $+30.4$ |

The accord between the Lick plate and Barnard's observation is satisfactory, and the Paris Ephemeris also represents the observations very well.

[1] *Astronomical Journal*, No. 484

# ON THE PRODUCTION OF A LINE SPECTRUM BY ANOMALOUS DISPERSION, AND ITS APPLICATION TO THE "FLASH SPECTRUM."

By R. W. Wood.

In a communication published in the *Proceedings of the Royal Academy of Sciences*, Amsterdam,[1] W. H. Julius makes the very brilliant suggestion that the "flash spectrum" seen immediately at totality may be due to photosphere light abnormally refracted in the atmosphere of metallic vapors surrounding the Sun: in other words, the light of the flash spectrum does not come from the reversing layer at all, but from the photosphere. The author shows that the light which will be thus abnormally refracted will be of wave-lengths almost identical with the wave-lengths which the metallic vapors are themselves capable of radiating. This beautiful theory not only explains the apparent shallowness of the reversing layer, a thing that has always puzzled astrophysicists, but it accounts for the extraordinary brilliancy of the lines.

I have succeeded in producing such a flash spectrum by an arrangement in which I have endeavored to imitate as closely as possible the conditions supposed to exist at the surface of the Sun: in brief, I have obtained a spectrum of bright lines, with light from a source showing a continuous spectrum, by means of anomalous dispersion in an incandescent metallic vapor.

The theory of Julius supposes the Sun to be surrounded by an atmosphere of metallic vapors, the density and refractive index of which decrease with increasing distance from the surface. In this atmosphere the rays of light coming from the photosphere will move in curved paths similar to those of rays in our own atmosphere.

The refractive index is, however, very small except for wave-lengths very near those which are absorbed by the vapor, consequently the light most strongly refracted, if it could be sorted

[1] See also Astrophysical Journal, 12, 185.

[illegible] This shows that [illegible] may account for the [illegible] attributed to the reversing layer. [illegible] of the solar atmosphere. [illegible] photosphere has been hidden by the Moon.

For the reproduction of the phenomenon in the laboratory it is necessary to form an atmosphere of metallic vapor in which the density changes rapidly from layer to layer. This I accomplished by allowing the flame of a Bunsen burner fed with metallic sodium to play against the under side of a white plaster plate. On looking along the surface of the plate it was seen that a dark space existed between the flame and the cold surface, resembling somewhat the dark space surrounding the cathode of a Crookes' tube. It seemed highly probable that, inasmuch as the temperature of the flame was lowered to such a degree by contact with the plate, the density of the sodium vapor would increase very rapidly from the surface of the plate downward. The change may of course be abrupt instead of progressive, though I am inclined to favor the latter supposition. In either case the action will be practically the same, the case being similar to the transition from a curved ray to a broken line ray, as the change of the index of the medium becomes less gradual. The under surface of the plaster plate being covered with a non-homogeneous layer of sodium vapor, a spot at the edge of the flame was illuminated with sunlight concentrated by a large mirror. This spot radiated white light in every direction and corresponded to the incandescent photosphere of the Sun (Fig. 1). A telescope provided with an objective direct vision prism was directed toward the white spot and moved into such a position that, owing to the reduction in the width of the source of light by foreshortening, the Fraunhofer lines appeared in the spectrum. This represented the stage of an eclipse when only the thin crescent of the Sun is visible. The sodium flame appeared superposed on the spectrum, of course.

On moving the spectroscope until it was well inside of the plane of the illuminated surface and feeding the flame with fresh sodium, the solar spectrum vanished and there suddenly blazed out two narrow bright yellow lines, almost exactly in the place of the sodium lines, as is shown in Fig. 2, in which the inverted

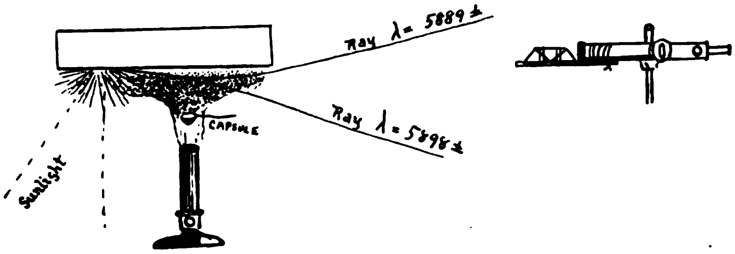



FIG. 1.

sodium flame appears on the continuous spectrum. Cutting off the sunlight with a screen caused the instant disappearance of the lines.

Repeating the experiment I found that the bright lines came into view on the sides of the sodium lines toward the blue, that is to say, it is light for which the medium has an abnormally low refractive index that is bent around the edge of the plate and enters the instrument. This is precisely what we should expect, for sodium vapor has a refractive index of less than 1 for waves slightly shorter than $D_1$ and $D_2$, as was shown by Julius in his paper. The rays then will be concave upward in a medium in which the refractive index varies

FIG. 2.
Flash Spectrum of Sodium produced by Anomalous Dispersion.

as I have supposed it to vary in the present case. If the sodium vapor is very dense we see only a single bright line bordering $D_2$, owing to the complete absorption of the light between the lines.

I next instituted a search for the light of a wave-length slightly greater than that of the sodium lines. For these waves the vapor has a refractive index greater than 1, consequently the rays will be concave downward in the layer of vapor. If we move our prismatic telescope down in a search for these rays the solar spectrum will appear and drown out everything, but if we set up a screen (shown in Fig. 3) in such a position as to

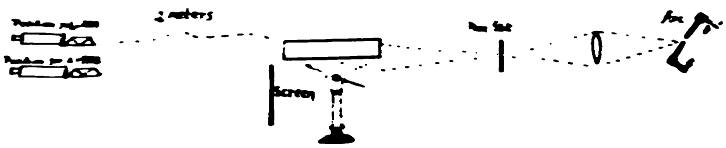


FIG. 3.

just cut off the light from the illuminated spot, and feed the flame with sodium, we shall presently see bright lines appear on the side of the sodium lines towards the red. In this case when the vapor is dense we get only a single line bordering $D_1$. The paths of these rays is indicated (on an exaggerated scale) in Fig. 1. The arrangement described is inconvenient in many ways to work with, and I accordingly modified it in the following way.

The light of an arc lamp is focused on a horizontal slit, and a flat metal plate supported so that the plane in which its under surface lies coincides with the plane of the slit. The plate should be an inch or so thick, with a fairly level surface. At a distance of about two meters a telescope provided with a prism (direct vision if possible), arranged so as to give a vertical spectrum, is placed at such a height that the prism barely catches the rays coming from the slit and grazing the surface of the plate, Fig. 3. On looking into the telescope we see a bright continuous spectrum, and the telescope is to be raised until this becomes quite faint. The Bunsen burner beneath the plate is now to be lighted and a

bit of sodium, in a small iron capsule, introduced into the center of the flame. The results obtained are practically identical with those which have been described. The flash spectrum of potassium has been obtained in a similar manner, consisting of lines in the extreme red from one to three in number according to the density of the vapor and position of the telescope. Fair results have also been obtained with thallium.

I am now arranging apparatus by which I hope to obtain similar flash spectra by the dispersion of vapors exhibiting more complicated absorption spectra than sodium. If these experiments are successful much can be learned by comparing the flash spectra with the emission spectra. If it be found that certain lines are absent in the flash which are present in the emission spectra, interesting comparisons can be made with photographs of the actual flash-spectrum of the Sun. Julius applies the anomalous dispersion theory to the prominences as well as to the reversing layer. I have succeeded in producing such a phenomenon with sodium vapor, a wavering flame of intense brilliancy perfectly sharp when seen through a prism of high dispersion, and yet shining wholly by refracted light coming from the crater of an arc light. I am also engaged in making accurate determinations of the dispersion of metallic vapors by means of a metal prism of 45 degrees furnished with mica windows. The prism is filled with hydrogen and the metal — say sodium — vaporized in this atmosphere by the application of heat. The results obtained in this way are far superior to those yielded by prismatic flames. The angle of the prism is accurately known, and it is filled with non-luminous sodium vapor of uniform density and under known conditions of temperature and pressure. With this I have obtained much greater curvature of the spectrum in the vicinity of the absorption lines than that figured by Julius, and the spectrum is perfectly steady, instead of fluttering, as is the case when the deviation is effected by means of a sodium flame of prismatic form. The work along these lines will be reported in a subsequent paper.

UNIVERSITY OF WISCONSIN,
December 22, 1900.

# THE NATURE OF THE SOLAR CORONA.

By R. W. Wood.

I PROPOSE in the following paper to discuss certain theories of the solar corona, and present the results of some recent experiments which I feel may have some bearing on the subject.

The most generally accepted theory of the corona attributes its continuous spectrum to light emitted in virtue of high temperature due to solar radiation,[1] with which is mixed a small amount of reflected sunlight producing traces of radial polarization. The recent work of Abbot, of the Smithsonian Institution, at Wadesboro, showing a cold corona, has set people to thinking about the old electrical theory, according to which the corona is regarded as a phenomenon akin to the aurora, and the light in Geissler tubes.

Certain of the experiments to be described later on indicate, it seems to me, that the absence of radiant heat offers no difficulty to the solid particle theory, being in fact precisely what we should expect. On the other hand, the presence of polarized light in the spectrum, and there is a good deal of it, judging from the strength of the Savart bands which I observed at the eclipse of last May, must be taken almost as proof positive that minute solid particles are present. Under no conditions has an electrically excited gas been found to emit polarized light, at least not to my knowledge. One or two cases in which the phenomenon was supposed to have been found, were shown to be spurious, the polarization resulting from reflection from the inner walls of the tube.

Going back now to the other theory: If the large amount of polarized light is reflected sunlight, why are not the Fraunhofer lines seen in the spectrum? I have called attention in a

[1] See paper by Scheiner and reply by Sir William Huggins in recent numbers of the ASTROPHYSICAL JOURNAL.

previous paper[1] to the fact that this may be because the oblique prism faces refuse transmission to the polarized light, which is the light in which the wave-lengths corresponding to the dark lines are absent. The certain knowledge that the lines are present or absent will be of great aid in formulating a satisfactory theory of the coronal light, and I trust that the long Sumatra eclipse will yield evidence in this direction. In the paper alluded to I suggested the use of a Nicol prism before the slit in such a position as to transmit the polarized radiations, the slit being set tangential to the Sun's edge, in which position the light entering the instrument is polarized in such a plane as to be transmitted by the prism faces. The light showing the dark lines would then be transmitted with undiminished intensity, while the emitted or non-polarized light would be reduced in intensity by one half. The great change in the ratio might easily be sufficient to bring out the dark lines. I feel sure that the experiment is worth trying at some future eclipse, for I have carried it out successfully in the laboratory with an artificial corona. It was found that a gas flame in a strong beam of sunlight shone with a pure bluish-white light, due to the reflection or rather scattering of the sunlight by the minute carbon particles.[2] A photograph of the flame with a spot illuminated by powerful convergent beams of sunlight is reproduced. It furnishes a beautiful proof of the existence of solid particles in the flame (Fig. 1.) The flame thus illuminated showed the Fraunhofer lines distinctly, but by reducing the intensity of the sunlight a point was reached at which they disappeared, and the spectrum appeared continuous. The light scattered by the flame was found to be *completely* plane-polarized in certain directions, giving us just the required conditions, namely particles emitting a continuous spectrum, and scattering a polarized solar spectrum. In front of the slit of the spectroscope a Nicol was arranged in

[1] "The Problem of the Daylight Observation of the Corona," ASTROPHYSICAL JOURNAL, November 1900.

[2] The reflection of light by flames I have since found has been observed before by Mr. Burch and also by Sir George Stokes.

such a manner that it could be drawn into and out of position by a cord. The Fraunhofer lines could be made to appear by sliding the Nicol in front of the slit, and disappear by drawing it away. While it does not by any means follow that the use of a Nicol on the actual corona will bring out the lines, the experiment seems to be well worth trying, as it would furnish further information regarding the relative intensity of the emitted and reflected light. Another interesting point is that the minute particles in the flame do not scatter the longer waves, the flame reflecting practically no red or orange light. Thus the Fraunhofer lines can only be traced up to about the D lines. By gradually reducing the intensity of the sunlight they disappear first in the yellow, then in the green, blue, and violet in succession. This indicates that our chances of detecting the lines in the spectrum of the corona will be greatest in the actinic part of the spectrum.

FIG. 1.

The preponderance of the shorter wave-lengths in the light scattered by the carbon particles in the flame can be shown in the following way. The light from the crater of an arc lamp is focused on a candle flame by means of a lens. We thus get a very small spot in the center of the flame illuminated with a powerful light rich in waves of all lengths. This candle flame is then photographed with an objective prism, on a plate made sensitive to the entire spectrum. We find that the spectrum of the flame has a bright line running through its center — the spectrum of the illuminated spot. This line, however, can only be traced as far as the yellow; there is absolutely no trace of it in the red and scarcely a trace in the orange. In the green it is many times brighter than the background, while in the violet it stands out strong on a black background, showing that the flame gives out very little violet light except in the spot where the image of the arc falls.

In making this negative a color filter was used during a part of the exposure to cut down the action of the blue and violet, while a record of the red end of the spectrum was being secured. The apparent absorption band between the yellow and green is due to the fact the plates are not absolutely orthochromatic, and has no bearing on the subject. Two of these photographs are reproduced in Fig. 2, a short and a long exposure.

FIG. 2.

This inability of the particles in the flame to scatter or diffract the longer waves may explain the absence of heat radiations in the corona's spectrum, for we have only to assume that the particles are small in comparison to the wave-length. A determination of the distribution of energy in the corona spectrum would be useful in this connection. It would seem as if the red ought to be relatively feebler than in the solar spectrum.

This explanation of the absence of radiant heat does not apply to the emission spectrum due to the incandescence of the particles, but it seems to me to be very probable that, if the incandescence is due to solar radiation alone, the scattered or reflected light will be greatly in excess of the emitted light. The temperature of a body near the Sun has been recently calculated by Scheiner[1] and found to be about 4000 degrees at a distance of one half of the solar radius from the Sun's surface. Doubtless a body at this temperature even if very minute will emit a powerful light, but the fact appears to have been overlooked that a body placed in a radiation intense enough to produce this high temperature will—if it be large—reflect, or if small diffract an amount of light which will be commensurate with the amount emitted. It occurred to me that any determinations of the ratio of emitted to scattered light of a body brought

[1] ASTROPHYSICAL JOURNAL, July 1900.

to incandescence by solar radiation would perhaps throw some light on the problem. The question could, it seems to me, be at once settled if we had a burning glass capable of giving a temperature of 4000 degrees at its focus, as this is the upper limit placed by Scheiner for a small black spherical particle at a distance of less than half a radius from the surface of the Sun. The corona can, however, be followed to a distance of from two to three diameters, consequently we are not obliged to limit ourselves to particles in such proximity to the radiating surface. Using the values for $W$ and $\sigma$ adopted by Scheiner, which will be more apt to give too large than too small values, I have calculated the temperatures and intensities of radiation at various distances from the Sun. These are given in the following table. The distances from the surface are given in the first column, the intensity of the radiation (the intensity at the Earth's distance being unity) in the second, and the corresponding temperature of a black spherical body in the third.

| Distance | Intensity of Radiation | Absolute Temp. |
|---|---|---|
| ½ Radius | 23000 | 4160° |
| 1 Radius | 11600 | 3500 |
| 1 Diameter | 5160 | 2870 |
| 2 Diameters | 1860 | 2200 |
| 3 Diameters | 923 | 1860 |

I had at my disposal a silver concave mirror of short focus, the area of which was 803 sq. cm, which gave an image of the Sun 9 mm in diameter, or of area 0.63 sq. cm. The ratio of these areas is the measure of the intensity of the radiation at the focus, and is 1270, corresponding to that at a distance of between two and three solar diameters. The sunlight, however, passed through the window and through the glass of the mirror, consequently a considerable amount of energy is lost by reflection and absorption. That due to the former can be calculated by Fresnel's formula, and is found to be, for four glass surfaces, at normal incidence about 0.2 of the whole. This will bring our intensity down to a little less than 1000, which, neglecting atmospheric absorption, corresponds to the intensity at a distance of three solar diameters.

The temperature at the focus was found to be 1100°, copper being melted with ease. The mirror was provided with a diaphragm, as shown in Fig. 3, by which the intensity of the radiation at the focus could be controlled. One side of the square aperture was graduated to centimeters, and the area of the exposed portion of the mirror could be at once determined. Determinations of the temperature at the focus, with different apertures, were made in the following way. A number of alloys and metals fusing at temperatures varying from 185° C. to 1100° C. were rolled into sheets of foil of uniform thickness, from which disks, a trifle smaller than the image of the Sun at the focus of the mirror, were cut. Each disk was supported by a very thin strip of the same metal, to lessen as much as possible losses by conduction. The disks were smoked in a candle flame and the number of square centimeters of mirror necessary to fuse each determined. The radiating surface was of course double the area of the surface receiving the radiation, while in the case of a spherical particle it is four times as large; consequently the temperatures taken by the disks were much higher than we should have in the case of solid particles. This was easily shown by a temperature calibration of the mirror made with a mercury thermometer. For example, 100 square centimeters of mirror surface were sufficient to melt the zinc disk (415°), while 200 sq. cm radiating against the thermometer bulb, produced a temperature of only 270°.

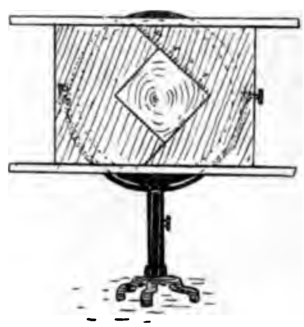


FIG. 3.

Though the temperature at the focus was not as high as would be desirable for a conclusive test, it seemed that some notion of the ratio between the emitted and diffracted light might be obtained by bringing bodies brought to full incandescence by other means into the focus. I have already mentioned the experiment of illuminating a flame with the concentrated beam. Here we have small black particles at a temperature of

say 2000° (based on the fact that fine platinum wire fuses in the flame). It was found that in the case of a candle flame the radiation from 25 sq. cm of the mirror's surface caused the Fraunhofer lines to appear in the spectrum of the flame. This means that solar radiation only forty times as intense as the radiation at the Earth's distance will be scattered to such an extent by small particles, themselves radiating in virtue of a temperature of at least 1200, and possibly 2000 degrees, that the diffracted light is at strong as the emitted. This last statement is based on Hastings' statement in his eclipse report that the Fraunhofer lines remain visible until the sunlight is diluted with rather more than an equal amount of continuous spectrum light.

In the case of a flame of illuminating gas a radiation of intensity 230 was necessary to bring out the lines in the spectrum. This seemed somewhat surprising at first sight, for the temperature cannot be very different in the two cases, and the color of the flame was about the same as that of the candle flame. I found by a little experimenting that the probable cause was to be found in the smaller size of the particles than in their higher temperature. A gas flame from a large aperture, say 3 mm in diameter would show the lines with an illumination from 121 sq. cm of mirror, while a flame from an aperture of say 0.5 mm, which could not be distinguished in appearance from the other flame, would not show the lines even when illuminated with the full radiation of the entire mirror (over 800 sq. cm). I can only explain this anomaly on the supposition that in one condition the carbon particles are set free in a much more finely divided state than in the other.

In Fig. 4 are reproduced photographs of a gas and candle flame close together, both traversed by the concentrated beam from the entire mirror. The gas flame is burning from a small aperture, and shows no trace of the illuminating beam, while the candle flame is intensely illuminated. The two prints are given of the negative to enable a better idea of the relative intensity of the candle flame and the spot of sunshine on it to be formed.

Summing up the results thus far it seems safe to assert that a radiation 40 times as strong as the radiation at the Earth's distance is sufficient to show the Fraunhofer lines in the spectrum of matter incandescent at say 1500° (to take an intermediate value). Now at the distance from the Sun taken by Scheiner, where the temperature will be 4000°, we have a radiation 23,000 times as intense as the radiation at the Earth's distance. The scattered light will increase directly in proportion to the intensity of the radiation, that is it will be 575 times as intense as in the case of a flame illuminated by light 40 times as bright as ordinary sunlight at the Earth's surface. Whether the intensity of the *emitted* light will increase in like proportion is the question on which the appearance of the Fraunhofer lines depends. If we could raise small particles to a temperature of 4000° and then illuminate them with sunlight of sufficient intensity to bring out the lines we should have a direct answer to the question, but this I have been unable to do.

FIG. 4.

The temperature of the arc has been estimated at about 4000°, but the intensity of the radiation at the focus of my mirror is insufficient to accomplish the desired result in this case. I have experimented with various sources of light and then by comparing their intensities have been able to arrive at a fairly definite conclusion. The results of these experiments are given in the following table. The intensity of the solar radiation necessary to cause the appearance of the dark lines in the spectrum of the illuminated spot on the incandescent source, is expressed as before in terms of the normal solar radiation at the Earth's surface.

| Source of Light | Intensity of Radiation |
|---|---|
| Welsbach mantle | 8 |
| Candle flame | 40 |
| Platinum at melting point | 32 |
| Calcium light | 100 |
| Carbon rod in oxy-hyd. flame | 100 |
| Illuminating gas flam | 230 |

The Welsbach mantle is at a temperature of probably 1500° more or less: being a white substance it reflects strongly and a surprisingly feeble solar radiation is sufficient to impress the solar spectrum on it. Probably the carbon particles in the gas flame are at a temperature nearly as high, but it does not seem to me that we are justified in comparing the light diffused by a solid with the light diffracted by small particles. How much stress can be laid on the results obtained with sources of light in the form of compact solid masses I do not feel prepared to say. It is interesting to see that the same intensity is required for a white lime cylinder and a black carbon rod in the oxy-hydrogen flame. The lime cylinder is many times as bright as the carbon rod, but being white it reflects strongly. I have made photometric determinations which are recorded in my notes as follows. The arc light is 6 times as intense as a lime cylinder illuminated with the full aperture of the mirror. The arc is 36 times as bright as a carbon rod illuminated in the same way. It would appear from this that a radiation 36 times as intense as that produced by my mirror would be necessary to cause the arc to show the Fraunhofer lines, whereas a white substance at the same temperature would show them under an illumination 6 times that produced by the mirror. At the distance from the Sun's surface taken by Scheiner in his calculations the radiation would be some 23 times stronger than that of the mirror. A black substance would then reflect and emit about the same amount, while a white substance would reflect much more than it emitted. Whether this will hold for very small particles or not I do not know. It is almost impossible to draw any conclusions regarding a body at such a high temperature as 4000°, but if we retreat to a greater distance from the Sun, say to a distance of 2 diameters, where the upper limit that we can assign to the temperature is only a little over 2000°, it is easier to apply the experimental data. Here the intensity of the solar radiation will be only about double that at the focus of the mirror, and practically everything emitting light at this temperature (melting platinum, carbon and lime in the oxy-hydrogen flame) showed

the Fraunhofer lines when illuminated with very much less than the full aperture of the mirror. The exception is the gas flame from a small aperture, but I am inclined to regard this as a case of particles too small to scatter any appreciable amount of light, rather than a direct temperature effect.

I am well aware that the results and arguments set forth in this paper are open to criticism from every side. There is probably no field of research in which there are so many pitfalls as that of radiation and emission. I feel that much more satissactory conclusions could be reached with a mirror which would give an intensity at the focus of say 10 times that with which I have worked. In the present case too much dangerous exterpolation is required in drawing conclusions. Some of the results seem to me to be interesting and if they prove suggestive in any way to others better qualified than myself to discuss the problem, I shall feel well repaid for the work.

The absence of radiant heat in the spectrum of the corona indicates apparently that the amount of light emitted in virtue of incandescence must be small. It has also been supposed to indicate an absence of reflected sunlight, but this we see can be explained by the small size of the particles. My present notion is that the path of least logical resistance is to assume that in the corona we have very minute particles shining principally by diffracted sunlight, and moving towards or away from the Sun with sufficient velocity to preclude the appearance of the Fraunhofer lines in the spectrum by Doppler's principle applied to the line of sight component of velocity.

The breadth of the "1474" corona line in Professor Campbell's photograph indicates the strong probability of internal motion in the corona, and I see no reason why this motion cannot be postulated for the diffracting particles as well as for the incandescent coronium vapor.

Much light can be thrown on the nature of the corona by a more complete study of its polarization. The Sumatra eclipse, on account of its long duration, will furnish exceptional opportunities for work of this sort. For preliminary work with

apparatus arranged for the study of the polarization of the corona, I believe that an artificial corona that I have recently devised will be found most useful. It resembles the real corona in a most striking manner, and is polarized in the same way. I have published a brief account of it in *Science*.

A rectangular glass tank about a foot square on the front and five or six inches wide, and a six-candle-power incandescent lamp are all that are necessary. The dimensions of the tank are not of much importance, a small aquarium being admirably adapted for the purpose. The tank should be nearly filled with clean water, and a spoonful or two (the right amount determined by experiment) of an alcoholic solution of mastic should be added. The mastic is at once thrown down as an exceedingly fine precipitate, giving the water a milky appearance. The wires leading to the lamp should be passed through a short glass tube, and the lamp fastened at a right angle to the end of the tube with sealing wax, taking care to make a tight joint, to prevent the water from entering the tube. Five or six strips of tin foil are now fastened with shellac along the sides of the lamp, leaving a space of from ½ to 1 mm between them. The strips should be of about the same width as the clear spaces. They are to be mounted in two groups on opposite sides of the lamp, and the rays passing between them produce the polar streamers. The proper number, width, and distribution of the strips necessary to produce the most realistic effect can be easily determined by experiment. A circular disk of metal a trifle larger than the lamp should be fastened to the tip of the lamp with sealing wax, or any soft, water-resisting cement; this cuts off the direct light of the lamp and represents the dark disk of the Moon. The whole is to be immersed in the tank, with the lamp in a horizontal position, and the metal disk close against the front glass plate. It is a good plan to have a rheostat in circuit with the lamp to regulate the intensity of the illumination. On turning on the current and seating ourselves in front of the tank, we shall see a most beautiful corona, caused by the scattering of the light of the lamp by the small particles of mastic suspended in the water. If we

look at it through a Nicol prism we shall find that it is radially polarized, a dark area appearing on each side of the lamp, which turns as we turn the Nicol. The illumination is not uniform around the lamp, owing to unsymmetrical distribution of the candle power, and this heightens the effect. If the polar streamers are found to be too sharply defined or too wide, the defect can be easily remedied by altering the tin-foil strips. The eclipse is not yet perfect, however, the illumination of the sky background being too white and too brilliant in comparison. By adding a solution of some bluish-green aniline dye (I used malachite-green) the sky can be given its weird color, and the corona brought out much more distinctly. If the proper amount of the dye be added, the sky can be strongly colored without apparently changing the color of the corona in the slightest degree, a rather surprising circumstance, since both are produced by the same means. We should have now a most beautiful and perfect reproduction of the wonderful atmosphere around the Sun, a corona of pure golden white light, with pearly luster and exquisite texture, the misty streamers stretching out until lost on the bluish-green background of the sky. The rifts or darker areas due to the unequal illumination are present, as well as the polar streamers. The effect is heightened if the eyes are partially closed.

FIG. 5.

A photograph of one of these artificial eclipses is reproduced in Fig. 5. Much of the fine detail present in the negative is lost in the print, and still more will doubtless go in the process of reproduction. The coronal streamers extend out much farther than is indicated by the photograph. No especial pains were taken to get the polar rays just right.

UNIVERSITY OF WISCONSIN.

# A PRELIMINARY DETERMINATION OF THE MOTION OF THE SOLAR SYSTEM.

By W. W. CAMPBELL.

THE first investigation undertaken with the Mills spectrograph, in May 1895, related to the determination of the radial velocities in the system of *Saturn*.[1] It confirmed, in all respects, the noted results announced by Professor Keeler a few weeks earlier. Determinations of stellar velocities were now undertaken, and results of considerable accuracy were at once obtained. The observed velocities of a bright solar type star could be depended upon to fall within a range of five or six kilometers. However, it soon became apparent that the instrument contained many defects. Some of these, with their remedies, have been described in my article on "The Mills Spectrograph," in this JOURNAL for October 1898; but the large majority were purely local, and do not call for special comment. The greater part of the first year was devoted to isolating and eliminating these defects; and it was not until the summer of 1896 that results considered satisfactory for publication were secured. Added precautions taken, and improvements made in the instrument and methods, have shown corresponding and gratifying increase of accuracy from year to year.

Following the methods of observation already described in this JOURNAL, two thousand spectrograms have been secured since the summer of 1896. These include: plates of the solar spectrum for determining the camera focus and scale values; plates of stellar spectra for determining the focus of the 36-inch objective at different temperatures; plates of comparison spectra, etc.; perhaps one hundred stellar spectrograms rejected for cause, without measurement; and in the neighborhood of fifteen hundred satisfactory spectrograms of about three hundred and

[1] ASTROPHYSICAL JOURNAL, August 1895, pp. 127–135.

twenty-five stars, situated between the North Pole and Declination $-30°$. At least three or four hundred of these photographs relate to spectroscopic binaries, for some of which, such as $\zeta$ *Geminorum*, nearly fifty plates were needed.

It is not practicable to publish the observed velocities at the present time, for two reasons:

(A) Many of the plates have been only partially measured, and reduced by approximate methods. Experience shows that these approximate results may be changed as much as 1½ km by the final measures and reductions, though the average change is much less.

(B) The reductions have not been based upon the definitive wave-lengths of the solar and comparison lines. These are not yet available, but they are expected soon.

Repeated requests have been made that the observations already secured should be used to determine the motion of the solar system with reference to the system of observed stars; and it is the purpose of this article to communicate the preliminary results of such an investigation.

Omitting several Type I stars whose lines could not be accurately measured, and some thirty spectroscopic and visual[1] binaries for whose centers of gravity the velocities are not yet known, there remain 280 stars available for determining the relative motion of our system. Inasmuch as this number is constantly increasing with the progress of the observations, and in a few years will, I hope, be doubled and include stars distributed over the entire sky, it did not seem necessary to form an equation of condition for each star. The 280 stars were divided into 80 groups, by combining neighboring stars into one group; taking the mean of their individual velocities as the velocity of the group. The data for each of the 80 groups are contained in the first four columns of Table I.

Let $v$ be the observed speed of a star with reference to the solar system; $V$ the Sun's speed with reference to the system of 280 observed stars; and $D$ the angular distance of a star from

[1] Such as *Sirius*, *Procyon*, etc.

the apex of the solar motion. Then each star, or each group of stars, furnishes an equation of condition having the form:

$$V \cos D - v = 0. \qquad (1)$$

Let $\alpha_0$, $\delta_0$ be the coördinates of the apex, and $\alpha$, $\delta$ those of the star; then we have $\cos D$ defined by the well-known equation for the distance between two stars or points,

TABLE I.

| No. of Stars | Mean R. A. | Mean Dec. | Mean Observed Velocity | No. of Stars | Mean R. A. | Mean Dec. | Mean Observed Velocity |
|---|---|---|---|---|---|---|---|
| | h m | | k | | h m | | k |
| 2 | 0 26.5 | −14°.0 | +17.0 | 3 | 12 20.4 | +43°.0 | − 4.7 |
| 6 | 0 40.0 | +57.8 | −10.3 | 3 | 12 35.7 | −22.5 | − 1.7 |
| 5 | 0 45.0 | +28.2 | −24.2 | 3 | 12 48.1 | + 4.9 | −17.0 |
| 3 | 1 23.0 | −10.1 | +12.7 | 2 | 13 6.2 | +23.2 | − 8.2 |
| 4 | 1 35.5 | +43.9 | + 0.2 | 2 | 14 9.2 | − 7.6 | + 3.0 |
| 3 | 1 43.7 | +10.6 | + 7.3 | 5 | 14 14.9 | +18.1 | − 3.9 |
| 2 | 1 47.4 | −19.0 | + 2.5 | 2 | 14 29.5 | −25.6 | +11.8 |
| 2 | 2 2.6 | +24.2 | −19.0 | 7 | 14 58.0 | +29.6 | −13.1 |
| 9 | 2 38.5 | −14.1 | − 3.2 | 4 | 15 2.4 | +46.0 | −28.5 |
| 3 | 2 54.4 | + 0.1 | +21.7 | 4 | 15 23.2 | + 4.8 | − 7.0 |
| 5 | 2 55.4 | +50.2 | +13.6 | 4 | 15 27.4 | +65.8 | −14.2 |
| 4 | 3 3.8 | +41.2 | + 4.5 | 4 | 15 41.4 | −12.9 | − 1.0 |
| 3 | 3 16.9 | +13.6 | + 6.7 | 4 | 16 17.1 | −20.8 | −11.1 |
| 3 | 3 45.8 | − 9.2 | −10.7 | 4 | 16 32.8 | − 8.5 | − 7.0 |
| 3 | 4 3.2 | −17.3 | +34.8 | 3 | 16 38.0 | +13.3 | −26.0 |
| 5 | 4 16.6 | +18.0 | +35.5 | 3 | 16 49.5 | +35.9 | −29.0 |
| 2 | 4 29.7 | +79.8 | − 3.0 | 3 | 17 44.7 | +53.6 | −22.3 |
| 4 | 4 46.8 | +44.7 | + 8.2 | 4 | 17 53.6 | +29.5 | − 9.9 |
| 3 | 5 15.7 | +37.4 | +30.7 | 3 | 17 53.8 | + 5.3 | − 7.5 |
| 2 | 5 17.0 | + 7.1 | +22.0 | 6 | 18 18.6 | − 7.2 | − 5.4 |
| 4 | 5 23.5 | −20.9 | + 1.8 | 6 | 18 36.4 | −23.5 | −23.5 |
| 3 | 5 32.6 | − 7.5 | − 0.7 | 3 | 18 38.6 | +19.0 | −28.5 |
| 3 | 5 51.5 | +57.7 | + 5.0 | 2 | 18 43.0 | +41.2 | −20.5 |
| 4 | 6 18.1 | +25.0 | +24.8 | 6 | 18 48.9 | +69.6 | + 6.4 |
| 2 | 6 41.0 | −15.6 | +50.5 | 4 | 19 42.1 | + 6.2 | −19.9 |
| 3 | 6 43.3 | +16.7 | + 6.3 | 5 | 19 42.6 | +23.6 | −11.6 |
| 3 | 7 32.4 | +27.0 | +11.3 | 4 | 20 3.6 | +55.4 | −44.2 |
| 2 | 7 33.8 | −25.1 | +42.0 | 3 | 20 19.3 | − 9.1 | − 8.7 |
| 2 | 7 47.0 | + 9.3 | +34.0 | 3 | 20 48.9 | +43.3 | + 1.0 |
| 2 | 7 50.6 | − 6.0 | −21.0 | 5 | 20 58.5 | +12.8 | −28.2 |
| 4 | 8 56.6 | +10.5 | −26.9 | 5 | 21 1.3 | +31.9 | − 2.6 |
| 2 | 8 57.8 | +32.0 | +25.2 | 4 | 21 24.6 | −15.8 | − 2.9 |
| 4 | 8 58.0 | +65.7 | − 4.0 | 3 | 22 00.2 | +12.7 | − 7.3 |
| 2 | 9 10.2 | +47.2 | +19.0 | 2 | 22 13.6 | +54.7 | −14.2 |
| 3 | 9 27.2 | − 3.7 | +12.3 | 4 | 22 32.6 | − 6.7 | −11.3 |
| 2 | 9 57.2 | +22.3 | −15.2 | 5 | 22 52.9 | +25.4 | − 0.2 |
| 5 | 10 40.2 | −15.2 | +19.6 | 2 | 23 10.6 | +71.4 | −26.5 |
| 5 | 10 44.7 | +38.1 | − 2.8 | 4 | 23 10.7 | −18.7 | + 5.6 |
| 2 | 11 11.6 | +66.1 | ± 0.0 | 2 | 23 16.0 | +43.8 | − 2.0 |
| 6 | 11 32.8 | + 6.2 | + 3.7 | 4 | 23 31.0 | + 5.0 | − 0.9 |

$$\cos D = \sin \delta_0 \sin \delta + \cos \delta_0 \cos \delta \cos (\alpha_0 - \alpha). \tag{2}$$

If we place

$$\left.\begin{aligned} x &= V \sin \delta_0 \\ y &= V \cos \alpha_0 \cos \delta_0 \\ z &= V \sin \alpha_0 \cos \delta_0 \end{aligned}\right\} \tag{3}$$

equations (1) take the form

$$\sin \delta . x + \cos \alpha \cos \delta . y + \sin \alpha \cos \delta . z - v = 0 \tag{4}$$

from which the values of $x$, $y$, and $z$ may be determined; and the values of $V$, $\alpha_0$, and $\delta_0$ may then be found from (3) by the relations

$$\left.\begin{aligned} V^2 &= x^2 + y^2 + z^2 \\ \tan \alpha_0 &= \frac{z}{y} \\ \sin \delta_0 &= \frac{x}{V} \end{aligned}\right\} \tag{5}$$

The values of $\alpha$ and $\delta$ for each group were substituted in equation (4), and the resulting equation was weighted in proportion to the number of stars on which it is based, as indicated in column 1 of the table. The eighty equations thus formed were combined and solved by the method of least squares, and the following elements of the solar motion were obtained:

$$\begin{aligned} V &= -19.89 \text{ km} \pm 1.52 \text{ km} \\ \alpha_0 &= \quad 277^\circ 30' \pm 4^\circ.8 \\ \delta_0 &= + \ 19^\circ 58' \pm 5^\circ.9 \end{aligned}$$

The list of stars employed in this investigation includes all that were available: none were rejected arbitrarily, on account of very high speed or otherwise. The results represent the solar motion relative to the entire system of observed stars. Had a dozen stars of great velocity been rejected, the speed and direction of the motion would have been only slightly different, but the computed probable errors would have been very much smaller than those appended.

On the basis of these elements, the component correction for the solar motion was computed and applied to each star. The 280 results obtained in this manner represent the individual stellar components of motion in the line of sight, with reference to the entire system. Of these there are

| | | Km per second |
|---|---|---|
| 151 | positive, average, | + 17.01 |
| 129 | negative, average, | —17.10 |
| 280 | numerical average, | 17.05 |

The average component velocity of each star in a plane at right angles to the line of sight is therefore

$$\frac{\pi}{2} \cdot 17.05 = 26.78 \text{ km per second};$$

and the average velocity *in space* of each star in the system is

$$2 \times 17.05 = 34.10 \text{ km per second}.$$

The Sun's relative velocity, 19.9 km, is therefore much smaller than that of the average star of the system, 34.1 km.

The 280 stars were classified roughly, according to their spectral types, in the following manner: the *Harvard Photometry* contains estimates of their brightness, based upon the visual radiations. The *Draper Catalogue* estimates their brightness by virtue of the photographic intensities of their spectra in the $H\gamma$ region. The difference between the visual and photographic magnitudes is very small in the case of the white stars, such as $\beta$ *Orionis;* it is usually from 1.5 to 2.0 magnitudes for the solar type stars, such as $\beta$ and $\gamma$ *Andromedae;* and is fully 2.5 magnitudes for red stars, such as $\alpha$ *Scorpii.*

In the system of stars observed, the difference of magnitude is equal to or greater than 1.0 for 144 stars. Subdividing these according as their component velocities in the line of sight are positive or negative, we have

| | | |
|---|---|---|
| 78 | positive, average component, | + 17.07 km |
| 66 | negative, average component, | — 14.99 |
| 144 | numerical average, - - - - | 16.12 |

For 136 stars the difference of magnitude is less than 1.0, as follows:

| | | |
|---|---|---|
| 73 | positive, average component, | + 16.94 km |
| 63 | negative, average component, | — 19.32 |
| 136 | numerical average, - - - - | 18.04 |

The discrepancy of 1.9 km in the results is hardly sufficient

to justify any statement as to the effect of spectral type upon velocity.

The relation between visual brightness and velocity was next investigated.

Of stars equal to or brighter than 3.0 magnitude, there are

| | | |
|---|---|---|
| 26 | positive, average component, | + 13.11 km |
| 21 | negative, average component | — 12.99 |
| 47 | numerical average, - - - - | 13.05 |
| | Corresponding velocity in space, - | 26.10 |

Of stars lying between magnitudes 3.1 and 4.0 inclusive, there are

| | | |
|---|---|---|
| 59 | positive, average component, | + 17.70 km |
| 53 | negative, average component, | — 14.42 |
| 112 | numerical average, - - - - | 16.15 |
| | Corresponding velocity in space, - | 32.30 |

Of stars fainter than 4.0 magnitude, there are

| | | |
|---|---|---|
| 66 | positive, average component, | + 17.93 km |
| 55 | negative, average component, | — 21.27 |
| 121 | numerical average, - - - - | 19.44 |
| | Corresponding velocity in space, - | 38.88 |

The progression in these results is so pronounced, and the differences are so large, that I think we are justified in drawing the important conclusion that the faint stars of the system are moving more rapidly than the bright stars. This apparent fact, derived quite independently of any assumption as to the relative distances of the stars of different magnitudes, should profoundly affect the question of, and the methods of determining, the structure of our sidereal system. If the fainter stars are moving relatively more rapidly than has been previously assumed, they must be relatively further from us than the investigations of their proper motions have led us to conclude.

This progression is in no wise due to an increase of probable error of a velocity determination with decreasing magnitude. The probable error of a single determination is well under half a kilometer for such ercellent stars as *Polaris* and *Procyon;* and it is not much greater for fifth magnitude stars whose spectra contain well-defined lines;

The elements of the solar motion deduced above depend upon the assumption that their most probable values are those which make the sum of the squares of the residual stellar components of speed in the line of sight a minimum. This in turn assumes that the magnitudes of these components are distributed according to the law of accidental errors. No doubt they are distributed according to a somewhat different law, which I hope to investigate fully a few years later, before making a definitive determination of the motion, based upon a much larger number of stars distributed over the entire sphere.[1]

The right ascension of the apex, 277°30′, agrees exactly with the value deduced by Professor Newcomb[2] from all the "proper motion" data available; and differs only 1°30′ from Professor Kapteyn's[3] assumed value, 276°. My value of the declination, $+19°58'$, differs widely from Newcomb's value, $+35°$, and Kapteyn's, $+34°$. It must be noticed that very few radial velocities are available for the region $-15°$ to $-30°$, and none whatever south of $-30°$ declination. Fully one third of the sky is unrepresented in the solution. The data for determining the declination of the apex are extremely unsymmetrical in arrangement. The data north of the line of motion are fairly complete, whereas the data to the south are very incomplete. To determine the declination therefrom is somewhat similar to flying with one wing very imperfect. The right ascension, on the contrary, is determined from data reasonably symmetrical in distribution.

A comparison of my results with those obtained by Stumpe[4] from proper motions is of great interest. He classified the stars of relatively large proper motions according to their visual magnitudes, with the following results for the position of the apex:

[1] An additional reason for delay arises from the fact that many years of observation are required to establish constancy of stellar velocities, in some cases: of the stars used in the above determination, two have since been discovered to have variable velocities.

[2] *Astronomical Journal*, No. 457, pp. 4, 5.

[3] *Astronomische Nachrichten*, No. 3487, p. 104.

[4] *Astronomical Journal*, No. 457, p. 5.

| No. of Stars | Magnitude | R. A. | Dec. |
|---|---|---|---|
| 284 | 1 to 5.5 | 263°.8 | +31°.1 |
| 473 | 5.6 to 7.5 | 290.7 | +37.5 |
| 238 | 7.6 to > | 286.7 | +46.9 |

In view of these widely different positions of the apex, it is perhaps not surprising that my result for declination, depending upon even brighter stars than his first group, should be smaller than any hitherto obtained.

The motion of the solar system is a purely relative quantity. It refers to specified groups of stars. The results for various groups may differ widely, and all be correct. It would be easy to select a group of stars with reference to which the solar motion would be reversed 180° from the values assigned above. It is perhaps unsafe to draw conclusions, from my value of the declination, concerning the drift of the brighter (and presumably nearer) stars until the data from the southern sky are available.

Before making the preceding solution for the solar motion by the method of least squares, I had already made an approximate determination of the speed of the solar system, by a different method, as follows: the apical distance $D$ of each star was computed from Newcomb's assumed coördinates of that point ($\alpha = 277°.5, \delta = +35°$). The stars were formed into groups according to their apical distances, as indicated in the first column of Table II. The number of stars in each group is given in column two. The mean apical distance of the group is $[D]$ and the mean observed velocity is $[v]$. It is interesting to note that each $[v]$ between apical distances 0° and 90° is negative, and each one between 90° and 180° is positive. Each radial velocity furnishes an equation of condition of the form

$$V - v \sec D = 0, \tag{6}$$

from which to determine $V$. We shall assume that the weight of each determination is equal to $\cos D$. The resulting value of $V$ will now be given by

$$V = \frac{\Sigma\,(n \cos [D]\,.\,[v] \sec [D])}{\Sigma\, n \cos [D]} = \frac{\Sigma\, n\,[v]}{\Sigma\, n \cos [D]}. \tag{7}$$

Substituting the values of $n$, $[v]$, and $[D]$ in this equation, I obtained

$$V = -20.4 \text{ km}.$$

TABLE II.

| Apical distances | $n$ | $[D]$ | $[v]$ | $\cos [D]$ |
|---|---|---|---|---|
| 0°— 10° | 4 | 7°.4 | − 9.9 | +0.992 |
| 10 — 20 | 10 | 15.5 | −24.0 | + .964 |
| 20 — 30 | 16 | 24.7 | −17.5 | + .908 |
| 30 — 40 | 24 | 34.8 | −12.9 | + .821 |
| 40 — 50 | 24 | 44.1 | −16.6 | + .718 |
| 50 — 60 | 29 | 54.6 | − 6.1 | + .579 |
| 60 — 70 | 29 | 64.4 | − 7.0 | + .432 |
| 70 — 90 | 47 | 79.7 | − 2.7 | + .179 |
| 90 —110 | 35 | 99.4 | + 8.0 | − .163 |
| 110 —120 | 19 | 116.3 | +14.4 | − .443 |
| 120 —130 | 18 | 124.4 | +13.8 | − .565 |
| 130 —140 | 10 | 134.4 | +16.4 | − .700 |
| 140 —150 | 5 | 145.1 | +14.6 | − .820 |
| 150 —160 | 6 | 156.0 | +29.3 | − .914 |
| 160 —170 | 4 | 164.0 | + 6.0 | −0.961 |
| | 280 | | | |

$$V = \frac{\Sigma\, n\, [v]}{\Sigma\, n \cos [D]} = \frac{-3010}{147.5} = -20.4 \text{ km.}$$

If we use this value of $V$ as a basis for further approximations to its true value, by the method of Kapteyn,[1] we shall obtain $V = -19$ kilometers; though it should be said that his method involves assumptions concerning proper motions.

The foregoing data bear decisively upon the question of stellar parallaxes and other fundamental problems; but these portions of the subject are reserved for a future paper.

The work with the Mills spectrograph has furnished many important by-products. Special mention may be made of the discovery of an unexpectedly great number of spectroscopic binaries. Two or more satisfactory observations have been secured for each of 285 stars of my program. From the Mills spectrograph observations alone, we have discovered that thirty-one[2] of these stars are spectroscopic binaries. To these we must add three binaries in the same list previously discovered

[1] *Astronomische Nachrichten*, No. 3487.

[2] Twenty-five of these have been announced in this JOURNAL, and six now await announcement.

by another observer,[1] making thirty-four in all. That is, of 285 observed stars, *more than one star in nine is a spectroscopic binary.* Further, five additional suspected binaries await verification, and it is altogether probable that many other stars in the list are binaries awaiting discovery. Two plates are not sufficient to detect variable velocity, even in many cases of short period; and still less are they sufficient in many cases of long period, now coming to light by virtue of our older observations. It is not improbable that at least one star in five or six will be found to be a spectroscopic binary; and I should not be surprised to see a still larger ratio established.

The proven existence of so large a number of stellar systems differing widely in structure from the solar system gives rise to a suspicion, at least, that our system is not of the prevailing type of stellar systems. The new field of astronomical research thus opened up is of great richness, and may well occupy the attention, for an indefinite period, of the large number of observers and institutions now engaging in its development. It is perhaps unnecessary to say that the measure of success attainable is dependent upon the degree of accuracy[2] realized in the observed velocities.

It is a pleasure to record that I have been assisted most efficiently in these investigations, since August 1897, by Mr. W. H. Wright, assistant astronomer.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
December 1900.

[1] DR. BÉLOPOLSKY, at Pulkowa, $\alpha_1$ *Geminorum;* the well-known variable stars $\delta$ *Cephei* and $\eta$ *Aquilae;* and the independent and prior discovery of the binary character of the well-known variable, $\zeta$ *Geminorum.*

[2] In the later observations of the *best stars* with the Mills spectrograph, an extreme range of two kilometers would afford strong suspicion of variable velocity; and the greater portion of a smaller range due to unavoidable errors would arise not from errors in the spectrograms, I believe, but from changes in the observer's personal habits of measuring the plates.

# THE MOTION OF ζ *GEMINORUM* IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

ζ *Geminorum* is a well-known variable star, discovered by Schmidt in 1847. It varies from a minimum of 4.5 magnitudes to a maximum of 3.7 magnitudes in 5.015 days, and returns to the minimum in 5.139 days. According to Chandler[1] the period of 10.154 days is well defined, and constant; but Miss Clerke states[2] that the period lengthened by ten minutes between the years 1847 and 1890. The light-curve of ζ *Geminorum* is approximately represented in the upper part of Fig. 1.

This star was placed on the regular observing list for the Mills spectrograph. The second photograph of its spectrum, secured in January 1899, led to the discovery that it is a spectroscopic binary. In response to my announcement of the discovery, published in this JOURNAL for February 1899, Dr. Bélopolsky kindly called attention[3] to his prior discovery of its binary character, in January 1898. He had announced his discovery in a lecture before the Russian Astronomische Gesellschaft; though so far as I am aware no published statement was made.

In the meantime, twenty-two spectrograms had been obtained here, preparatory to determining the orbit of the bright component; and it seemed advisable to continue the observations with the Mills spectrograph.

Forty-four spectrograms were secured between 1898, November 11, and 1900, February 11. The Greenwich mean times of observations are contained in the accompanying table, column one, and the observed velocities, in kilometers, in column three.

[1] "Third Catalogue of Variable Stars," *Astronomical Jour.*, No. 379, pp. 148, 149; and by private letter, June 1900.

[2] *The System of the Stars*, p. 133.

[3] *Astronomische Nachrichten*, No. 3565, May 1899.

| | Greenwich Mean Time | | Interval after Min. | Observed Velocity | Computed Velocity | O—C |
|---|---|---|---|---|---|---|
| | d | h | d | k | k | k |
| 1900—February .... | 11 | 16.0 | 0.017 | +21.0 C | +19.0 | +2.0 |
| " .... | 11 | 18.7 | 0.129 | +19.4 C | +18.3 | +1.1 |
| 1898—November.... | 11 | 23.4 | 0.246 | +19.9 C | +17.5 | +2.4 |
| 1899—February..... | 21 | 16.3 | 0.408 | +17.9 W | +16.2 | +1.7 |
| " .... | 21 | 18.2 | 0.488 | +16.3 W | +15.6 | +0.7 |
| April........ | 13 | 16.8 | 0.662 | +13.3 W | +14.3 | −1.0 |
| September ... | 13 | 0.7 | 0.688 | +14.6 C | +14.0 | +0.6 |
| October...... | 23 | 23.9 | 1.038 | +10.2 W | +11.3 | −1.1 |
| " .... | 4 | 0.9 | 1.388 | + 5.2 C | + 8.7 | −3.5 |
| February..... | 22 | 16.5 | 1.417 | + 7.0 W | + 8.5 | −1.5 |
| " .... | 22 | 18.3 | 1.492 | + 4.7 C | + 8.0 | −3.3 |
| " .... | 22 | 18.3 | | + 4.7 W | + 8.0 | −3.3 |
| October...... | 25 | 0.7 | 2.071 | + 2.9 C | + 4.4 | −1.5 |
| February..... | 13 | 16.0 | 2.550 | + 1.7 W | + 1.9 | −0.2 |
| April........ | 5 | 15.6 | 2.767 | + 2.5 W | + 0.9 | +1.6 |
| December.... | 25 | 20.6 | 2.979 | + 1.3 C | ± 0.0 | +1.3 |
| January...... | 24 | 20.3 | 3.038 | + 0.7 C | − 0.2 | +0.9 |
| December.... | 26 | 0.1 | 3.125 | + 0.7 W | − 0.3 | +1.0 |
| 1900—January...... | 15 | 16.3 | 3.492 | + 0.5 C | − 1.7 | +2.2 |
| 1899—December.... | 26 | 18.3 | 3.883 | + 0.0 C | − 2.7 | +2.7 |
| " .... | 26 | 18.3 | | − 0.2 C | − 2.7 | +2.5 |
| January...... | 25 | 21.0 | 4.067 | − 1.8 C | − 3.1 | +1.3 |
| September ... | 27 | 0.0 | 4.504 | − 3.8 C | − 3.7 | −0.1 |
| February..... | 15 | 18.3 | 4.646 | − 4.3 W | − 3.8 | −0.5 |
| " .... | 15 | 19.6 | 4.700 | − 4.7 W | − 3.8 | −0.9 |
| December.... | 27 | 23.1 | 5.083 | − 6.2 W | − 3.9 | −2.3 |
| " .... | 28 | 0.2 | 5.129 | − 5.9 C | − 3.8 | −2.1 |
| 1900—February..... | 6 | 20.7 | 5.367 | − 6.7 C | − 3.6 | −3.1 |
| 1899— " .... | 6 | 16.6 | 5.729 | − 2.9 W | − 3.0 | +0.1 |
| " .... | 6 | 17.7 | 5.775 | − 2.6 W | − 2.9 | +0.3 |
| January...... | 27 | 20.2 | 6.033 | − 2.4 C | − 2.1 | −0.3 |
| 1900—February..... | 7 | 17.2 | 6.221 | − 0.4 C | − 1.4 | +1.0 |
| 1899— " .... | 7 | 16.2 | 6.712 | + 3.6 W | + 1.2 | +2.4 |
| January...... | 28 | 20.2 | 7.033 | + 5.4 C | + 3.5 | +1.9 |
| 1900—January...... | 29 | 18.4 | 7.425 | + 8.8 C | + 7.0 | +1.8 |
| 1899—November.... | 30 | 0.4 | 7.600 | +11.2 C | + 8.8 | +2.4 |
| April........ | 10 | 16.4 | 7.800 | +12.6 W | +11.0 | +1.6 |
| January...... | 29 | 18.4 | 7.958 | +13.0 W | +12.7 | +0.3 |
| 1900—January...... | 30 | 18.1 | 8.412 | +16.2 C | +17.6 | −1.4 |
| " .... | 10 | 19.4 | 8.775 | +18.9 C | +20.6 | −1.7 |
| 1899—January...... | 30 | 19.8 | 9.017 | +20.5 W | +22.0 | −1.5 |
| " .... | 30 | 21.1 | 9.071 | +20.1 W | +22.1 | −2.0 |
| 1900—January...... | 21 | 16.5 | 9.500 | +21.2 C | +22.3 | −1.1 |
| 1899—September... | 22 | 0.0 | 9.658 | +24.2 W | +21.9 | +2.3 |
| " .... | 12 | 0.5 | 9.833 | +23.5 C | +21.1 | +2.4 |
| April........ | 12 | 17.8 | 9.858 | +23.5 W | +21.0 | +2.5 |

They are arranged in the order of the intervals after the instants of minima, as indicated in column two. The letters C and W in column three indicate that the plates were measured by Campbell, and Wright, respectively. Some time before the

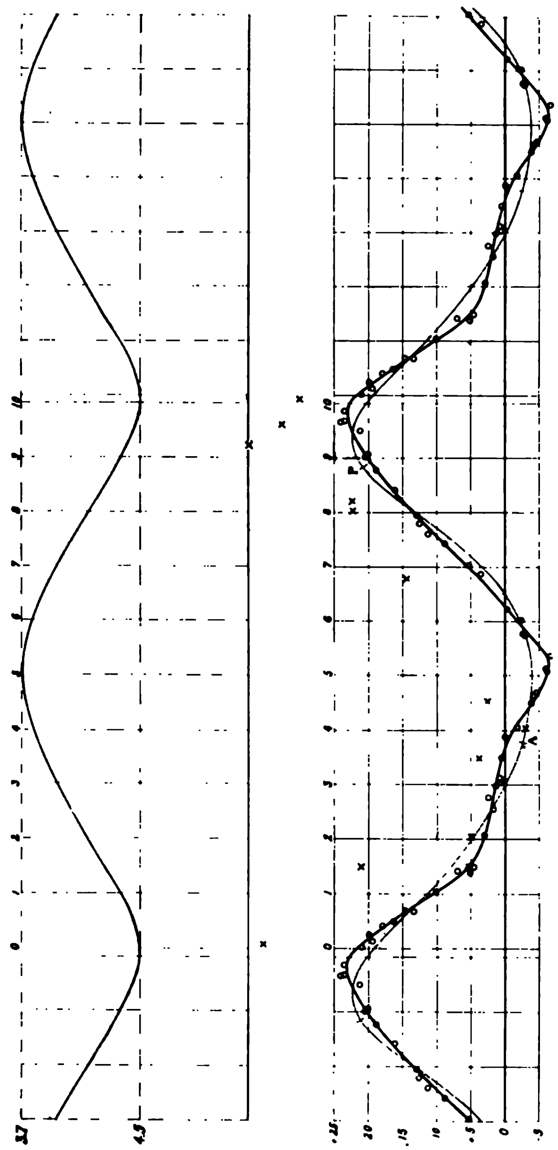


FIG. 1

observations were completed it was noticed that the velocity curve includes some curious irregularities. The later observations were planned, in point of time, so as to fill in the small gaps of the curve; and they leave no doubt that the observed irregularities are real.

In Fig. 1 the unit of abscissæ is one day, and the unit of ordinates is five kilometers. The Mills spectrograph observations are represented by small circles, o; each result being plotted twice in such a way as to show two complete cycles of velocity variations. The heavy irregular line drawn through the observed points is the curve of observed velocities. Its irregular character is well defined, and obviously does not represent motion in an ellipse. After some half a dozen trials of different systems of elliptic elements, the following were adopted, as they appear to afford the best possible elliptic representation of the observed curve.

ELEMENTS.

$V = +6.8$ km, velocity of the system;
$A = 15.7$ km, greatest positive velocity of the bright component;
$B = 10.7$ km, numerical value of greatest negative velocity;
$e = 0.22$, eccentricity of orbit;
$\omega = 333°$, position of periastron;
$T = +1.313$ days, time of periastron passage, referred to the instant of minimum brightness;
$a \sin i = 1,797,800$ km, projection of semi-major axis.

The velocities computed from these elements for the instants of observation are contained in column four of the table; and the corresponding curve is the lighter line in the lower portion of Fig. 1. There is no satisfactory basis on which to compute the probable error of a single observation; but a simple inspection of Fig. 1 will show that it is in the neighborhood of four or five tenths of a kilometer. Except in good atmospheric conditions, it was difficult to secure a satisfactory spectrogram when the brightness was a minimum.

The observed velocity curve is alternately above and below the elliptic curve, and the intersections of the two occur at

approximately equal intervals of time. There are six of these intersections, corresponding to three complete periods or cycles in exactly one period of the light curve. The observations extend over fifteen months, or about forty-five complete periods; and there is no reason to doubt that the apparent velocity curve repeats itself faithfully during each light period.

It would be possible to explain fairly well the observed irregularities in the velocity by assuming that $\zeta$ *Geminorum* is a triple system; that the bright component and a dark companion are revolving around their center of gravity in a period of 3.385 days, with a velocity double amplitude of about 4.5 km for the bright component; and that these two bodies are revolving around a third component in a period of 10.154 days. It is questionable, however, whether such a system would be a stable one. If the short period is exactly commensurate with the longer one, as it appears *on the average* to be, it would seem that the stability of the system would be open to question, especially since the amplitudes 4.5 and 26.4 are not very unequal.

The constancy of the light period, the equality of the light and (apparent) velocity periods, and the satisfactory representation of the *general features* of the velocity curve, leave no doubt, I think, that the system is at least binary, and that the light variations are due in some manner to the influence of the companion. The *form* of the orbit of the bright component, and its relation to the line of sight, are shown in Fig. 2. $O\,E$ is the line of sight, $O$ is the center of gravity of the two components of $\zeta$ *Geminorum*, $P$ is the periastron, and $A$ the apastron. The points of the velocity curve corresponding to periastron and apastron are likewise marked $P$ and $A$. When the star's brightness is a minimum the bright component is at the point of the orbit marked Min, and the companion is somewhere on the line Om. It is certain, therefore, that the light variation is not the result of an eclipse.

Granting that the variation in brightness is due to the influence of the companion, the most satisfactory explanation available seems to be that it arises from tidal disturbances in the

bright star's atmosphere, produced by the gravitational attraction of the dark component. In all probability the two components are only a few millions of kilometers apart, and their atmospheres may approach comparatively near to each other. Recalling that the eccentricity of the orbit is 0.22, and that the tide-raising force varies inversely as the cube of the distance between the masses, it is plain that the disturbances in the bright star's atmosphere

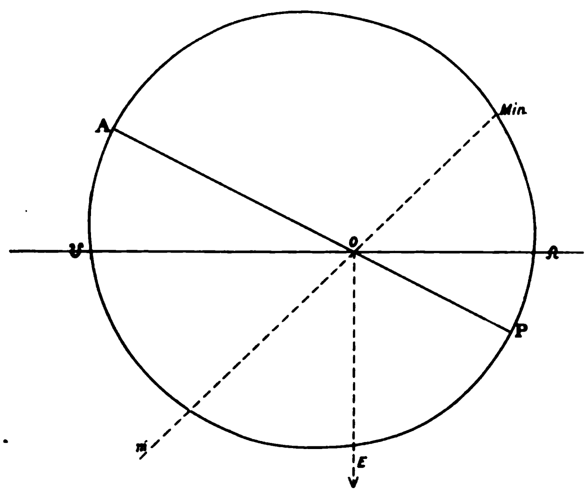



FIG. 2

may easily be on an enormous scale, and be very different in different parts of the orbit.

In this connection attention should be called to the fact that the minimum brightness occurs 1.3 days after periastron passage, and maximum occurs 6.3 days after periastron. It may be urged that if the disturbance is of tidal origin the maximum brightness should occur soon after periastron; and perhaps this is true. But is it not also possible that the maximum tidal forces may lead to a strong expansion of the gases of the atmosphere and to a reduction of brightness? The question must be left in an unsatisfactory state; and a consideration of the phenomena of two other variable stars of the same class, δ *Cephei* and η *Aquilae*, affords

little assistance. In the case of each of these stars the brightness rapidly increases at the time of periastron passage and reaches a maximum very soon thereafter. This is perhaps what would be expected in the case of tidal disturbances, but it is just the opposite of the effect observed in $\zeta$ *Geminorum*. However, the orbits of $\delta$ *Cephei* and $\eta$ *Aquilae* are very eccentric, whereas that of $\zeta$ *Geminorum* is much less so. The eccentricities in the three cases are 0.46, 0.47, and 0.22, respectively, and their periods are 5, 7, and 10 days. The first two are subjected to vastly more rapid changes of tidal forces than the last; and that fact may account for very divergent effects. It is of interest to note that the brightness of $\delta$ *Cephei* and $\eta$ *Aquilae* varies 1.2 magnitude in five and seven days, respectively, whereas that of $\zeta$ *Geminorum* varies only 0.8 magnitude in ten days.

By way of explanation of the deviations of the observed velocity curve from the elliptic curve, I think the probability is very strong that they are minor tidal effects. Terrestrial tides run through their cycle in one period of the Moon — neglecting the second-order quantities in the positions and distances of the Sun and Moon. They are profoundly modified by the rotation of the earth, increasing their number from (roughly) two per month to two per day. If tidal disturbances, with a double period of ten days, are sufficient to account for the light curve of $\zeta$ *Geminorum*, I think it is not impossible that the irregularities in its apparent velocity arise from modifications in the tides caused by the rotation of the star. These modifications might affect the apparent velocities in the line of sight in either or both of two ways, viz.:

(A) By producing an actual movement of considerable velocity within the atmosphere; and

(B) By producing considerable variations of pressure within the absorbing layer, and consequent displacement of the spectral lines. The remarkably interesting results obtained by Professor Wilsing, of Potsdam, in his study of the phenomena of "new stars," are sufficient justification for the belief that enormous changes of pressure may occur in such disturbed stars as

*ζ Geminorum.* In this connection it is very desirable that the light curve of *ζ Geminorum* be determined with great accuracy, to ascertain whether the irregularities in the velocity curve may not have their exact counterparts in the light curve.

These short-period irregularities have not been detected as yet in the velocity curves of *δ Cephei* and *η Aquilae;* but perhaps in these five and seven-day, and very eccentric, systems the periods of revolution and rotation synchronize.

I recognize that the hypotheses advanced above are not now capable of proof, and I have endeavored to state them from that point of view.

Fifteen observations by Dr. Bélopolsky in 1898 and 1899 are plotted in the lower portion of Fig. 1, being represented by an *X*. His observations, on the average, are six kilometers above my curve.

Acknowledgments are due to Mr. Wright for skillful assistance throughout the investigation.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
December 1900.

# SOME STARS WITH LARGE RADIAL VELOCITIES.

By W. W. CAMPBELL.

WHILE pursuing the regular program of observation with the Mills spectrograph, it was found that the following stars have large velocities in the line of sight, as indicated below:

$\epsilon$ ANDROMEDAE ($\alpha = 0^h\ 33^m$, $\delta = +28^\circ\ 46'$).

| | | | |
|---|---|---|---|
| 1898 | October 4 | − 83.4 km | Wright |
| | October 9 | − 83.3 | Wright |
| 1899 | August 29 | − 84.6 | Wright |
| 1900 | August 22 | − 83.4 | Wright |
| | Mean | − 83.7 | |

$\mu$ CASSIOPEIAE ($\alpha = 1^h\ 0^m$, $\delta = +54^\circ\ 20'$).

| | | | |
|---|---|---|---|
| 1900 | September 9 | − 97.2 km | Wright |
| | September 18 | − 97.0 | Wright |
| | December 11 | − 98 | Campbell |

The proper motion of $\mu$ *Cassiopeiae* is 3.″75 per year. Jacoby's parallax determined from the Rutherfurd photographs is 0.″275. These correspond to a motion at right angles to the line of sight of 66 km per second, though this includes nearly the full component of the motion of the solar system.

$\delta$ LEPORIS ($\alpha = 5^h\ 47^m.0$, $\delta = -20^\circ\ 54'$).

| | | | |
|---|---|---|---|
| 1900 | December 24 | + 95 km | Campbell |
| | December 25 | + 96 | Campbell |
| | December 30 | + 94 | Campbell |

$\theta$ CANIS MAJORIS ($\alpha = 6^h\ 50^m$, $\delta = -11^\circ\ 55'$).

| | | | |
|---|---|---|---|
| 1897 | December 15 | + 96 km | Campbell |
| 1899 | October 16 | + 96.0 | Wright |
| 1900 | October 9 | + 95.5 | Wright |

I PEGASI ($\alpha = 21^h\ 17^m$, $\delta = +19^\circ\ 23'$).

| | | | |
|---|---|---|---|
| 1900 | July 3 | − 75.7 km | Wright |
| | July 8 | − 74.9 | Wright |
| | July 16 | − 77.1 | Wright |

$\mu$ SAGITTARII ($\alpha = 18^h\ 7^m.8$, $\delta = -21°\ 05'$).

| | | | |
|---|---|---|---|
| 1899 | June 19 | − 75 km | Wright |
| 1900 | May 30 | − 76 | Wright |

These measures are subject to an uncertainty of several kilometers, on account of the character of the spectrum.

LICK OBSERVATORY,
December 1900.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

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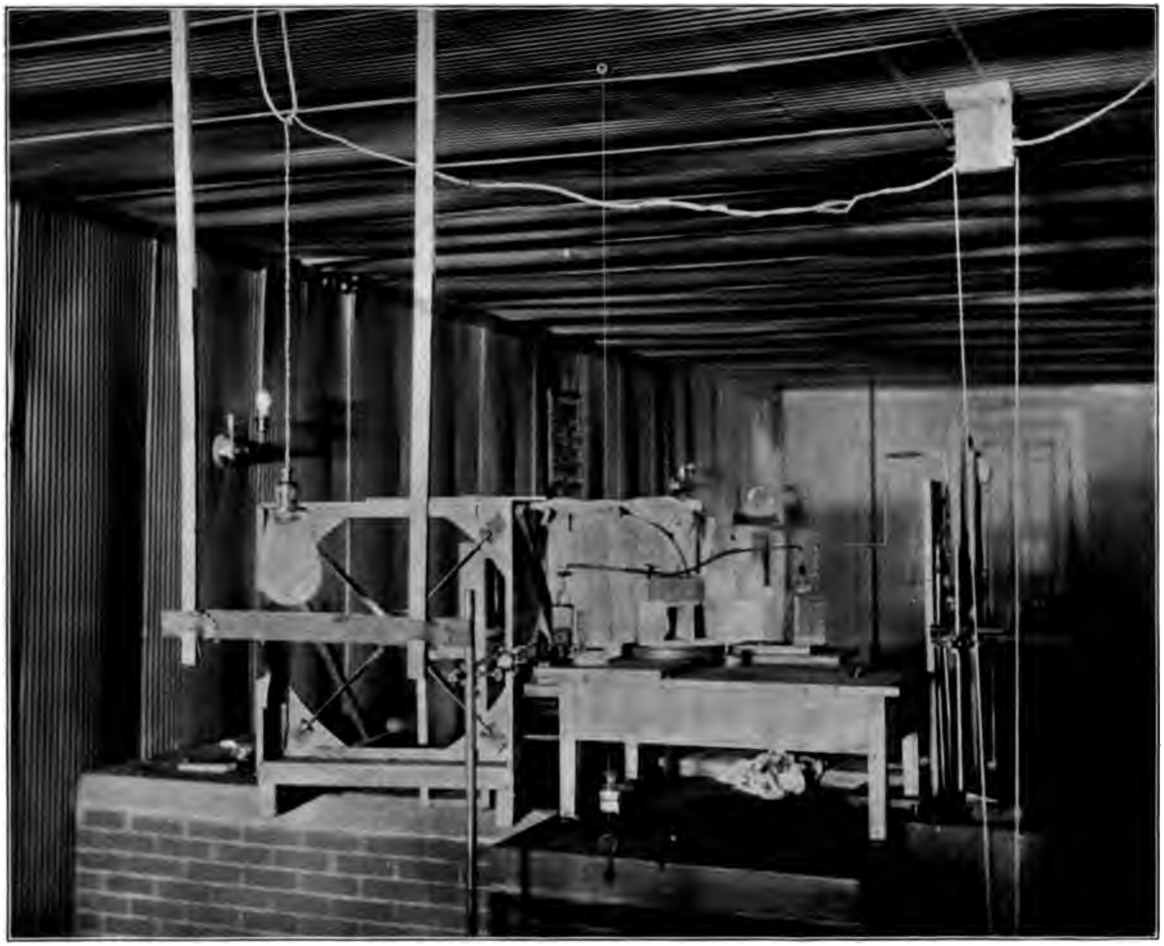
PLATE I.



RADIOMETER AND 24-INCH MIRROR USED IN MEASURING THE HEAT RADIATION OF STARS AND PLANETS.

THE

# ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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## ON THE HEAT RADIATION OF *ARCTURUS*, *VEGA*, *JUPITER*, AND *SATURN*.

By E. F. NICHOLS, assisted by A. L. COLTON and C. E. ST. JOHN.

IN 1888 Professor C. V. Boys,[1] equipped with his newly invented radiomicrometer, began a series of measurements to determine the heat radiation of the brighter stars. The published results of the work of Dr. Huggins,[2] in 1869, and of Stone's experiments[3] on the same subject, in 1870, had encouraged him to hope that the far greater sensitiveness to heat radiation, gained in the new radiomicrometer over the thermopile (or thermojunction) and galvanometer used by Huggins and Stone, might make possible a comparison of even the fainter red stars, comets and nebulæ. The failure which attended the earlier observations on the brightest stars and planets led to an effort toward greater delicacy in the apparatus, and a higher degree of sensitiveness in the radiomicrometer was reached.

In Boys' apparatus the receiving surface of a radiomicrometer suspension was mounted in the focus of a 16-inch reflecting

[1] C. V. BOYS, *Proc. Roy. Soc.*, **47**, 480, No. 291, 1890.

[2] W. HUGGINS, *Proc. Roy. Soc.*, **17**, 309, 1868–9.

[3] E. J. STONE, *Proc. Roy. Soc.*, **18**, 159, 1869–70.

telescope of 67.8 inches focal length. The most delicate suspension employed presented a blackened receiving surface of 4 sq. mm at the sensitive junction, and was hung from a quartz fiber of $\frac{1}{5780}$ inch diameter. The period of vibration was 10 seconds and the sensitiveness was such that without mirrors or lenses for concentration, the heat from a candle at a distance of 60 inches gave a deflection of 60 mm. A further test of the sensitiveness of the radiomicrometer connected with the reflector showed that on a dry, clear night a deflection of 38 mm could be obtained from a single candle at a distance of 250.7 yards.

Professor Boys' observations were continued, at intervals, from September 1888 to April 1890, and the objects observed were the Moon, *Venus*, *Jupiter*, *Saturn*, *Arcturus*, *Vega*, *Capella*, *Altair*, and other stars. With the sensitiveness used, $\frac{1}{150000}$ of the heat sent by the full Moon to the mirror could have been detected. Slight deflections, which Boys himself regarded as of questionable origin, were obtained on one occasion from *Venus*. From none of the other planets nor stars were any indications of heat observed; certainly not as much, according to Boys' reckoning, as would be received by the mirror from a candle at the distance of 1.71 miles, were it not for atmospheric absorption. Boys' results show conclusively that the heat effects obtained by Huggins and by Stone must have been from accidental sources and could not have been due to the radiations from the stars, to which they were attributed. The measurements to be given later in the present paper will be seen to justify Boys' conclusions concerning his own measurements, and to confirm (were further confirmation necessary) his opinion of the accidental origin of the heat measured by Huggins and Stone.

Mr. T. A. Edison,[1] while with the Draper eclipse party at Rawlins, Wyoming, in 1878, made a short series of observations on *Arcturus* to test the sensitiveness of his micro-tasimeter. The tasimeter was placed in the principal focus of a 4-inch Dolland telescope. The result of five successive exposures gave consistent deflections on the side of heat. No very trustworthy

[1] T. A. EDISON, *Am. Jour. Sci.*, 67, 52, 1879.

statements of the sensitiveness of the micro-tasimeter were accessible to the writer. One statement[1] gives the sensitiveness as such that the "heat of the hand at a distance of six or eight inches threw the galvanometer light-spot off the scale," a feat not beyond the powers of a moderately good thermopile in connection with a sensitive galvanometer. Again,[2] it is stated that the instrument, in Mr. Edison's hands, was capable of "measuring the one fifty-thousandth of a degree of heat."

The radiometer used in the present study is shown later to have been at least twelve times more sensitive than Boys'[3] radiomicrometer, which would show the one one-millionth of a degree rise of temperature.

It appears, therefore, that an instrument capable of measuring temperature differences of the order of one ten-millionth of a degree Centigrade, placed in the principal focus of a mirror of two feet aperture, is required to show any indication whatever of heat from *Arcturus*. For a 4-inch aperture, a sensitiveness corresponding to the one one-hundred-millionth of a degree, or, more probably, the one one-thousand-millionth of a degree, would be necessary. That measurements with such a sensitiveness would be practically impossible with any except a compensating instrument, my own experience makes certain. That it is very easy to mistake deflections from accidental sources for legitimate ones, the experiments of Sir W. Huggins and Dr. Stone, just cited, are ample evidence. It appears likely, therefore, that Mr. Edison was deceived in his supposed indications of heat from *Arcturus*.

Later, Minchin,[4] working with a selenium photo-electric cell in the focus of a two-foot reflector, measured electromotive forces due to radiations from several planets and stars. It will appear later, in a comparison between Minchin's values and corresponding values given in the present paper, that the photo-electric cell seems to be strongly selective in its action outside the visible spectrum; so that its indications are probably not in

[1] *Chem. News*, 38, 57. [2] *Chem. News*, 38, 26. [3] C. V. BOYS, *loc. cit.*, p. 496.
[4] G. M. MINCHIN, *Proc. Roy. Soc.*, 58, 142, 1895.

proportion to the total radiant energy received from the star: a conclusion which Boys' results serve equally well to establish.

## THE APPARATUS.

1. *The radiometer construction.*—The heat measuring instrument used in the present study was of the same type as the compensating torsion radiometer, of which a description has already appeared.[1] The case of the instrument was made from a block of bronze $5 \times 5 \times 10$ cm, the long axis of which was bored out from the top to within 7 mm of the bottom, by a hole 3 cm in diameter (Fig. 1). Communicating with this axial boring were three lateral borings. Into a boring in the middle of the front face was soldered a tube 22 mm in diameter and 22 mm long, capped at the inner end by a circular brass plate with a central circular opening 13 mm in diameter. A screw thread cut inside this tube near the inner end was fitted by a ring nut. This window was closed by a fluorite disk 21 mm in diameter and 3.41 mm thick, with plane parallel faces. The air-tight packing used during the summer of 1898 consisted of rubber washers smeared with Ramsey's preparation of paraffin, india-rubber and vaseline. These were placed before and behind the fluorite window. Pressure against the packing was produced by screwing up the ring nut. In the work two years later the window was simply cemented into place by Chatterton wax. The apex of the cone of star rays from the condensing mirror entered the radiometer by traversing the fluorite window, and could be directed to fall on one of the blackened surfaces of the suspension directly behind the window. On the opposite side of the bronze case a circular boring 11 mm in diameter was made, coaxial with the boring for the front window just described. The hole was earlier covered by a plate of glass cemented on the back of the case, but later by a plate of fluorite to fit the instrument for the purpose of another study. Through this window at the back of the case, the star image in the radiometer, and the blackened vanes of the suspension, could be seen at the same time. The

[1] E. F. NICHOLS, *Physical Review*, **4**, 297; also *Wied. Ann.*, **60**, 401, 1897.

third boring, 17 mm in diameter, entered the bronze well 25 mm lower than the other two, and on the left hand face of the block, as seen from the front. A piece of good plate glass was cemented over this opening, through which the deflections of the suspension were read by the telescope and scale method. To the top surface of the bronze case, a circular glass plate 73 mm in diameter, with a central circular opening 35 mm in diameter, was cemented. Upon the upper surface of this plate rested a small bell of glass terminating in a tube. The flange of the bell was well ground upon the upper surface of the circular plate. In the tube a short distance above the bell, was a stopcock with oblique bore. Beyond the stopcock the tube was attached by a rubber connection to a glass tube leading from a drying bottle, which contained phosphoric anhydride. Another glass tube leading away from the drying bottle was connected by a short rubber connector to a Geissler mercury air pump. The rubber connections were all smeared on the outside with the Ramsey preparation, which made them nearly enough air-tight for the purpose in hand. In the interior of the bronze case near the top, a narrow brass ring was soldered, and upon this ring rested a light bridge, *c* (Figs. 1 and 2). A torsion head, *a*, carrying the upper end of the suspension, was in turn carried in a small square brass block *b*, free to slide in a slot in the bridge *c*, permitting the suspension to be brought closer to, or withdrawn from, the fluorite window in front of it. The radiometer case was mounted on a tripod with leveling screws (not shown in the cuts).

The radiometer suspension was built up on a whip of fine drawn glass 32 mm long, to the lower end of which was attached a small plane mirror 2.2 by 3 mm, made by silvering a fragment of very thin microscope cover glass. On the axis 22 mm above the mirror, and in a plane at right angles to it, a delicate cross arm of drawn glass was fastened, bearing on its extremities the two blackened radiometer vanes *dd*. The sensitive vanes were circles approximately 2 mm in diameter, which, to secure lightness and uniformity, were stamped out of thin mica

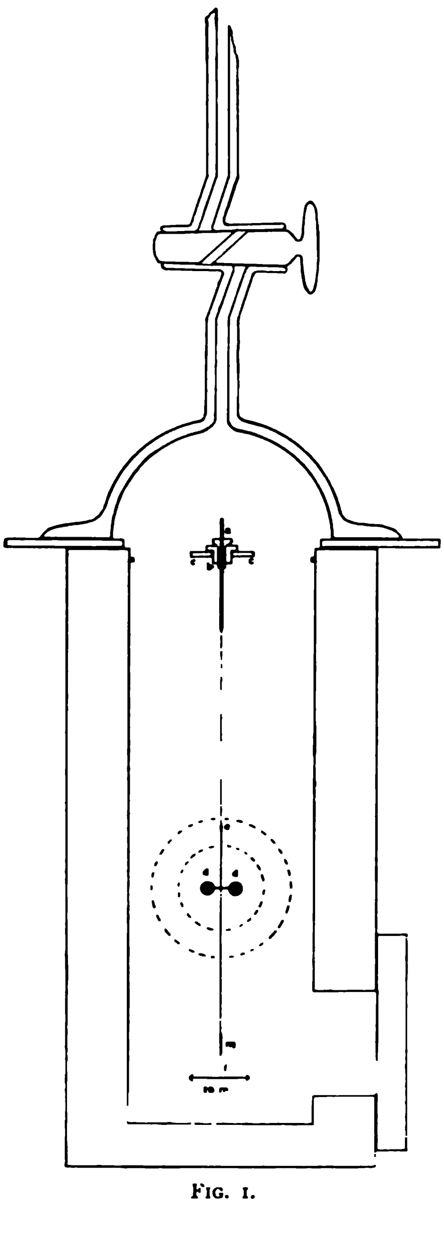


FIG. 1.

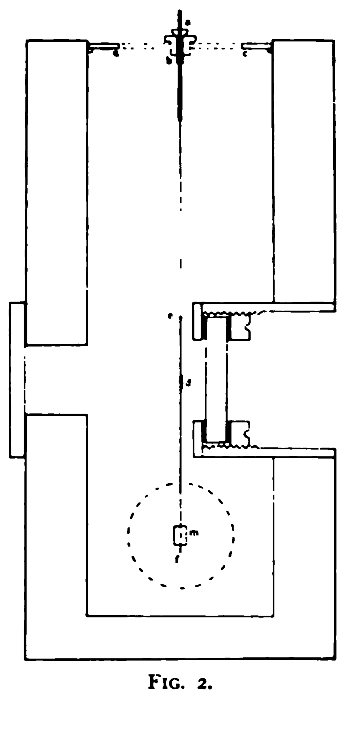


FIG. 2.

with a circular steel punch, made for the purpose. These vanes were uniformly coated with lampblack, and mounted as symmetrically as possible with reference to the axis of rotation *ef*. To the upper end of *ef* a very fine quartz fiber 32 mm long was attached. The upper end of the fiber was made fast to a bit of steel wire, which passed up through a small hole in the axis of the torsion head *a*. The distance between the centers of the vanes was 4.5 mm. The attachments of the fiber, and of the small glass rods and mica disks were all made with shellac. No delicate balance was available, so the exact weight of the suspension could not be ascertained, but from comparison with several similar suspensions built earlier, its weight ought not to have exceeded 6 or 7 mg.

2. *The radiometer as a measuring instrument.*—It is hardly probable that the form and dimensions of the radiometer here described could have been of the most favorable proportions for the greatest sensitiveness. The whole matter of the radiometric activity is too little understood to make theoretical speculations on the best form or proportions of parts of much practical value. My own experience, furthermore, has been too short to lead to any exact quantitative rules for construction. The instrument, as described, was built with the following considerations in mind: (1) It appeared from earlier experience that the maximum sensitiveness of the radiometer, as it changes with the pressure of the enclosed gas, increases as the vanes are brought nearer the window in front. They cannot be brought too near the window, however, and still keep the deflections closely proportional to the energy received for deflections widely different in magnitude. In the present instrument the vanes were from 2.5 to 3 mm behind the fluorite window. For this distance it has been proved in an earlier paper[1] that the deflections are proportional to the energy causing them. (2) The instrument must have as short a period as is consistent with a high sensitiveness. In the radiometer used in 1898, the period was 11 seconds, so that the maximum effect of an

[1] E. F. Nichols, *loc. cit.*, p. 300.

exposure to a source of radiation was reached in 5½ seconds. To accomplish this, as well as to insure a more constant zero by giving nearly equal exposure of both vanes to all objects in front of the radiometer, the vanes were brought close together. (3) The sensitiveness seems very closely related to the damping, and this in an inverse order. For that reason the vanes were made as small as possible, and still be large enough to receive the whole planet or star image. In this way, with vanes of $\pi$ sq. mm exposed surface, a sensitiveness per sq. mm was obtained, six times as great as that (with nearly the same period) in a suspension with rectangular vanes $2 \times 16$ mm. It would doubtless have been better to still further reduce the size of vanes, if the work could have been done with sufficiently well figured reflectors so that perfect images had been possible. (4) Sensitiveness of the vanes under all circumstances is most intimately connected with the pressure of the surrounding gas. The pressure corresponding to maximum sensitiveness could not be measured, because no pressure gauge was at hand. In a similar instrument, but with larger vanes, a pressure of 0.05 mm of mercury gave maximum sensitiveness. Fortunately the curve of sensitiveness, as dependent upon pressure, is a curve somewhat flattened at its maximum. Small changes of pressure at this point affect the sensitiveness but little. Some standard of radiation, such as a Leslie cube, or in very crude work, a candle, is needed as a reference, because there is no way of referring the sensitiveness in the radiometer, as in the bolometer, to anything as definite as the period of a galvanometer and the current through a bridge. It ought to be possible, however, by improved construction, to make the radiometer case more completely air-tight, or else to calibrate the sensitiveness as a function of the pressure, and measure this. I have not so far used the radiometer in a way which made such precautions necessary.

As compared with the bolometer or thermopile, the present form of radiometer has the following advantages: (1) It is uninfluenced by all magnetic and thermoelectric disturbances, which beset a sensitive galvanometer. (2) The radiometer is

free from any disturbances corresponding to the convection currents which arise about a heated bolometer strip. It has, however, the following disadvantages: (1) It is not so easily portable as the thermopile or bolometer. (2) All rays to be measured must traverse the window of the radiometer and be subject to its selective absorption and reflection.

3. *The arrangement of reflectors.*—Observations were made in the month of August in the summers of 1898 and 1900. The arrangement of mirrors in the two series differed sufficiently to make separate description of the schemes desirable. The two plans are shown in Figs. 3 and 4.

The room in which the experiments were made was the heliostat room [1] of the Yerkes Observatory. This room could hardly be improved in its appointments for the work in hand, and was in fact designed purposely for work of a similar nature. The gallery to the left of the double partition is provided with a movable roof and sides, which slide back between the walls of the enclosed room to the right, leaving only a low parapet above the level of the floor. The only openings through the double partition are a window large enough to admit the beam from the heliostat at *H*, and a passage way closed by double doors. The beam from the heliostat fell upon a two-foot (61 cm) concave mirror *M* of 7 ft. 9 in. (233 cm) focal length, figured and silvered by Mr. G. W. Ritchey. The converging cone was caught on a small 45° flat mirror, *f*, 4 × 6 inches (10.2 × 15.4 cm), and directed thence into the radiometer case through the fluorite window, the focal point lying in the plane of the vanes. The heliostat, *H*, used in 1898, was of a modified Foucault type, built by Adam Hilger, and belonging to the Allegheny Observatory. It was made to carry a 17-inch flat mirror, and was earlier used by Professor S. P. Langley in his work on the temperature of the Moon. As it was necessary to get a larger beam to fill the aperture of the 2-foot concave mirror, Mr. Ritchey selected from many pieces a sheet 30 × 36 inches (76.2 × 91.4 cm) of best commercial plate glass ⅜ in.

[1] GEORGE E. HALE, ASTROPHYSICAL JOURNAL, 5, 260, 1897.

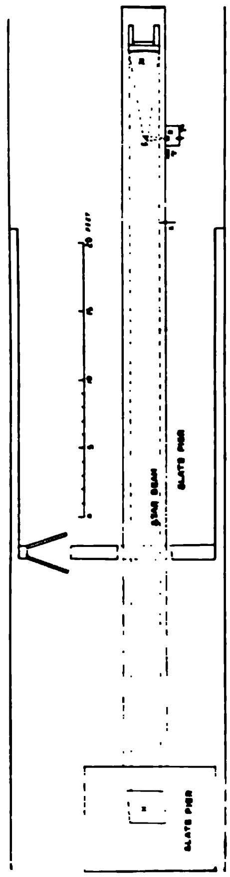


Fig. 3.

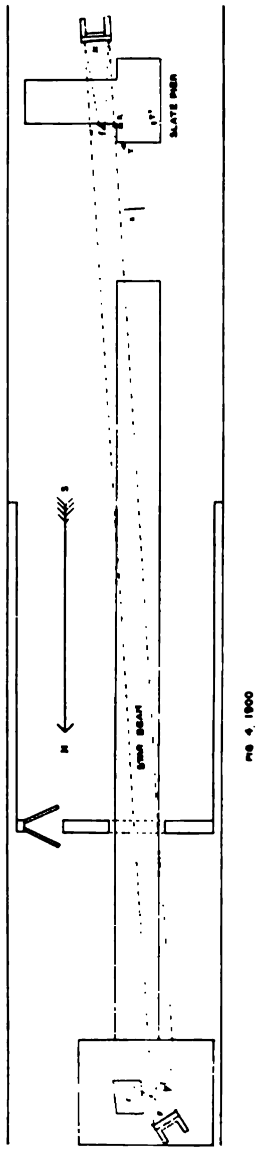


Fig. 4.

thick. This plate was silvered by Mr. Ritchey, and mounted in a frame attached to the metal cell of the heliostat. Its thickness was not great enough to support its weight, and preserve a plane surface. Mr. Ritchey accordingly introduced a system of three levers in the bed of the frame, each lever pivoted at the middle, and carrying a cushion on either end. The glass rested on the cushions in such a way that each cushion was at the geometrical center of each sixth part of the mirror. By this means images sufficiently good for the purpose were in most cases obtained. The heliostat had unfortunately been a good deal worn and racked by use, and the load of the large mirror was more than it was built to carry, so that the manner of its driving and adjustment left much to be desired. The radiometer, *R*, was mounted on a wooden table, standing on an overhang built out from the long slate pier shown in the diagram. An observer at the telescope, *T*, read the deflections of the radiometer in millimeter divisions on a scale at *S*, behind and above him at a distance of about 6 feet (183 cm) from the radiometer. Cords connected to the slow motions on the heliostat were brought to a point within convenient reach of a second observer at the telescope *T′*, which was focused on the sensitive vanes, as seen through the window in the back of the radiometer case. The latter observer could keep the star image constantly in sight, except when it fell upon one of the vanes, in which case a very small quantity of stray light in the image showed its position.

### THE AUGUST 1898 OBSERVATIONS.

The first series of observations was made on *Vega*, the night of August 3, 1898. From July 5, when the work was begun, the time had been spent in building the radiometer and in assembling and adjusting the auxiliary apparatus. The work on the radiometer suspension was too hastily done, with the result that although the sensitiveness was in the main sufficient, the compensation was so poor that small disturbances in the sky and temperature changes in the room had too great an effect on

the zero. A light cardboard house was built around the radiometer case, which partly shielded it against sudden draughts, but its restlessness was in greater part due to unsteady sky conditions. It was found later that the coat of lampblack on the inside wall of the case, directly opposite the fluorite window, had been marred in a way to expose a portion of the cylindrical surface of the metal. Rays which came from the mirror in a particular direction and were focused in the plane of the vanes, were reflected by the exposed wall and brought to a focus on the back surface of one vane. This discovery accounted for some of the restlessness of the suspension in use.

*The method of observing.*— As a result of the experience of others, it was obvious at the outset that no form of shutter could be used in making the observations, and that to obtain results of any value the radiation from the star must be directly compared with that from the open sky, at a point very near the star. It was further desirable that the motion of the mirror, by which the star image was exchanged for a sky image, should be small, and take place as far from the radiometer as possible. With an observer at each of the telescopes $T$ and $T'$, the observer at $T$ watched the motion of the radiometer, and waited for a period of comparative quiet which would bring the image of the scale to rest; then signaled to the observer at $T'$ to throw the star image on the vane, or off it, as the case might be, by means of the cords running to the slow motion of the heliostat. Had the heliostat always obeyed the cords with a consistent action, the difficulties of observation would have been greatly simplified. Any delay in bringing the star image on, or off, the vane at the signal of the first observer, was disastrous, for the radiometer rarely maintained the almost absolute quiet necessary for a successful measurement of such minute deflections, longer than the five or six seconds required for the legitimate deflection to take place. Under the most favorable circumstances, very few uninterrupted deflections were obtained, and many contradictory single deflections were the rule in every series. These harassing disturbances were often considerably

greater than the deflection caused by the star; so that the only hope lay in the fact that they were strictly accidental and not systematic, and that therefore they would neutralize in making up the average of a long series of deflections. This has proved to be the case. At a given signal from the observer at *T*, the image of the star was thrown on one of the vanes, if off, or *vice versa*, and after a suitable time the radiometer deflection was read. In this way the throw in one direction only was taken. If the external conditions could have been depended upon to remain stationary, it would have been advantageous to read the return swing by undoing the previous change at the first reading. This, however, was out of the question. So that if the star image was thrown on one of the vanes, it was in general, though not invariably, kept there after the deflection was taken, to await another season of comparative quiet. In collecting and comparing the deflections at the close of a series, they were divided into two sets, called "on" observations, when the star image was thrown on one of the vanes, and "off" observations, corresponding to the deflection obtained when the image was moved off the vane. Thus the average of the "on" observations should show a repulsion between the fluorite window and the vane on which the star image was thrown, and the "off" observations should show an apparent attraction between the vane and the window. The direction of the apparent movement of the scale thus produced would depend on which vane was used; one vane giving, for the same treatment, a deflection opposite in direction to that of the other.

In averaging a series of observations to determine the quantity of heat received, the "on" observations and the "off" observations were averaged separately. The average of the "on" observations in all series, save the series of August 4 on *Vega*, showed repulsion between the vane on which the star image fell and the window in front. The averages of the "off" observations in the fourteen series made, without exception, showed attraction between the vane and window.

In computing the probable error, given in the combined

average column in the tables, the residuals of the individual "on" observations were made up with reference to the average of the "on" observations in each series, and the residuals of the individual "off" observations, with reference to the average of the "off" observations. The probable error was then computed from the sum of the squares of both sets of residuals and the combined number of "on" and "off" observations, in the usual way. The reason for this method of treatment lies in the consideration that during many of the series there was a very small but persistent drift of the zero in one or the other direction, the effect of which would be to augment either the "on" or the "off" observations, at the expense of the other. The fact that only one set of the "on" or "off" observations, in a total of 28, gave a contradictory result, shows the average drift in all but this one case to be less than the counter influence of the star.

It was extremely important, in measuring such small deflections under disturbed conditions, that the observer at $T$ should be wholly unprejudiced. This was accomplished by Mr. Colton, who sat behind the radiometer, arranging in his own mind a series of signals, which were used to call a change by the observer at $T$, who was kept in ignorance as to which signal meant "off" and which "on;" further, he did not know which vane of the radiometer was in use. On his part he kept the deflections read to himself, so that the observer at $T'$ did not know what results the radiometer was showing in response to his management of the star image. At the end of a series, the record of deflections was compared with the signals and its indication of heat or cold for the star image was determined. To prevent the large deflections which were sometimes obtained (in which the star could obviously have had but little part) from exerting an overwhelming influence upon the average, it was decided to throw out all deflections greater than 2 mm, irrespective of their direction, as well as deflections which were spoiled by an evidence of some extraneous disturbance, setting in before the signal could be executed.

*Sensitiveness of the apparatus.*—It was found impracticable, without spending too much time, to make the radiometer case perfectly air-tight. It was consequently necessary, once in two or three days, to pump it out in order to maintain the requisite sensitiveness, which ran down slowly from the maximum point, as the pressure increased through leakage. As has been already mentioned, it was advisable to keep track of the sensitiveness from time to time, by measuring the throw obtained from some reasonably constant source. As the work on the stars was so very roughly quantitative, due to changing atmospheric absorption, as well as to the difficulty in accurately measuring such minute deflections under the uncontrollable conditions which surrounded the work, it was decided that with reasonable care and attention to the flame, a candle at a distance from the instrument would most conveniently serve the purpose. It should be borne in mind, that the *total radiation* from a candle is subject to smaller variations than the *photometric intensity*. No standard candles were to be had, but a uniform grade of paraffin candles was obtained, which normally burned 7.6 grams of paraffin per hour.

Such a candle was placed at a distance of 830 cm (27 ft. 3 in.) from the radiometer. A small silvered flat mirror was interposed in the path to direct the candle rays into the radiometer. In determining the sensitiveness, the candle was lighted and left to burn until its flame had assumed approximately normal height and diameter, and then five or six exposures were made by raising a shutter near the candle. Deflections were taken in the one direction only, *i. e.*, not counting the return swing. The average was taken as the sensitiveness. The changes in sensitiveness with time were so gradual that one determination was sufficient for an evening's work.

The sensitiveness of the radiometer during each series of observations, in terms of a candle 830 cm distant, is given in the "sensitiveness" column in the tables. The sensitiveness for direct comparison with the star heat can perhaps be best gotten at roughly, by computing the deflection which should be caused by all the heat from a candle one meter distant, which fell on a

surface equal to the effective aperture of the 2-foot mirror. From the law of inverse squares, a deflection of 7.5 mm from a candle 830 cm distant (the mean of the sensitiveness actually used) would correspond to a deflection of 520 mm for a candle one meter distant. The ratio of the surface of the radiometer vane to the effective aperture of the concave mirror was approximately 1 : 94968. The corresponding deflection for all the heat from a candle one meter distant, incident upon the aperture of the concave mirror, should be $520 \times 94968 = 49380000$. A deflection of 1 mm, for a star at this sensitiveness, would thus signify that the intensity of the star's radiation was about one forty-nine-millionth part of that of a candle at a distance of one meter. The mean of the results for *Arcturus*, in the combined average column reduced (without correction for atmospheric absorption) to sensitiveness 7.5, was 0.53 mm; the heat from *Arcturus* would thus be something greater than the one one-hundred-millionth part of the heat from a candle at a distance of one meter.

The column headed "Sens. $10^{-8}$ meter-candle," in Tables I and II, contains the "Combined average," reduced to sensitiveness 15.4 mm, for comparison. For this sensitiveness, 1 mm would correspond to the one one-hundred-millionth part of the heat from a candle 1 meter distant, incident upon a disk equal to the effective aperture of the concave mirror. The last column in the table shows the average zenith distance of the star during each series of observations. The column headed "No. neg. obs.," gives the total number of deflections of wrong sign obtained in each series.

*Description of the nights on which observations were made.*— August 3. Sky cleared after several days of stormy weather; fairly transparent; full Moon.

August 4. Sky thick and whitish; thermal conditions very unsteady.

August 5. Sky somewhat more transparent than on previous night and thermal conditions steadier; sky clouded over by 10 P.M.

August 7. Sky, between clouds, very transparent; break of

one half hour in series on *Arcturus* because of clouds; completely clouded over at 9:45.

August 8. Sky clear and fairly transparent; gentle east wind.

August 9. Sky thick and white; fresh east wind; thermal conditions unsteady.

August 11. Sky clear, but only moderately transparent.

August 13. Sky thick; conditions during series on *Arcturus* much disturbed; lightning in the west; western sky whitish.

TABLE I.

*ARCTURUS.* AUGUST 1898.

| Date | Time P.M. | "On" obs. | | "Off" obs. | | No. neg. obs. | Combined average | Sens. | Sens. $10^{-8}$ meter candle | Zenith dist. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | No. | Av. | No. | Av. | | | | | |
| 4 | 8.00–9.30 | 18 | 0.32 mm | 20 | 0.48 mm | 11 | 0.40 mm ± 0.08 | 8.4 | 0.65 | 49° 40′ |
| 5 | 8.00–9.05 | 24 | 0.42 | 21 | 0.50 | 11 | 0.45 ± 0.11 | 6.6 | 1.06 | 47° 20′ |
| 7 | 8.40–9.45 | 14 | 0.94 | 12 | 0.61 | 5 | 0.78 ± 0.11 | 7.5 | 1.60 | 56° 10′ |
| 8 | 7.45–9.45 | 7 | 0.67 | 13 | 0.61 | 5 | 0.63 ± 0.11 | 7.5 | 1.30 | 55° 10′ |
| 9 | 8.30–9.45 | 15 | 0.50 | 32 | 0.43 | 12 | 0.45 ± 0.11 | 7.1 | 0.98 | 56° 50′ |
| 11 | 8.30–9.15 | 14 | 0.57 | 14 | 0.25 | 9 | 0.41 ± 0.13 | 4.6 | 1.36 | 60° 30′ |
| 13 | 7.45–9.45 | 14 | 0.40 | 8 | 0.30 | 9 | 0.36 ± 0.17 | 8.2 | 0.68 | 55° |
| | | | | | | | | | Mean | 54° 45′ |

TABLE II.

*VEGA.* AUGUST 1898.

| Date | Time P.M. | "On" obs. | | "Off" obs. | | No. neg. obs. | Combined average | Sens. | Sens. $10^{-8}$ meter candle | Zenith dist. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | No. | Av. | No. | Av. | | | | | |
| 3 | 9.45–11.00 | 36 | 0.45 mm | 38 | 0.28 mm | 19 | 0.36 mm ± 0.06 | 10.0 | 0.55 | 8° |
| 4 | 10.30–12.00 | 15 | -0.14 | 17 | 0.46 | 9 | 0.17 ± 0.07 | 8.4 | 0.31 | 18° 40′ |
| 8 | 10.00–11.30 | 21 | 0.16 | 24 | 0.44 | 14 | 0.31 ± 0.08 | 7.5 | 0.64 | 19° 50′ |
| 9 | 9.45–11.00 | 17 | 0.12 | 23 | 0.17 | 16 | 0.15 ± 0.11 | 7.1 | 0.33 | 12° 40′ |
| 11 | 10.15–10.50 | 10 | 0.11 | 10 | 0.24 | 9 | 0.18 ± 0.12 | 4.6 | 0.60 | 15° 40′ |
| 12 | 10.00–10.30 | 10 | 0.09 | 11 | 0.36 | 5 | 0.21 ± 0.08 | 6.5 | 0.50 | 13° 20′ |
| 13 | 9.45–11.00 | 23 | 0.37 | 24 | 0.36 | 15 | 0.36 ± 0.09 | 8.2 | 0.68 | 14° 3c′ |
| | | | | | | | | | Mean | 14° 45′ |

On the night of August 12 the method of observing was tested in two ways, to discover whether any heat effects could be detected, due to the movement of the heliostat mirror in throwing

the star image on and off the vane. A series of sixteen observations (sensitiveness 6.5), was made on a star too small to affect the radiometer, and used only as a mark in the sky. The measurements were carried out, and the results treated, in precisely the same way as in the series on *Arcturus* and *Vega*. The resulting "combined average" for the star was 0.03 mm, indicating cold. Immediately following, another series of sixteen observations was made with the same star in the field, but only moved back and forth underneath the vane, not touching it. The combined average was likewise 0.03 mm, indicating heat when the star image was directly under the vane. The smallness of these values is purely accidental, as the probable errors in the regular series run much higher. The tests serve merely to show that no considerable systematic error was introduced in the working of the apparatus.

Table III shows a comparison between the different series on *Arcturus* and on *Vega*, for the five nights on which both were observed. It must be borne in mind, however, that the candle heat in terms of which the sensitiveness was measured suffered but one reflection on silver, while the star heat was reflected from three surfaces before entering the radiometer; further, that the heat in all cases lying beyond 8 $\mu$ was *largely*, and that beyond 9.4 $\mu$ *entirely*, absorbed by the fluorite window. No correction is here introduced for atmospheric absorption, which was greater in the case of *Arcturus* than of *Vega*, as the zenith distances in Tables I and II show.

TABLE III.

*ARCTURUS* AND *VEGA* COMPARED.

| Date | *Vega* | *Arcturus* | *Arcturus* / *Vega* |
|---|---|---|---|
| Aug. 4 | 0.31 mm | 0.65 mm | 2.1 |
| 8 | 0.64 | 1.30 | 2.0 |
| 9 | 0.33 | 0.98 | 3.0 |
| 11 | 0.60 | 1.36 | 2.3 |
| 13 | 0.68 | 0.68 | 1.0 |
| Means | 0.51 mm | 0.99 mm | 2.1 |

In Tables I and II the range between the highest and lowest values for *Arcturus* and *Vega* is seen to be as 2 to 1. Different conditions of the sky on different nights, and at different times during the same night, doubtless account for much of this; also the difficulty of accurate measurement of such minute deflections under disturbed conditions. Lastly, the astigmatism of the heliostat mirror was not always the same. The consequent form of the star image changed in such a way that it was often impossible to center it on the vane, and frequently it was slightly larger than the vane. In Table III the ratios are as accordant as it is reasonable to expect. In the observations of August 13 *Arcturus* was so low in the west that a thunder-storm, which was gathering in that quarter, would explain the low value obtained if a host of other accidental reasons might not have caused it. The observers in the measurements so far given were invariably Mr. Colton and the writer.

The 1898 series was here necessarily brought to a close by the pressure of other engagements.

THE AUGUST 1900 OBSERVATIONS.

*Changes in the apparatus.*—In the second summer's work, the heliostat which had been the cause of so much annoyance was replaced by the heavily mounted coelostat used by the Yerkes Observatory party in the May 28, 1900, eclipse observations at Wadesboro, N. C. The coelostat was driven by the clock of the 12-inch Kenwood telescope. The same plane mirror earlier used on the heliostat was resilvered and mounted on the polar axis of the coelostat. The change to the coelostat made the use of an additional plane silvered surface necessary, to direct the beam to the 24-inch concave mirror. The position of this new vertical plane mirror depended upon the declination of the star observed. The arrangement of the mirrors is schematically shown in Fig. 4, in which $C$ represents the coelostat, $F$, the approximate position of the vertical flat in the *Jupiter* and *Saturn* observations, and $F'$, its relative position for *Arcturus*. The lettering of the remaining parts of the diagram corresponds to that in Fig. 3.

The remainder of the apparatus (Plate I) was mounted further back in the covered gallery than in the arrangement used in 1898. The use of selected commercial plate glass silvered had worked so well in the case of the heliostat mirror, that a plate 27 × 40 inches was chosen and mounted in an upright wooden frame, to be used at F. The observations on *Jupiter* of August 5, 6, 8, and 9, were made with this, and although the images obtained were far from satisfactory, showing a marked astigmatism. This fault was so greatly increased at the greater incidence necessary for the observation of *Arcturus*, as to render its employment out of the question. In meeting this difficulty, Director Hale kindly put at my disposal a fine 24-inch circular flat mirror, freshly silvered and very perfectly figured by Mr. Ritchey. This mirror was used in the subsequent measurements. The angle of incidence of the *Jupiter* beam from the coelostat upon this mirror was 30°, and the reflected beam filled only 96 per cent. of the aperture of the 24-inch concave mirror. A correcting factor of 1.04 was consequently to be applied to the heat measurements on *Jupiter* and *Saturn* to reduce them to full aperture. In the observations on *Arcturus* an incidence angle of 50° occurred, and a consequent factor of 1.22 was used to reduce the observations to full aperture. Unfortunately, *Vega* was beyond the limited range of the new apparatus, so that no further measurements upon that star were possible.

The connection between the driving-clock and the coelostat axis was through a worm and driving sector. This was set for a small lost motion, the axis counterpoised, and a cord attached to a lever on the axis; thus, by pulling the cord sharply the axis and the mirror attached to it could be rotated through an angle equal to the lost motion. In this way the star image was easily and promptly thrown on and off the radiometer vane by the observer at T. This device worked faultlessly.

From the summer of 1898 the radiometer had stood undisturbed in the position where it had been earlier used. It was necessary to open the case to recoat the inside surface with lampblack, and by an unfortunate accident in entering the case,

the fiber of the suspension was broken. A new fiber was attached, and the blackened surfaces of the vanes marred by the accident, were repaired. Several days were spent in equalizing the two vanes so that the compensation was much more complete, and the instrument was steadier in its action than before. The new fiber proved to be slightly finer than the old one, so that the period was raised from 10 or 11 to 13 seconds, and the sensitiveness correspondingly increased. The radiometer used under the new conditions was much more efficient than before.

*Sensitiveness of the apparatus.*—The sensitiveness in the different series of observations ranged between 9.9 and 12 mm for the heat from a candle 811 cm distant, after one reflection. The averages of the various series are given as they were observed, and the probable error computed as already described. A deflection of 11 mm (the average sensitiveness) for a candle 811 cm distant, would mean a deflection of $11 \times (8.11)^2 = 724$, for a candle one meter distant. Taking into account the ratio of the surface of the radiometer vane to the effective aperture of the mirror, 1 : 94968, we have 68750000. A deflection of 1 mm would be caused then by $\frac{1}{68750000}$ of the heat received on a surface equal to the aperture of the concave mirror, from a candle 1 meter distant. If the averages are to be expressed in terms of the unit $10^{-8}$ meter candle, they must be reduced to a sensitiveness corresponding to 16 mm for a candle at a distance of 811 cm. The two years' observations will then be reduced to the same unit.

*Sensitiveness of the apparatus and atmospheric absorption.*—Although the processes by which the above equivalence of a 1 mm deflection to the $\frac{1}{68750000}$ part of the heat from a candle 1 meter distant, when concentrated by the 24-inch concave mirror, seemed legitimate, still no allowance was made for the reflection on two additional surfaces. It was therefore desirable to make actual tests of the apparatus upon a candle at so great a distance that the radiometer sensitiveness could be determined, so far as possible, in connection with the same mirrors used in the star observations. In such a test, it was plain that the

absorption of a long layer of intervening air would be unavoidable. In order to obtain a correction factor for this absorption, it was decided to establish two stations at different distances from the Observatory.

From the parapet of the heliostat gallery, the ground with very slight undulations stretches away to the westward for several miles in nearly level fields of pasture and cultivated ground, with no intervening trees or objects to obstruct the view (Fig. 6, Plate II). From the base of the parapet some 25 feet below, a line was run due westward, and a distance of 2000 feet chained off. At this point the first station was established, and the same line continued 2500 feet further, or 4500 feet, from the parapet, where the second station was located (Fig. 5, Plate II). A tent was set up at each station, and inside each, a wooden box mounted four feet or more from the ground on stakes. Vertical slits 2½ inches wide, and a foot or more long, were cut in the boxes on the side toward the Observatory. The boxes were otherwise well ventilated, so that a candle placed inside burned quietly with a flame of normal height. At first the slits were provided with light shutters which could be raised and lowered by the drawing of a cord by an assistant at a considerable distance from the box. It was found, however, that moving the shutter, with no candle in the box, often caused a small deflection. It was therefore decided to stand the candle on a small sliding carriage on the floor of the box, by which it could be drawn up behind the opening, or allowed to slide to one side of the slit, so that the wall of the box concealed it. It was found that moving the candle back and forth produced no deflection when the candle was not lighted, nor did it seriously disturb the flame when lighted.

FIG. 7.
(2½ times natural size.)

From the parapet the line of sight to the further tent passed directly over the nearer tent. The 24-inch flat mirror *F*, Fig. 4, was turned until its plane lay N. W. and S. E., and so adjusted that the whole of the reflected beam fell on the 24-inch concave

PLATE II.

FIG. 5.
TAKEN WITH TELEPHOTO LENS.

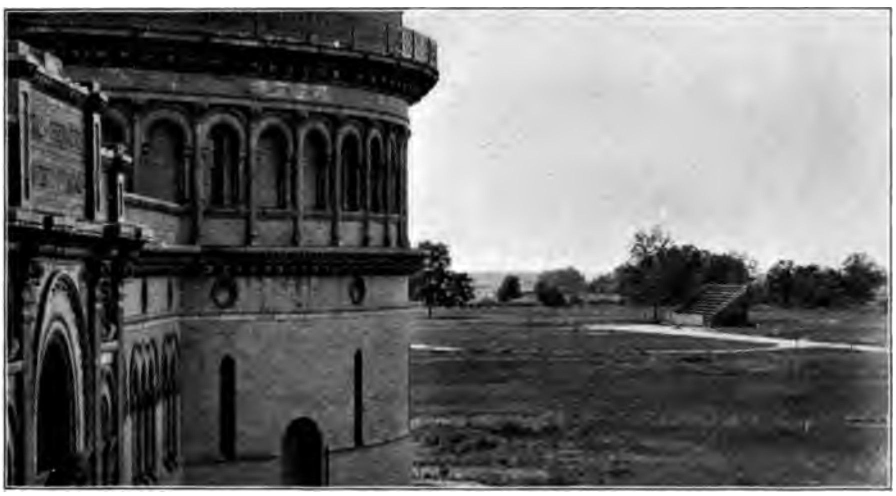


FIG. 6.
STATIONS FOR MEASURING ATMOSPHERIC ABSORPTION.
Distances from Observatory: 2000 feet and 4500 feet.

mirror, which formed an image of the candle in the box of the nearer tent upon one of the radiometer vanes (Fig. 7). The coelostat mirror was not used in these measurements. The candle heat suffered three reflections before entering the radiometer, as in the 1898 star heat measures, instead of four, as in the case of the 1900 measurements. The image of the candle in the box in the further tent could easily be brought upon the vane by slipping a small wedge into the space between the frame carrying the mirror *F*, and its support behind it, to which it is hinged at the top.

It was found that the air layer along the ground, which transmitted the radiation from the candles in the two tents, was so disturbed during the day that it was necessary to make the measurements at night; and even then the radiometer was less quiet than when the mirrors faced the sky, as in the star measures.

In making the observations one of the observers stationed himself at the nearer tent, lighted and trimmed the candle, and then, with cord in hand, withdrew to such a distance that, though he could watch the candle, his own image could by no possibility be reflected into the radiometer. A box, with the shutter facing the tents, was placed on the parapet of the heliostat gallery with a lighted lantern inside to serve as a signal to the man in the tent. A cord which operated this shutter was carried in to the observer at the radiometer. When all was ready he drew this cord, which exposed the lantern to the assistant in the tent, who replied by exposing the candle so that its image fell upon the radiometer vane. When the suspension had reached the extreme of its first swing, the lantern shutter was dropped as a signal to conceal the candle in the tent. After a satisfactory series of measurements had been obtained for the candle in the nearer tent, the assistant was signaled to go to the further tent, where the same procedure was followed until a satisfactory series had been obtained from that station. On the way in from the further tent, a second series of measurements was made on the candle in the nearer tent. By this method measurements on the

heat of one candle at a distance of 633.6 meters, and on the heat of two candles at a distance of 1396 meters from the 24-inch concave mirror, were made on August 26 and 27. The series obtained on each of these two evenings showed such large fluctuations in the transmission of the atmosphere as the night advanced, that in each case the mean of the first and last series in the near tent could hardly be depended upon to represent the atmospheric conditions during the middle series in the further tent. On August 29 it was decided to make three series of observations in as rapid succession as possible from the nearer tent, and in between, two from the further tent.

Professor St. John stationed himself at the further tent and Mr. Ellerman at the nearer one. The observations were made as rapidly as possible. The results for the three evenings' observations on candles in the two tents were as follows:

| Date | Heat of 2 candles at 1396 meters / Heat of 1 candle at 633.6 meters |
|---|---|
| August 26 | 19.3 per cent. |
| August 27 | 25.0 " |
| August 29 | 20.2 " |
| Mean | 21.5 per cent. |

If there had been no loss by atmospheric absorption the ratio, according to the law of inverse squares, should equal $\frac{2\,(633.6)^2}{(1396)^2}$ $=41.2$ per cent. The mean transmission of candle heat of an air layer of 762.4 meters along the ground, for the three evenings, is thus $\frac{21.5}{41.2}=52.3$ per cent. Applying the general absorption equation, in which $I$ equals the original intensity and $a$ the absorption of a unit layer, the intensity after traversing $n$ such layers will be $I(1-a)^n$. Taking 100 meters as the unit layer, and calling the original intensity 1, we have $(1-a)^{7.62}$ $=0.523$; whence $a=0.081+$, or the transmission of 100 meters $=0.918$.[1] In connection with his study on the temperature of

[1] It will appear from this transmission coefficient that an air layer along the ground, 250 meters in length, will absorb as large a percentage of the total radiation from a candle as the percentage of starlight absorbed by the depth of the whole

the Moon, Professor Langley[1] measured the transmission of an air layer of 100 meters along the ground for rays from a large blackened Leslie cube filled with boiling water. The measurements on four different evenings in June and August gave absorptions varying between 14.6 per cent. and 32.9 per cent., depending on the quantity of precipitable water in the air. The extreme range of absorption noted in the case of the present measurements occurred between the two series of observations made the evening of August 26 on the candle in the near tent. The first series was made at 8:30 P. M.; the second at 10:30 P. M. During this interval the temperature fell from 68° Fahr. to 60°, accompanied by the formation of an unusually heavy dew. The mean deflection of the earlier series was 36 mm; of the later, 92 mm. The average absorption given above for 100 meters corresponds to a deflection of 61 mm, at the same sensitiveness, for a candle in the nearer tent.[2] Unfortunately for the comparison, it was not convenient to make any accurate determination of the precipitable atmospheric moisture in the foregoing measurements, but if the results can be taken to represent average conditions, it would appear that atmospheric absorption is less for the heat of a candle than for that from a black radiator at 100°[3]. The maximum in the energy curve of a black radiator at 100° was found by Langley to fall in the region of the spectrum corresponding to the greatest atmospheric absorption. Hence, the higher the temperature, the smaller will be the percentage of the whole emission lying in the region of these

atmosphere. In the course of the observations on the heat from a candle in the nearer tent, the absorption of a sheet of plate glass, held about six inches in front of the candle, was found to be 55 per cent. The absorption, by the same glass plate, of the heat from a candle in the same room with the radiometer was measured and found to be practically the same percentage as in the tent.

[1] S. P. LANGLEY, *Mem. Nat. Acad. Sci.*, Vol. IV, Pt. 2, p. 183.

[2] During a series of measurements on a single candle in the near tent made August 25, Professor St. John extinguished the candle and placed his head in front of the candle box at the signal, instead of exposing the candle. The uniform deflection obtained was 25 mm. The candle gave a deflection of 62 mm.

[3] It is possible that the absorption of the fluorite window may have exerted a small influence on the present measurements of atmospheric absorption.

wave-lengths, and the smaller the percentage of absorption by moist air for the total radiation. This raises a question pertinent to the present study, namely, the fallacy of using the same correction factor for atmospheric absorption in comparing two bodies of as obviously different energy spectra as *Arcturus* and *Vega*.

At the average sensitiveness (11 mm for the test candle 811 cm distant), the mean deflection for a candle in the nearer tent, 633 meters distant, was 67 mm. To correct for absorption $(.918)^{6.33} I = 67$, whence $I = 115$. The angle of incidence on the 24-inch circular flat mirror at *F*, Fig. 4, was 47°; consequently, the deflection, if the full aperture of the 24 in. concave mirror could have been employed, would have been $\frac{115}{\cos 47^\circ} = 169$ mm; whence the computed deflection for a candle 1 meter distant would be $169 \times (633)^2 = 67660000$ mm. This agrees very closely with the sensitiveness value, 68750000 mm deduced in the earlier computation. As there were two more reflecting silver surfaces in the path of the beam in the latter than in the former case, the value ought to have been 6 or 7 per cent. smaller. The discrepancy may have been due to error in measuring the diameter of the radiometer vane, which was given as 2 mm. A diameter of 1.94 mm would make the ratio of surface of vane to surface of 24-inch concave mirror enough larger than 1 : 94968 (the value adopted in the previous calculation) to bring both values into agreement.

*The sensitiveness of the radiometer and radiomicrometer compared.* —We now have a double means for comparing the sensitiveness of the radiometer with that of the radiomicrometer used by Professor Boys, which gave a deflection of 60 mm for a candle 152 cm (60 in.) distant. This corresponds to a deflection of 1.7 mm for a candle at a distance of 811 cm. The radiometer under these circumstances gave a mean deflection of 11 mm. The ratio of the area of the receiving service in the radiomicrometer to that of the radiometer was as $4 : \pi$. The ratio of the effective sensitiveness would thus be $1.7\pi : 4 \times 11 = 1 : 12+$. In other words, the radiometer was 12 times as sensitive as the radiomicrometer.

The radiomicrometer was used with a concave reflector of 16 in. diameter; the radiometer with one of 24 in. diameter. The ratio of the respective apertures is as 1 : 2.2. Thus the radiometer combined with its mirror wherever the full aperture could be utilized, was over twenty-six times as sensitive as the apparatus used by Professor Boys. The sensitiveness of the two sets of apparatus may be roughly compared as they were used on the stars, by comparing the deflection of 38 mm., which Professor Boys obtained from a candle 229.2 meters (250.7 yards) distant, with the deflection of 67 mm obtained by the present apparatus for a candle 633 meters away, using only two thirds of the aperture of the condensing mirror. Assuming the average transmission coefficient for a 100 meter air layer and deducing from it the deflection for a candle 1 meter from the 16-inch concave mirror, the value 1923000 mm is obtained. As compared with the corresponding deflection of 67000000+ mm, the advantage in sensitiveness of the radiometer over the radiomicrometer comes out in the ratio of 35 to 1. The uncertainty of the atmospheric absorption in Professor Boys' measurement makes this result of no value except as a rough check upon the earlier computation of the sensitiveness ratio. It may be further added that with the telescope and scale method employed in measuring the radiometer deflections it was possible to read to the tenth millimeter while Professor Boys attempted no closer estimation than to a fourth millimeter. The small amounts of heat detected from *Jupiter*, *Arcturus*, *Vega*, and *Saturn* with the present apparatus readily accounts for the negative results of the radiomicrometer measurements. With no atmospheric absorption, the number of candles in a group at a distance of $\sqrt{680000000} = 26000$ meters (about 16.2 miles) could be determined by the mean of a series of measurements. Using the atmospheric absorption for 100 meters given above, we have the formula $\frac{680 \times 10^6 \times 0.981^n}{(10n)^2}$ for computing the distance in kilometers along the ground at which the number of candles in a group could be thus determined. For $n = 43$, or a distance of 4.3 kilometers, each candle would

give a deflection of only 0.1 mm, the smallest recognizable deflection. About 2.7 miles along the ground then, would be the practicable limit of measurement for the heat from a single candle.

*Changes in the method of observing.*—The method of making the observations was but slightly changed from that already described under the 1898 observations. As the coelostat was more easily managed to bring the star image on and off the radiometer vane, the observer at $T'$ (Fig. 4) was less constantly occupied with his work, so that, to still further remove the effects of prejudice on the part of the observer who read the deflections, he took over the work of recording the observations as well. When the radiometer suspension had come to rest, the observer at $T$ gave a signal to shift the star image. The recorder changed the image and counted slowly up to seven. The counting was found to involve a period slightly under seven seconds. At the seventh count the deflection was read aloud to the recorder, who entered it with a sign to indicate to what change on his part the deflection corresponded. He often made the same change over and over, such as throwing the image on the same vane several times in succession instead of throwing it first on, and then off at the next signal. The observer at $T$ might as well have been a strip of photographic paper, so far as any knowledge of the significance of what he was reading was concerned. His only chance to use judgment was to decide when the index was quiet enough to risk a deflection.

*Character of the nights on which measurements were made.*—August 5. Sky thick and white; light wind. Series on *Jupiter.* E. F. N., observer; C. E. St. J., recorder.

August 6. Sky same as on preceding night; no wind. Series on *Jupiter.* E. F. N., observer; C. E. St. J., recorder.

August 8. Sky thick; light clouds forming and dissolving constantly; light breeze. Series on *Jupiter.* E. F. N., observer; C. E. St. J., recorder.

August 9. Sky hazy; later developing light cirrus clouds in

neighborhood of *Jupiter*. No wind. Series on *Jupiter*. E. F. N., observer; C. E. St. J., recorder.

August 10. Sky more transparent than on previous evenings, but not brilliant. Series on *Arcturus*. E. F. N., observer; C. E. St. J., recorder.

August 13. Sky clearer than on previous night. Series on *Arcturus* and *Jupiter*. E. B. Frost, observer; E. F. N., recorder.

August 14. Sky only fairly transparent; lightning in west and southwest; conditions disturbed and radiometer unsteady. Series on *Arcturus* and *Jupiter*. Latter cut short by clouds. E. F. N., observer; C. E. St. J., recorder.

August 15. A brilliantly transparent sky after dissipation of clouds in early evening. Series on *Jupiter*. G. E. Hale and H. M. Goodwin, observers; C. E. St. J., recorder.

August 18. Thick sky; fairly uniform in early evening; increasing in transparency later. Series made on *Arcturus*, *Jupiter*, and *Saturn*. E. F. N., observer; C. E. St. J., recorder.

August 19. Very transparent sky. Series made on *Arcturus*, *Jupiter*, and *Saturn*. E. F. N., observer; C. E. St. J., recorder.

August 26. Measurement of atmospheric absorption by observations on candles in distant tents. Early part of evening partly cloudy. Lantern signals on Observatory parapet, as seen from tents, showed marked twinkling. Atmospheric conditions improved steadily during progress of observations and at close sky was unusally transparent. At beginning, wet bulb thermometer 67° and dry bulb 68°.8 F. At close, wet bulb 60°, dry bulb 60° F., at nearer tent six feet from ground. Air free from dust.

August 27. Observations of previous evening repeated. Night began transparent, but later thickened. Candle in further tent appeared noticeably redder than candle in nearer tent. At beginning, dry bulb 70°, wet bulb 67°.5; at close, dry bulb 69°, wet bulb 67°.3. No dust in air.

August 28. Sky transparent, very steady and uniform throughout the evening. At close of last series heavy fog began to rise from ground. Series on *Jupiter* and *Saturn*. C. E. St. J., observer; E. F. N., recorder.

August 29. Night dry and clear; no dust; very light dew. Measurement of atmospheric absorption by observations on candles in distant tents.

*The tables.*—The headings of the columns of the tables which follow have the same signification as in the tables of the 1898 observations, except that in the next to the last column the values, in addition to a reduction to $10^{-8}$ meter candle, have also been reduced to the full aperture of the mirror. No such correction was necessary in the 1898 measurements, because the heliostat reflected a beam large enough to fill the whole aperture of the 24-inch concave mirror.

The first four series on *Jupiter*, August 5, 6, 8, and 9, were made with a defective mirror at *F*, which gave very badly distorted images, so that it is doubtful whether all of the heat which should have reached the concave mirror was concentrated on the vane; hence a heavy line is drawn in the table separating these values from those that follow.

TABLE IV.

*JUPITER.* AUGUST 1900.

| Date | Time P.M. | "On" obs. No. | "On" obs. Av. | "Off" obs. No. | "Off" obs. Av. | No. neg. obs | Combined average | | Sens. | Sens. $10^{-8}$ meter candle | Zenith dist. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | [illegible] | 14 | [illegible] mm | 23 | 0.54 mm | 12 | 0.42 mm | ± [illegible] | [illegible] | [illegible] | [illegible]° 40′ |
| 6 | [illegible] | [illegible] | 0.21 | [illegible] | [illegible] | 5 | 0.47 | ± [illegible] | [illegible] | [illegible] | 70° 20′ |
| 8 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 10′ |
| 9 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | 0.44 | ± [illegible] | [illegible] | [illegible] | [illegible]° |
| 13 | [illegible] | [illegible] | [illegible] | [illegible] | 0.45 | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 50′ |
| 14 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 50′ |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 40′ |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 50′ |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 30′ |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | ± [illegible] | [illegible] | [illegible] | [illegible]° 10′ |
| | | | | | | | | | | Mean | [illegible]° |

TABLE V.

*ARCTURUS.* AUGUST 1900.

| Date | Time P.M. | "On" obs. No. | "On" obs. Av. | "Off" obs. No. | "Off" obs. Av. | No. neg. obs. | Combined average | Sens. | Sens. $10^{-8}$ meter candle | Zenith dist. |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 8.00–8.30 | 16 | 0.61 mm | 16 | 0.42 mm | 10 | 0.51 mm ± 0.09 | 10.9 | 1.45 | 55° 50′ |
| 13 | 8.10–8.50 | 14 | 0.13 | 14 | 0.52 | 8 | 0.32 ± 0.09 | 9.9 | 0.99 | 55° 50′ |
| 14 | 7.50–8.45 | 15 | 0.04 | 16 | 0.46 | 11 | 0.25 ± 0.11 | 11.1 | 0.70 | 52° 20′ |
| 18 | 7.45–8.18 | 17 | 0.45 | 18 | 0.34 | 11 | 0.38 ± 0.09 | 11.2 | 1.06 | 51° 40′ |
| 19 | 7.55–8.35 | 11 | 0.73 | 12 | 0.03 | 9 | 0.38 ± 0.12 | 10.2 | 1.15 | 55° 40′ |
| | | | | | | | | | Mean | 54° |

TABLE VI.

*SATURN.* AUGUST 1900.

| Date | Time P.M. | "On" obs. No. | "On" obs. Av. | "Off" obs. No. | "Off" obs. Av. | No. neg. obs. | Combined average | Sens. | Sens. $10^{-8}$ meter candle | Zenith dist. |
|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 9.35–10.15 | 16 | 0.09 mm | 16 | 0.27 mm | 10 | 0.18 mm ± 0.13 | 11.2 | 0.27 | 71° 30′ |
| 19 | 9.50–11.00 | 19 | 0.20 | 16 | 0.02 | 15 | 0.11 ± 0.10 | 10.2 | 0.18 | 74° 30′ |
| 28 | 8.40– 9.30 | 19 | 0.20 | 18 | 0.15 | 13 | 0.17 ± 0.09 | 11.9 | 0.24 | 70° 10′ |
| | | | | | | | | | Mean | 72° |

To test the method, after the series on *Saturn*, August 28, the coelostat was turned back a short distance in right ascension, so that the field in the radiometer was a patch of uniform bare sky. The coelostat was then oscillated back and forth, in the same manner in which the star image was thrown on and off the radiometer vane, and the deflections were read with the same care. The result of a series of twenty-five observations is given below.

| No. "on" obs. | Av. "on" obs. | No. "off" obs. | Av. "off" obs. | Comb. av. | Sens. |
|---|---|---|---|---|---|
| 12 | −0.12 | 13 | 0.15 | 0.05 ± 0.12 | 11.9 |

There was a heavier drift than usual during this series, so that the averages of the "on" and the "off" observations were contradictory, both indicating a deflection in the *same* direction instead of in *opposite* directions. One set, therefore, had to be interpreted negatively, giving the above combined average.

Two series are given below copied entire from the Observatory notebook, which show the deflections in the order in which they were observed. Of the two series, the one on *Jupiter* is one of the best, and the one on *Saturn* among the worst, obtained. *o*, indicates star image thrown on the vane; *f*, image thrown off the vane.

August 15. *Jupiter:* image on right vane of radiometer. G. E. Hale and H. M. Goodwin, observers, C. E. St. John, recorder. 8:55 P. M.

| | Cont. | Conclu. |
|---|---|---|
| *f*+1.5 mm | *f*+1.2 | *f*+0.3 against drift |
| *o*−1.4 | *o*−1.6 | *o*−1.2 |
| *o* neg.[1] | *f*+1.5 | *f*+1.3 |
| *f*+1.1 | *o*−1.7 | *f*+1.1 |
| *o*−1.3 | *f*+0.8 | *o*−1.5 |
| *f*+1.5 | *o*−1.2 | *f*+1.4 |
| *o*−0.5 | *f*+0.8 against drift | *f*+1.6 |
| *o*−1.2 | *o*−1.2 | *o*−1.9 |
| *o*−1.2 | *o*−0.7 | *f*+1.7 |
| *f*+1.2 | *f*+0.9 | *o*−1.8 9:25 P. M. |

*o* negative and *f* positive indicate heat.

August 18. *Saturn:* image on left vane of radiometer. 9:35 P. M. E. F. N., observer, C. E. St. J., recorder.

| | Cont. | | Conclu. |
|---|---|---|---|
| *f*+0.3 mm | *f* pos. | Radiometer | *f*−1.1 |
| *o*—0.4 first to −1, | *o*−1.9 | much | *f*−1.9 |
| *f*—0.7 | *f*−0.2 | disturbed | *f*−1.0 |
| *o*+0.8 | *o* pos. | | *f*—0.4 |
| *f*—0.4 | *o*−1.0 | | *o*−1.1 |
| *o*+1.9 | *f*+1.5 | | *o*+1.4 |
| *f*+1.8 | *o*−1.2 | | *o*−1.4 |
| *o*+0.5 | *f*−0.2 | | *f*—0.3 |
| *f*—0.1 | *o*+0.6 | | *o*+0.0 |
| *o*—0.5 | *f*—0.8 | | *o*+1.1 |
| | *f*+0.9 | | *o*+1.2 |
| | *o*+0.6 | | *f*−1.7 10:15 P.M. |

*f*, negative and *o*, positive indicate heat.

[1] "Neg. or pos." indicate a deflection of more than 2 mm in the negative or positive direction.

### RESULTS.

Table VII displays all the results thus far obtained, with the exception of the four doubtful series on *Jupiter*, reduced to $10^{-8}$ meter candle with no correction for atmospheric absorption.

The very close agreement between the means of the two sets of observations on *Arcturus* cannot but be accidental, for one high or one low value, more or less, in either series, would completely upset the coincidence. No correction has been applied to the 1900 observations in comparing them with those of 1898, on account of the one additional reflection in the 1900 series.

TABLE VII.

| Date 1898 | *Vega* | *Arcturus* | *Jupiter* | *Saturn* | *Arcturus* / *Vega* |
|---|---|---|---|---|---|
| Aug. 3 | 0.55 | | | | |
| 4 | 0.33 | 0.65 | | | 2.1 |
| 5 | | 1.06 | | | |
| 7 | | 1.60 | | | |
| 8 | 0.64 | 1.30 | | | 2.0 |
| 9 | 0.33 | 0.98 | | | 3.0 |
| 11 | 0.60 | 1.36 | | | 2.3 |
| 12 | 0.50 | | | | |
| 13 | 0.68 | 0.68 | | | 1.0 |
| Means | 0.52 | 1.09 | | | 2.1 |
| 1900 | | | | | *Jupiter* / *Arcturus* |
| Aug. 10 | | 1.45 | | | |
| 13 | | 0.99 | 0.92 | | 0.93 |
| 14 | | 0.70 | 0.89 | | 1.3 |
| 15 | | | 1.70 | | |
| 18 | | 1.06 | 1.58 | 0.27 | 1.5 |
| 19 | | 1.15 | 1.87 | 0.18 | 1.6 |
| 28 | | | 1.93 | 0.24 | |
| Means | | 1.07 | 1.48 | 0.23 | 1.33 |

*Reduction of the observations to the zenith. Final results.*—In comparing the heat effects of planets and stars at such widely different zenith distances as 14° and 75°, some correction for the differences in atmospheric absorption in the two positions ought obviously to be applied. I have tried to find trustworthy

pyrheliometric or actinometric measurements of the solar constant for high and low Sun, made at some station about 1000 feet above the sea (an altitude corresponding to that of the Yerkes Observatory), but so far have not succeeded.

Müller's[1] photometric extinction coefficients for Potsdam correspond well so far as concerns altitude, but they do not take account of energy outside of the visible spectrum. From observations made at Allegheny and Mt. Whitney, Langley[2] finds that the atmosphere lets through infra-red wave-lengths in greater proportion than visible rays, except in the region of the infra-red cold bands, where very heavy absorption occurs. The same experiments show that there is a considerable fluctuation in the diathermancy of the atmosphere with the seasons, absorption being greatest in the summer and least in the winter. The whole matter of zenith reduction is unsatisfactory, because tables must be made out for an abstraction called "an average night." As the range of atmospheric diathermancy for nights which can be called clear is at least as great as one to two, it is only at rare intervals that a strictly average night is to be had, and then average conditions maintain for an hour or two at most. One cannot be sure that the mean of four, five, or even a greater number of nights, will represent these average conditions. Further, as has already been pointed out, stars of different types at the same zenith distance for simultaneous observations ought not, in strictness, to have the same reduction factor applied. Some of these considerations, however, involve refinements beyond the accuracy of the present measurements, and under the circumstances there is no choice but to apply Müller's coefficients in the zenith reductions.

The correction for *Vega* is so small that the values in Table VII may be taken as zenith values. Table VIII shows the relative intensities of the means expressed in $10^{-8}$ meter candle, after the zenith reductions have been made. Because of the

[1] G. MÜLLER, *Photometrie der Gestirne*. Leipzig, 1897.

[2] S. P. LANGLEY, "Report on Mt. Whitney Expedition," *Sig. Serv. Profess. Papers*, 15, p. 211.

variation in atmospheric absorption, the averages in Table VIII were made up from series gathered on nights when at least two of the bodies compared were observed.

TABLE VIII.

| *Vega* | *Arcturus* | *Jupiter* | *Saturn* |
|---|---|---|---|
| 0.51 | 1.14 | 2.38 | 0.37 |

Thus the thermal intensity of:

*Vega*: *Arcturus*: *Jupiter*: *Saturn* :: 1 : 2.2 : 4.7 : 0.74.

The ratio of the zenith photometric intensities is:

*Vega*: *Arcturus*: *Jupiter* :: 1 : 1 : 7.8.[1]

The ratio greater than 2 to 1, of the total radiation of *Arcturus* to that of *Vega* (stars which by most observers are estimated to be of nearly equal photometric magnitude) indicates a proportionately more intense infra-red spectrum for the former than for the latter star. The greater intensity of *Arcturus* in the infra-red may be accounted for in two ways. The photosphere of *Arcturus* may be at a lower temperature than that of *Vega*, but the star be of sufficiently greater angular diameter, as seen from the Earth, to equal *Vega* in light intensity and surpass it in total radiation. This would be, without doubt, the first explanation to suggest itself. Recently, however, Sir William and Lady Huggins[2] have brought forward evidence to show that the photospheres of solar type stars are actually hotter than in stars of the first type, and that the color difference is due to the absorption of the stellar atmosphere, which, in solar stars, is denser and further developed. If this theory of Sir William and Lady Huggins be accepted, it will not be necessary to assume a greater angular diameter for *Arcturus* than for *Vega* to explain the present results.

The thermal intensity of *Arcturus* to *Jupiter* is 1 : 2.2, while the light ratio is 1 : 7.8. So far as the present results are trustworthy, this may be explained in any one or more of three ways: an infra-red spectrum of great extent and intensity for

[1] The photometric intensity of *Jupiter* for August 18, 1900, was computed from Müller's value for a mean opposition (*loc. cit.*, p. 384).

[2] *Atlas of Representative Stellar Spectra*, London, 1899, p. 79 *et seq.*

*Arcturus;* a comparatively low temperature of the outer envelope of *Jupiter;* or a strongly selective albedo for *Jupiter* in the infra-red.[1] That *Jupiter* emits no light rays is rendered probable by the fact that Professor Barnard[2] was unable to follow any of the satellites into the planet's shadow, even with the light-gathering power of the 36-inch Lick telescope.

In an endeavor to further test the matter, the transmission of *Jupiter* rays through a piece of plate glass 3.4 mm thick, was measured in connection with the other heat measurements. The plan followed was to take a series of eight or ten deflections on *Jupiter*, then take about the same number through the glass, repeat the first set, and so on. The dates of the observations, together with the transmission percentages and probable errors, follow.

August 9, $70 \pm 3$ per cent. August 19, $77 \pm 9$ per cent. August 28, $78 \pm 3$ per cent.

Of these values the last is the most, and the first the least, trustworthy.

The transmission of the same plate for various sources of heat follows in Table IX.

TABLE IX.

TRANSMISSION OF GLASS PLATE 3.4 mm THICK.

| Source | Per cent. | Remarks |
|---|---|---|
| Leslie cube 100° | 0 | Or at least less than 1 per cent. |
| Candle flame | 40 | |
| Full Moon | 48 | |
| *Jupiter* | 75 | Zenith distance, 75°. |
| Sun | 80 | Zenith distance, 59°. |

The very high transmission obtained for the Moon, as compared with the results of Langley,[3] Lord Rosse[4] and Boys,[5] is

[1] Any error attributable to the zenith reduction is wholly inadequate to account for the discrepancy between the ratios of the thermal and photometric intensities.

[2] E. E. BARNARD, *Astr. Nachr.* Bd. 144, No. 3453, p. 330.

[3] S. P. LANGLEY, *Mem. Nat. Acad. Sci., loc. cit.*

[4] LORD ROSSE, *Phil. Trans.*, 1873, II, p. 587. [5] C. V. BOYS, *loc. cit.*

easily explained because a very large part of the Moon's emission was shown by Langley to consist of wave-lengths greater than $9\mu$, which were stopped under all circumstances by the fluorite window. It was impossible to work with the Moon's image on either vane of the radiometer alone because the heat violently drove the suspension beyond the scale, but the image was thrown on both vanes and the radiometer used differentially. The high transmission of *Jupiter* rays, and its close resemblance to the solar transmission, is in part doubtless explicable by *Jupiter's* abnormally high albedo, which is more than 4.5 times that of the Moon; yet there seems to be surprisingly little heat radiation present, such as a candle or Leslie cube sends out; *i. e.*, heat rays in large part stopped by the glass and let through by fluorite.

The results of a study, soon to be undertaken, to investigate the absorption spectrum of the glass plate, will give added definiteness to any conclusions to be drawn from these observations concerning the temperature of *Jupiter's* outer envelope.

### MINCHIN'S EXPERIMENTS.

In a preliminary paper before the Royal Society, Professor G. A. Minchin[1] described a very interesting series of experiments on the effect of radiation from the stars on a photo-electric cell of special construction, used in connection with a very sensitive electrometer. The action of radiant energy on the cell is such that the electric potential and the electrometer deflection increase as the square root of the incident intensity. Fortunately for a comparison of Minchin's results with those presented here, he used the sensitive surface of the cell in the focus of a 24-inch concave mirror. A part of Minchin's results are given in Table X.

TABLE X.

| Object | Deflection $d$ | Energy $\propto d^2$ |
|---|---|---|
| *Arcturus* | 8.2 mm | 67. |
| *Saturn* | 5.6 | 31. |
| *Vega* | 11.5 | 132. |
| Candle ten feet away | 10.0 | 100. |

[1] G. A. MINCHIN, *loc. cit.*

Thus *Vega* shows half again the intensity of *Arcturus*, and more by a third than a candle ten feet away (presumably without concentration by the 24-inch mirror). Were the above intensities proportional to the total radiant energies received from the bodies compared, the radiometer at average sensitiveness should have given a deflection of 100 mm for *Vega* instead of the bare ¼ mm observed. It would appear, therefore, that the photo-electric cell is not sensitive in the infra-red spectrum.

INSTRUMENTAL REQUIREMENTS FOR FURTHER EXPERIMENTS.

The object for which the present study was primarily undertaken has been in a measure realized in gaining more or less trustworthy estimates of the heat from four stars and planets. Although the results so far gained can be considered only in the roughest sense quantitative, they indicate a way to a more extensive knowledge of the heat radiation of the brighter stars, by supplying a basis upon which the requisite apertures of condensing mirrors may be computed and by suggesting further refinements in the radiometer. A concave mirror five feet in diameter would possess a gathering power more than six times that of the two-foot mirror used in the present work, and with a suitably modified Coudé mounting, such a mirror could be effectively employed, even with no greater radiometric sensitiveness than that already realized. By its aid white stars down to the second magnitude, and red stars possibly to the third, could be arranged in the order of the thermal intensity of their radiations. It ought also to be possible to study roughly the distribution of energy in the spectra of stars like *Sirius*, *Arcturus*, *Capella*, *Vega*, and possibly others.

Wien's[1] law of the distribution of energy in the spectrum of a black body furnishes the relation $\lambda_{max} T =$ const. (in which $\lambda_{max}$ is the wave-length of the maximum energy in the spectrum, and $T$ the absolute temperature) which, applied even to crude determinations of energy distribution in stellar spectra, should

[1] W. WIEN, *Ber. d. Berl. Akad.*, 1893.

afford rough estimates of stellar temperatures.[1] The radiations from stars which reach the Earth have, however, suffered selective absorption and selective reflection or scattering in traversing first the gaseous envelope of the star and later our own atmosphere. The studies of Langley, and others, have afforded some knowledge of the distribution of this kind of absorption in the Earth's atmosphere, and later work will doubtless make this knowledge more definite. But it is in dealing with the absorption of the stellar atmosphere that the greatest ultimate difficulty is to be expected. Wherever any considerable selective absorption or scattering exists in the atmosphere about the star, a determination of the wave-length of the maximum of the initial energy can be at best but uncertain, and the formula, which would be of comparatively easy application to the blue-white stars, because of their supposed small selective absorption, will not so aptly apply to stars belonging to other groups. This difficulty has already been dealt with in the problem of the Sun,[2] but in this case, in addition to being able to measure rays from different portions of the Sun's disk, we have such abundant energy to work with that the study of any desired number of isolated portions of the Sun's spectrum can be undertaken. In the case of the brightest stars and five-foot apertures, on the other hand, the whole stellar spectrum could not be divided into more than five or six regions for such a study. The problem will consequently be vastly more difficult and uncertain. That no very close approximation to the true stellar temperature of yellow and red stars is to be hoped for by this method is very plain, although the knowledge of the Sun already gained might be applied to stars of the solar type.[3] Approximations to the effective temperatures, though necessarily interpreted between rather wide limits, will be serviceable so long as no better are available. It

[1] This statement assumes that a stellar photosphere may be regarded as a "black body," a fact which, although doubtless approximately true, has as yet (so far as the writer is informed) received no direct confirmation.

[2] J. SCHEINER, *Strahlung und Temperatur der Sonne*, Leipzig, 1899.

[3] Compare J. SCHEINER, *loc. cit.*, pp. 39 and 60.

is well to remember in this connection how recent are any concordant results concerning the solar temperature, although the fault here is attributable to the lack of any trustworthy law of radiation.

Could stellar temperatures be roughly determined, approximate values of the relative angular semi-diameters among stars of the same spectral type might be computed in accordance with Stefan's law. It will be not without interest, in a later paper, to use the present material for an estimate of the angular semi-diameter of *Arcturus*, as follows: The relation of *Arcturus* to a candle has been measured and the same comparison can be made between a similar candle and the Sun at the same time of year. Assuming that the total radiant intensity of unit area of *Arcturus* differs little from that of the Sun,[1] then if $E_a : E_s$ equal the ratio of the heat quantities measured, $\theta_a : \theta_s$ that of the angular diameters, $\theta_a = \theta_s \sqrt{\frac{E_a}{E_s}}$. Further, if $D_a$ and $D_s$ represent the distances of *Arcturus* and the Sun, and $V_a$ and $V_s$ their respective volumes, then $V_a = \frac{V_s D_a^3}{D_s^3} \left(\frac{E_a}{E_s}\right)^{\frac{3}{2}}$. In this equation I believe it is safe to assume that the probable error in $D_a$ is likely to be as large as that of the thermal quantities involved.

Considering the possibilities of a large reflecting telescope of the Coudé type in the photography of the fainter nebulæ and star clusters, and in the photography of star spectra, as well as its use in measurements of star heat, and many other astrophysical researches where laboratory conditions are essential, it is safe to predict that the addition of such an instrument to the equipment of one of our leading observatories would provide the means for solving a greater number of outstanding problems than the addition of any other single instrument which it is now possible to build.

Before closing I wish to acknowledge my widespread obligations to others for aid in the furtherance of the present study.

[1] The assumption of equality for total radiant intensity involves a smaller error than if an equivalence of luminous intensity had been assumed.

To Director George E. Hale I am indebted for the generous invitation to make use of the unique resources of the Yerkes Observatory for the foregoing experiments. I have received from him every possible assistance, the most valuable suggestions and advice, and have benefited by his most enthusiastic interest and coöperation. To my two assistants, Mr. A. L. Colton, formerly an assistant at the Lick Observatory, and Professor Charles E. St. John, of Oberlin College, I am indebted for many suggestions in dealing with the experimental difficulties which arose during the progress of the observations, and for skillful and patient assistance in the course of the measurements. I am, further, indebted to Professor Edwin B. Frost, and Messrs. G. W. Ritchey and Ferdinand Ellerman, for suggestions and assistance, and in some degree to every other member of the Observatory staff.

DARTMOUTH COLLEGE,
January 24, 1901.

# THE ATMOSPHERIC ABSORPTION OF THE VISIBLE RAYS, DETERMINED FROM SPECTROSCOPIC OBSERVATIONS OF THE EIFFEL TOWER ELECTRIC LIGHTS IN 1889.

By A. CORNU.

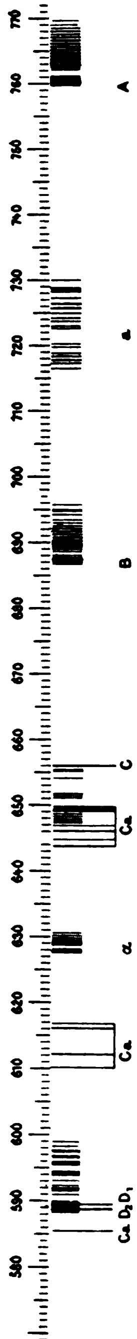
IT is natural to suppose that the terrestrial atmosphere would absorb in a horizontal direction the same rays and would produce the same spectral lines (the so-called *telluric* lines) that are observed in the solar spectrum. The existence of several telluric groups in the spectrum of an electric light projected from the Eiffel Tower to the Meudon Observatory has in fact been pointed out by M. Janssen,[1] and brought forward as a demonstration of the terrestrial origin of the A and B groups as well as of certain bands due to water vapor.

I undertook to carefully examine under high dispersion the series of dark lines visible in the spectrum of the electric lights at the summit of the tower, and to compare them with those that appear in the spectrum maps which I had previously published.[2] The observations also constituted a direct and valuable test of the *method of oscillating lines*, which had led me to distinguish between solar lines and those of terrestrial origin, in the most complicated groups of the solar spectrum.

The investigation was begun at the École Polytechnique, in the same place and with the same instruments which had been employed in my researches on solar spectroscopy. The observations were begun on October 24, 1889, with the aid of the light from one of the beacons at the summit of the tower, and were continued with one of the 90 cm Sautter and

[1] *Comptes Rendus*, **108**, 1035.

[2] "Sur les raies telluriques qu'on observe dans le spectre solaire au voisinage des raies D" (*Journal de l'École Polytechnique*, LIII, pp. 175–212, 1883). "Études des bandes telluriques α, B et A du spectre solaire." (*Annales de Chimie et de Physique* [6], 7, 5–102, 1886.)



Lemonnier search-lights, which M. Eiffel kindly caused to be directed toward the École Polytechnique between eight and ten o'clock, from October 27 to November 6, the day on which the Exposition of 1889 was closed and the search-lights extinguished. The distance from the tower to the École Polytechnique, as measured on a map of Paris of $\frac{1}{12500}$ scale, is about 4350 meters.

It was essential that the attendant in charge of the search-light should be able to recognize at once the point on the horizon toward which the light must be directed. For this purpose I had mounted near the window in the Pavillon des Elèves, where my instruments were installed, a large lens 24 cm in diameter and of 45 cm focal length; it was so adjusted during the day that the focal image of the highest gallery of the tower coincided with the central plane of the flame of a lamp, which was lighted at sunset and permitted the adjustments to be verified.

The reciprocity between the conjugate foci of the lens assured the projection of a beam of light which would cover the entire gallery with its battery of search-lights; the attendant in charge would thus perceive in the required direction an extremely bright disk, which could not possibly be confused with the scintillating points on the horizon. A red glass inserted near the flame rendered the distinction even easier.

I employed, according to circumstances, four spectroscopes, with increasing dispersion:

(1) A Duboscq direct-vision spectroscope with lateral scale.

(2) A Brunner goniometer, provided with two quartz prisms and quartz-fluorite objectives of 50 cm focal length, for photographing spectra.

(3) The same goniometer, provided with a flint prism and crown and flint objectives of 45 cm focal length.

(4) Finally, a large plane Rowland grating, observed with a collimator and telescope of 1 m and 1.40 m focal lengths respectively.

The collimator slit was illuminated by the image of the search-light of the tower, concentrated by an astronomical objective of 16 cm aperture and 2.30 m focal length.

The results fully corresponded with my expectations; during favorable evenings I was able to make a complete study of the telluric groups A, *a*, B, and D, at the outset with medium dispersion. But what seemed to me of the most importance was the possibility of using the great dispersion of the second order grating spectra; I succeeded in doing this several times, as is indicated in the résumé below of the results obtained each evening.

I should have wished to measure with the micrometer all the dark lines visible with this great dispersion; but unfortunately the sky became more and more hazy, the rain and fog constantly increased, and I could therefore only partially carry out this portion of my program, the settings becoming each day more difficult and more trying, because of the decreased intensity of the light.

Fortunately this long investigation turned out to be in large measure unnecessary, thanks to the characteristic configuration of the groups, which precisely reproduced those of my maps, so that a small number of settings sufficed to render certain their complete identification; this was precisely the end which I had in view.

## RÉSUMÉ OF RESULTS.

I transcribe from the notebook the principal results obtained each evening:

October 24, 25, 26, 1889.—First observations with the Duboscq direct-vision spectroscope: a small collecting lens, then the 16 cm objective projecting the linear image of the tower light upon the slit.

Recognized and measured various bright lines of the metallic vapors in the electric arc (sodium, calcium, magnesium), as well as several dark lines, on the continuous spectrum of the carbons in the red part of the spectrum.

A comparison of these determinations with those made with sunlight during the day of the 27th shows that these dark lines correspond with A, *a*, and B (A and B are due to absorption by the oxygen of the air; *a* by water-vapor).

October 27.—Observed the beam from the 90 cm search-light. Brightness remarkable. A newspaper can easily be read by the light from the search-light. Brilliant spectrum. Collecting lens of 50 cm focal length. Observed many details in A, *a*, B, in the water-vapor lines near C and near D, which is double and reversed. Tried an experiment with the Rowland grating. Detected the flutings of B.

October 29.—Brunner goniometer. Flint prism (60°). 16 cm objective to concentrate the image of the search-light upon the slit. Measured on the divided circle the principal lines in the groups A, *a*, B, and several water-vapor lines near D; also several bright lines of calcium.

The brightness of the beam is so great that the Rowland spectroscope can be used. The two D lines (sodium vapor) are magnificent even in the second order: they are reversed at the middle and bright at the end. Near them can be observed all the water-vapor lines on my map (only the metallic solar lines are naturally absent): I examined them one by one in the first order.

I can also follow in detail the structure of the B group as far as the eighth doublet; beyond this the intensity is not great enough.

I had intended to measure all these lines with a micrometer; but I am so familiar with their arrangement, and the agreement with my map is so perfect, that I do not consider it worth while to lose time and tire my eyes in order to make these settings.

The $\alpha$ group (due to oxygen) is faint; its ordinary appearance is considerably modified by the intensity of the water-vapor lines which it contains: nevertheless it can be recognized. I can also detect the groups of water-vapor lines between B and C, which I have indicated as such on Fievez's atlas.

October 30.—The search-light is very bright. The B group is admirably seen in the second order. I can follow it to at least the 11th doublet, and can see the succeeding water-vapor lines, particularly the very strong line $\lambda$695 ($\lambda$6955.8 on my map). Verification of the water-vapor lines near C. Spent the entire evening in carefully identifying the $\alpha$ group, which is very faint and modified by the predominance of the water-vapor lines. I placed a micrometer wire on one of the characteristic lines of $\alpha$; on the following day, at 2:50 P. M., I found with sunlight that it is certainly the line $\lambda$6276.8 of $\alpha$ on my map.

During the whole evening of October 30 the intensity of the light was so great that, without noticing it, I made all my observations in the second order, thinking it to be the first: the definition was so good that in the group near D I resolved the water-vapor line $\lambda 5922.6$ of my map.

October 31.—The sky is clearing; the air is growing cold and foggy; the image is not so bright as yesterday. Through the great kindness of the management the light from two projectors was sent to me simultaneously; but I could use only one of them, their angular separation being too great. The water-vapor lines are much less marked: they are almost altogether lacking near the two D lines. The $\alpha$ group, on the contrary, is much more easily seen. The B group is hardly visible in either the first or second orders. In spite of the increasing fog the violet lines H and K are visible with a Brunner goniometer, and even the ultra-violet band of carbon. The smoke of the electric power-house at the Place du Panthéon is very troublesome.

November 2.—Beautiful evening. Addition of a hydrogen Geissler tube to produce the C line as a standard in the field of the Rowland spectroscope. Verification of the water-vapor lines near C by comparison with this line and the bright lines of calcium.

$\alpha$ group very well seen; brightness sufficient to show as far as the fourth doublet of $\alpha$ and to permit settings to be made. Measured the distance between the strong line $\lambda 6276.8$ (oxygen) and the water-vapor line $\lambda 6291.4$ at the middle of the second doublet of $\alpha$. Two measures gave 3.10 and 3.11 turns of the filar micrometer; on November 4 the same measure, made with sunlight, gave 3.105 turns: the identification is thus perfect.

The aqueous vapor group near D is admirably shown, exactly as on my map; I resolved $\lambda 5922.6$.

November 3.—Rainy evening; nevertheless the light is fairly bright. Attempted to photograph the more refrangible part of the spectrum. Brunner goniometer. Collecting lens of quartz-fluorite. Double quartz prism set at minimum deviation on the violet calcium line $\lambda 423$. Obtained ten violet and ultra-violet spectra on four gelatine plates, the exposures varying from 5 seconds to 2 minutes. No telluric bands. The plates show only the continuous spectrum of the carbons, the two bright carbon flutings, the bright lines H and K of calcium, the two intervening lines of aluminium, and a few others. Contrary to what one would have expected from the meteorological conditions, the ultra-violet spectrum is quite extensive and appears to stop at $\lambda 329$ only because of the absorption of the glasses which cover the aperture of the searchlight, and the defective reflection in the ultra-violet of the silvered concave mirror.

November 4—A little fog, and smoke from the Panthéon power-house. The water-vapor lines near D are once more clearly visible; micrometer settings for identification. Verified the existence of the water-vapor line $\lambda 5882.7$,

which almost exactly coincides with an iron line on my map and is rendered visible by the oscillation of the latter line. $a$ group well seen, but no better than before. The B group is very beautiful when the slit is widened. Observed the water-vapor line $\lambda$ 6925.7 between the tenth and eleventh doublet and those which enclose the eleventh, *i. e.*, $\lambda\lambda$ 6928.1, 6928.3, 6928.9 on my map.

November 5—Rain all day; fog in the evening; the light appears very yellow. Nevertheless the red part of the spectrum is bright enough to permit me to make a long series of settings between B and C.

The thirty-four micrometer settings have been reduced to wave-lengths, using as standards the C line ($\lambda$ 6561.8) observed with the Geissler tube and a bright line of calcium ($\lambda$ 6438.1); six other bright lines of calcium have been identified with metallic lines in the Sun, and the other dark lines with those which I have marked telluric on the plate in Fievez's atlas, and on my unpublished map which I had previously made with the assistance of M. Obrecht.

November 6—Hazy evening; light faint and yellow; water-vapor lines very faint; they were already faint at 2 o'clock that afternoon. No satisfactory observations.

After the close of these evenings of observation I requested the Central Meteorological Bureau to supply me with the data obtained at the top of the tower for the state of the atmosphere from October 28 to November 6.

The variations of temperature and humidity are too slight to be of service in the discussion of the visibility of the spectral lines; the direction and intensity of the wind seem to have exercised more influence.

The following data were transmitted to me:

| Date | Temperature | Vapor tension | Wind | |
|---|---|---|---|---|
| | | | Direction | Velocity |
| 1899 Oct. 28 | 11.0 | 7.1$^{mm}$ | SSW | 7.8$^{m}$ |
| 29 | 9.8 | 7.4 | SSW | 10.3 |
| 30 | 10.5 | 7.3 | SSW | 12.3 |
| 31 | 7.0 | 4.9 | WNW | 1.8 |
| Nov. 1 | 7.1 | 6.1 | WNW | 7.0 |
| 2 | 7.0 | 5.5 | W | 9.3 |
| 3 | 9.0 | 8.6 | SW | 20.0 |
| 4 | 8.9 | 7.7 | SW | ? |
| 5 | 6.6 | 5.6 | NNE | 9.8 |
| 6 | 7.2 | 5.9 | NNE | 7.7 |

In general, the water-vapor lines were sometimes more clearly visible and sometimes less clearly visible than those of the dry atmosphere (bands A, B, $a$); variations in the humidity of the air and the direction of the wind readily explain this effect.

The lines of the dry atmosphere were always less marked than in the solar spectrum: this is due to the short distance (4350 m) traversed by the beam of light as compared with that traversed by the sunlight, even if the Sun were at the zenith. It can easily be shown that the absorbing mass at 4350 m is hardly more than half of that contained in a vertical column of equal base rising vertically to the limits of the atmosphere. The weight of the atmosphere on a square meter is well known to be equal to that of a column of mercury of equal base and 76 cm in height, or $0.76 \times 13596\,\text{kg} = 10333\,\text{kg}$. As a cubic meter of air at the surface of the Earth weighs 1.293 kg, the vertical height of a column of air of uniform density would be $\frac{10333}{1.293} = 7991$ meters.

The horizontal column of 4350 m having the same base thus contains a mass of air smaller in the ratio of 4350 to 7991, or 1 to 0.544, a ratio a little greater than one half. It is therefore not surprising that the lines in the bands A, B, and $a$ are relatively less dark than in solar observations when the Sun is near the zenith, and, with even greater reason, when near the horizon.

## CONCLUSION.

It follows from the spectroscopic observations given above that more than 200 dark lines, produced by the atmospheric absorption of radiations from a terrestrial source of light, have been identified, one by one, with the so-called *telluric* lines observed in the solar spectrum. The atmospheric origin of these lines is thus verified beyond all question.

# OBSERVATIONS OF THE SOLAR ECLIPSE OF MAY 28, 1900.

By H. C. LORD.

IN the July 1900 number of this JOURNAL, Professor Brown gives my preliminary report on the photographs of the flash spectrum secured at Barnesville. In that report the apparatus and method of observing is fully described, but a brief description here may not be out of place. On the eye-end of a four-inch Clark telescope was mounted the large star spectroscope of the Emerson McMillin Observatory. This instrument is provided with two dense 60° prisms. The slit being removed, the image of the solar crescent formed by the four-inch objective acted as the source of light for the spectroscope. The spectroscope and objective were rigidly mounted together and held in a jacket capable of rotation through a small angle about a line perpendicular to the common axis of the four-inch objective and collimator of the spectroscope. The axis of rotation was set at right angles to the line joining the points of second and third contacts, as seen in the six-inch coelostat which reflected the Sun's image into the instrument. The object of this rotation was to shift the instrument during totality, so that the crescents at second and third contacts could be made to fall nearly at the point occupied ordinarily by the slit. In place of an ordinary plate-holder a carriage was provided whereby four exposures could be made on one plate, the plate being shifted the proper amount by simply pressing a rubber bulb. During totality the plate-holder was changed and the carriage set for a second lot of four exposures, and the instrument set as above for third contact. The exposures were made by pulling a string which operated a shutter in front of the four-inch objective. In this way seven photographs were secured, two of which, numbers 3 and 6, were of the flash and will form the subject of this paper.

Plate No. 6 is reproduced, considerably enlarged, in the paper above referred to.

For the measurement of these plates the following plan was adopted. The instrument employed was a Zeiss comparator No. 10. This consists of a stand carrying two micrometer microscopes; under one is placed the plate to be measured and under the other a scale graduated on silver to 1/5 mm. The least reading of the instrument is 0.0001 mm. The scale is 100 mm long, but has only been investigated for division error from 40 to 60. The maximum division error found is 0.0021 mm. In order to confine the measurements to that portion of the scale the plate was measured in two positions. Thus division errors could be applied as far as λ4549.78, above which point the definition of the plate fell off rapidly and the error of pointing on the lines themselves became so large in comparison to any division error found in the part of the scale investigated that it did not seem advisable to use a third position. All lines from $D_1$ to λ4549.78 are freed from division error.

The two lines at λ5197.56 and λ5188.78 were taken as zero lines. The program of measurement was as follows: Three pointings were made on each of the zero lines; then three pointings on each of the lines to be measured, except where these lines were either wide groups or very hazy, when a single pointing was made (in this case the wave-lengths are carried out only to the nearest Ångström unit); then three more pointings on each of the two zero lines. In this way, not only could the several days' work and the two positions be reduced to a common zero, but any accidental displacement of the plate during a series of measurements could be detected. The zero adopted was the point midway between the two zero lines, and the corresponding micrometer reading was taken as 47.438 mm. Thus the micrometer reading for any other line is given by the equation

$$M = S + m + \Delta + me + 47.438 - \frac{M_1 + M_2}{2},$$

where $S$ is the scale division, $m$ the micrometer reading, $\Delta$ the

division error, $e$ the error of runs, and $M_1$ and $M_2$ the value of $M$ for each of the zero lines for any day's work, omitting the last two terms of the above formula. The micrometer was read to 0.0001 mm, and after applying $me$ and $\Delta$ the ten thousandths were dropped as being meaningless. The means of the three pointings were all made in duplicate. Each plate was measured in duplicate, with an independent estimate of the intensity and character. Table I gives in column 2 the mean intensity of the four estimates on a scale of 20, column 4 the several estimates of the character of the line, column 5 the mean value of $M$ from the two measures of plate 3, and column 8 the corresponding quantity from plate 6.

## TABLE I.

**Note.—In the column "Character," *S* indicates sharp; *H*, hazy; *B*, broad; *B G*, broad group; $V^2$ *H*, very, very hazy, and similarly for $V^3$ *H*; *? H*, doubtful, hazy line; *S B*, sharp band; *B H G*, broad, hazy group; *B P D*, broad, probably double; *W*, wide; *R S V H*, sharp on red side, hazy on violet.**

*** Normal lines.**

**Nos. 103 and 157 are very peculiar, being much more intense at the horns than at the vertex of crescent. Vertex too faint to be bisected, set by estimating distance at horns from near companion.**

| No. | I | Elev. | Character | M Plate 3 | Reduced to 6 | M Plate 3 Reduced to 6 | M Plate 6 | Mean | Computed Wave-length | Young's Chrom. Lines λ | F | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| * 1 $D_1$ | 4 | 4½ | S-H-HR-H | 60.034 | −.032 | 60.002 | 60.002 | 60.002 | 5896.16 | 5896.16 | 25 | 2-10 |
| * 2 $D_2$ | 4 | 4½ | S-S-H-S | 59.950 | −.032 | 59.918 | 59.914 | 59.916 | 5890.00 | 5890.18 | 25 | 2-10 |
| 3 $D_3$ | 10 | 5 | S-HR-HR-S | 59.780 | −.031 | 59.749 | 59.746 | 59.748 | 5878.05 | 5876.00 | 100 | 90 |
| 4 | 1 | 1 | ?H-VH | | | | 59.410 | 59.410 | 5854.27 | 5853.90 | 8 | 2 |
| 5 | 1 | 1 | Hazy group | 57.25 | −.02 | 57.23 | 57.24 | 57.24 | 5710 | 5709.62 | 1 | 1 |
| 6 | 1 | 1 | Hazy group | | | | 56.86 | 56.86 | 5686 | 5688.43 | 2 | 1 |
| 7 | 1 | 1 | ?H-H | 56.527 | −.023 | 56.504 | | 56.504 | 5663.56 | 5662.75 | 15 | 2 |
| 8 | 1 | 1 | Very large & broad ?H | | | | 56.478 | 56.478 | 5661.96 | | | |
| 9 | 1 | 1 | S-S-S-S | 56.436 | −.022 | 56.414 | 56.415 | 56.414 | 5658.02 | 5658.10 | 8 | 3 |
| 10 | 1 | 1 | S-S | 56.160 | −.022 | 56.138 | | 56.138 | 5641.17 | 5641.66 | 1 | 1 |
| 11 | 1 | 1 | VBH-?H | | | | 55.870 | 55.870 | 5624.98 | 5624.77 | 2 | 1 |
| 12 | 1 | 1 | H-H-S-S | 55.732 | −.021 | 55.710 | 55.714 | 55.712 | 5615.52 | 5615.88 | 2 | 1 |
| 13 | 1 | 1 | HG-HG-HG-H | 55.50 | | | 55.494 | 55.494 | 5602.57 | | | |
| 14 | 1 | 1 | S-S | | | | 55.350 | 55.350 | 5594.07 | | | |
| 15 | 1 | 1 | HPD-BH-HPD-H | 55.269 | −.020 | 55.249 | 55.248 | 55.248 | 5588.09 | 5588.98 | 2 | 2 |
| 16 | 1 | 1 | H-H | | | | 54.992 | 54.992 | 5573.17 | | | |
| 17 | 1 | 1 | H-H | | | | 54.933 | 54.933 | 5569.76 | | | |
| * 18 | 2 | 2 | S-S-S-S | 54.348 | −.017 | 54.331 | 54.328 | 54.330 | 5535.30 | 5535.07 | 50 | 12 |
| 19 | 2 | 3 | S-H-H-B | 54.208 | −.017 | 54.191 | 54.198 | 54.194 | 5527.65 | 5528.64 | 40 | 5 |
| 20 | 1 | 1 | H-H-S-S | 53.830 | −.016 | 53.814 | 53.821 | 53.818 | 5506.69 | 5507.0 | 2 | 1 |
| 21 | 1 | 1 | H-H-S-H | 53.740 | −.016 | 53.724 | 53.727 | 53.726 | 5501.62 | 5501.69 | 2 | 1 |
| 22 | 1 | 1 | H-H-H-S | 53.666 | −.015 | 53.651 | 53.644 | 53.648 | 5497.32 | 5497.73 | 2 | 1 |
| 23 | 2 | 1 | S-S-S-S | 53.288 | −.014 | 53.274 | 53.272 | 53.273 | 5476.86 | 5477.13 | 1 | 1 |

TABLE I.—*Continued.*

| No. | I | Elev. | Character | M Plate 3 | Reduced to 6 | M Plate 3 Reduced to 6 | M Plate 6 | Mean | Computed Wave-length | Young's Chrom. Lines λ | F | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 24 | 1 | 1 | H-H-S-S | 53.024 | −.014 | 53.010 | 53.010 | 53.010 | 5462.70 | 5463.49 | 1 | 1 |
| * 25 | 3 | 2½ | S-S-S-S | 52.894 | −.014 | 52.880 | 52.883 | 52.882 | 5455.86 | 5455.83 | 10 | 4 |
| * 26 | 2 | 2½ | S-S-S-S | 52.731 | −.013 | 52.718 | 52.716 | 52.717 | 5447.08 | 5447.13 | 10 | 4 |
| 27 | 2 | 2 | S-S-S-S | 52.496 | −.012 | 52.484 | 52.481 | 52.482 | 5434.69 | 5434.74 | 2 | 2 |
| * 28 | 2 | 2 | S-S-S-S | 52.400 | −.012 | 52.388 | 52.384 | 52.386 | 5429.65 | 5429.9 | 8 | 3 |
| 29 | 1 | 1 | S-S-S-S | 52.308 | −.012 | 52.296 | 52.300 | 52.298 | 5425.06 | 5425.4 | 25 | 6 |
| 30 | 1 | 1 | S-S-H-?H | 52.198 | −.012 | 52.186 | 52.183 | 52.184 | 5419.12 | 5419.0 | 5 | 2 |
| 31 | 1 | 1 | H-S-H-S | 52.116 | −.012 | 52.104 | 52.106 | 52.105 | 5415.03 | 5415.42 | 2 | 2 |
| 32 | 1 | 1 | S-S-S-S | 52.020 | −.011 | 52.009 | 52.006 | 52.008 | 5410.02 | 5410.0 | 2 | 2 |
| 33 | 2 | 2 | S-S-S-S | 51.936 | −.011 | 51.925 | 51.927 | 51.926 | 5405.79 | 5405.99 | 2 | 1 |
| 34 | 1 | 1 | S-?S | | | | 51.896 | 51.896 | 5404.25 | 5404.1 | 5 | 3 |
| 35 | 1 | 2 | S-S-S-S | 51.772 | −.011 | 51.761 | 51.761 | 51.761 | 5397.34 | 5397.35 | 4 | 2 |
| 36 | 1 | 1 | ?H-S | | | | 51.686 | 51.686 | 5393.51 | 5393.38 | 2 | 1 |
| 37 | 1 | 1 | HG-HG-HG-BG | 51.46 | −.01 | 51.45 | 51.44 | 51.44 | 5381 | 5381.2 | 3 | 2 |
| 38 | 3 | 2½ | S-S-S-S | 51.262 | −.009 | 51.253 | 51.250 | 51.252 | 5371.58 | 5371.69 | 10 | 3 |
| 39 | 2 | 2 | H-H-V$^{2}$H | 51.090 | −.009 | 51.081 | 51.077 | 51.079 | 5362.92 | 5363.01 | 20 | 5-10 |
| 40 | 1 | 1 | H-H-S-S | 50.906 | −.008 | 50.898 | 50.887 | 50.892 | 5353.65 | 5353.59 | 2 | 2 |
| 41 | 1 | 1 | H-H | 50.820 | −.008 | 50.812 | | 50.812 | 5349.70 | | | |
| 42 | 1 | 1 | H-S-S-S | 50.748 | −.008 | 50.740 | 50.736 | 50.738 | 5346.06 | 5346.0 | 1 | 1 |
| 43 | 1 | 1 | S-S-S-S | 50.644 | −.008 | 50.636 | 50.632 | 50.634 | 5340.95 | 5341.3 | 2 | 1 |
| 44 | 1 | 1 | S-S-S-S | 50.558 | −.007 | 50.551 | 50.549 | 50.550 | 5336.84 | 5336.9 | 5 | 2 |
| * 45 | 4 | 3½ | S-S-S-S | 50.382 | −.007 | 50.375 | 50.375 | 50.375 | 5328.33 | { 5328.7<br>28.2 | 3 | 2 |
| 46 | 1 | 1 | H-H | | | | 50.290 | 50.290 | 5324.21 | 5325.4 | 2 | 2 |
| 47 | 4 | 3½ | S-S-S-S | 50.146 | −.006 | 50.140 | 50.135 | 50.138 | 5316.88 | 5316.79 | 100 | 2-20 |
| 48 | 1 | 1 | H-??H | 49.939 | −.006 | 49.933 | | 49.933 | 5307.05 | | | |
| 49 | 1 | 1 | H-H-S-H | 49.841 | −.006 | 49.835 | 49.835 | 49.835 | 5302.38 | | | |
| 50 | 1 | 1 | H-H-B & H-H | 49.742 | −.005 | 49.737 | 49.746 | 49.742 | 5297.97 | | | |
| 51 | 2 | 1 | Note HV-H-H | 49.450 | −.005 | 49.445 | 49.444 | 49.444 | 5283.92 | 5284.2 | 10 | 2-6 |
| 52 | 1 | 1 | ? H-H | | | | 49.390 | 49.390 | 5281.39 | | | |
| * 53 | 4 | 3½ | S-S-S-S | 49.284 | −.004 | 49.280 | 49.276 | 49.278 | 5276.16 | 5276.21 | 10 | 10 |
| * 54 | 5 | 3½ | S-S-S-S | 49.153 | −.004 | 49.149 | 49.144 | 49.146 | 5270.02 | 5270.50 | 5 | 2 |
| 55 | 1 | 1 | Band | 49.08 | ±.00 | 49.08 | 49.07 | 49.08 | 5267 | 5266.73 | 10 | 4 |
| 56 | 1 | 1 | | 48.96 | ±.00 | 48.96 | 48.96 | 48.96 | 5261 | 5264.10 | 1 | 1 |
| 57 | 1 | 1 | S-S-S-H | 48.820 | −.003 | 48.817 | 48.822 | 48.820 | 5254.98 | 5255.4 | 1 | 1 |
| 58 | 1 | 1 | H-S-S-S | 48.730 | −.003 | 48.727 | 48.724 | 48.726 | 5250.68 | 5249.8 | 3 | 1 |
| 59 | 1 | 1 | S-S-S-S | 48.638 | −.002 | 48.636 | 48.651 | 48.644 | 5246.94 | 5247.8 | 2 | 1 |
| 60 | 2 | 3 | H-SB-S-S | 48.364 | −.002 | 48.362 | 48.374 | 48.368 | 5234.42 | 5234.8 | 10 | 10 |
| 61 | 1 | 1 | S-S | | | | 48.334 | 43.334 | 5232.89 | 5233.12 | 1 | 2 |
| 62 | 1 | 1 | S-S-S-S | 48.206 | −.001 | 48.205 | 48.199 | 48.202 | 5226.95 | 5226.7 | 5 | 5 |
| 63 | 3 | 4 | H-H-H-V$^{2}$H | 47.963 | −.001 | 47.962 | 47.959 | 47.960 | 5216.15 | 5216.5 | 3 | 2 |
| *64 | 3 | 4 | S-S-H-S | 47.796 | +.000 | 47.796 | 47.788 | 47.792 | 5208.70 | { 5208.8<br>5208.6 | 4 | 5 |
| *65 | 1 | 1 | B-Prob. double | 47.723 | +.000 | 47.723 | 47.714 | 47.718 | 5205.43 | 5205.9 | 4 | 5 |
| 66 | 1 | 1 | S-S-S-S | 47.540 | +.000 | 47.540 | 47.538 | 47.539 | 5197.56 | 5197.8 | 15 | 10 |
| 67 | 1 | 1 | H-S | | | | 47.487 | 47.487 | 5195.29 | 5195.1 | 2 | 2 |
| 68 | 1 | 1 | S-S-H-S | 47.414 | +.001 | 47.415 | 47.413 | 47.414 | 5192.10 | | | |
| 69 | 1 | 1 | S-S-S-S | 47.337 | +.001 | 47.338 | 47.338 | 47.338 | 5188.78 | 5189.00 | 10 | 5 |
| *70$b_1$ | 8 | 5 | S-S-S-S | 47.232 | +.001 | 47.233 | 47.230 | 47.232 | 5184.18 | 5183.79 | 50 | 35 |
| *71$b_2$ | 5 | 5 | S-S-S-S | 46.979 | +.002 | 47.981 | 46.974 | 46.978 | 5173.22 | 5172.87 | 50 | 30 |
| *72$b_3$ | 5 | 4 | S-S | 47.866 | +.002 | 47.868 | 46.883 | 46.865 | 5169.14 | 5169.22 | 40 | 25 |
| *73$b_4$ | 5 | 4 | S-S | | | | 46.842 | | 5167.39 | 5167.57 | 20 | 10 |

TABLE I.—*Continued.*

| No. | I | Elev. | Character | M Plate 3 | Reduced to 6 | M Plate 3 Reduced to 6 | M Plate 6 | Mean | Computed Wave-length | Young's Chrom. Lines λ | F | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 74 | 1 | 1 | ?H-??H | | | | 46.717 | 46.717 | 5162.05 | 5161? | 1 | 1 |
| 75 | 1 | 1 | H-VH-H-H | 46.531 | +.003 | 46.534 | 46.527 | 46.530 | 5154.12 | 5153.5 | 2 | 2 |
| 76 | 1 | 1 | { ? Hazy-V² doubtful V-VH } | 46.456 | +.003 | 46.459 | 46.459 | 46.459 | 5151.12 | 5151.03 | 1 | 1 |
| 77 | 1 | 1 | ? H-? H | | | | 46.388 | 46.388 | 5148.12 | 5149.2 | 2 | 2 |
| 78 | 1 | 1 | H-H-S-H | 46.248 | +.003 | 46.251 | 46.258 | 46.254 | 5142.49 | 5143.04 | 2 | 1 |
| 79 | 1 | 1 | S-S-S-H | 46.178 | +.004 | 46.182 | 46.181 | 46.182 | 5139.48 | | | |
| 80 | 1 | 1 | H-? H | | | | 46.116 | 46.116 | 5136.72 | | | |
| 81 | 1 | 1 | H-H-S-VH | 45.928 | +.004 | 45.932 | 45.931 | 45.932 | 5129.07 | 5129.8 | 1 | 1 |
| 82 | 1 | 1 | H-H | 45.789 | +.005 | 45.794 | | 45.794 | 5123.36 | 5123.5 | 2 | 2 |
| 83 | 1 | 1 | S-S-S-S | 45.478 | +.005 | 45.483 | 45.482 | 45.482 | 5110.56 | 5112.3 | | |
| 84 | 1 | 1 | S-S-S | 45.398 | +.006 | 45.404 | 45.408 | 45.406 | 5107.46 | 5107.8 | 1 | 1 |
| 85 | 1 | 1 | H-H | 45.180 | | | 45.24 | 45.24 | 5101 | 5098.8 | 1 | 1 |
| 86 | 1 | 1 | | | | | 45.14 | 45.14 | 5097 | 5097.18 | 1 | 1 |
| 87 | 1 | 1 | S-S-S-S | 44.902 | +.007 | 44.909 | 44.912 | 44.910 | 5087.43 | 5087.6 | 2 | 1 |
| 88 | 1 | 1 | ·S-S-H-H | 44.798 | +.007 | 44.805 | 44.807 | 44.806 | 5083.28 | 5084.3 | 1 | 1 |
| 89 | 1 | 1 | H-H-H-H | 44.706 | +.007 | 44.713 | 44.716 | 44.714 | 5079.61 | 5079.0 | 1 | 2 |
| 90 | 1 | 1 | ? H-S-S | 44.583 | +.008 | 44.591 | 44.590 | 44.590 | 5074.69 | | | |
| 91 | 1 | 1 | H-? H-S-H | 44.516 | +.008 | 44.524 | 44.530 | 44.527 | 5072.19 | | | |
| 92 | 1 | 1 | H-H-VH | 44.437 | +.008 | 44.445 | 44.439 | 44.442 | 5068.84 | | | |
| 93 | 1 | 1 | H-H-S-H | 44.324 | +.008 | 44.332 | 44.337 | 44.334 | 5064.59 | | | |
| 94 | 1 | 1 | G | | | | 44.21 | 44.21 | 5060 | | | |
| 95 | 1 | 1 | BHG | 44.02 | +.01 | 44.03 | | 44.03 | 5053 | | | |
| 96 | 1 | 1 | | 43.89 | +.01 | 43.90 | | 43.90 | 5048 | 5048.2 | 2 | 2 |
| 97 | 2 | 1 | | | | | 44.000 | 44.000 | 5051.54 | | | |
| 98 | 2 | 1 | H-H-S-S | 43.724 | +.010 | 43.734 | 43.739 | 43.736 | 5041.33 | 5041.80 | 2 | 2 |
| 99 | 1 | 1 | H-H-S-S | 43.574 | +.010 | 43.584 | 43.588 | 43.586 | 5035.56 | | | |
| 100 | 2 | 1½ | S-S-S-H | 43.460 | +.011 | 43.471 | 43.472 | 43.472 | 5031.20 | 5031.3 | 4 | 3 |
| 101 | 1 | 1 | S-S | | | | 43.376 | 43.376 | 5027.54 | | | |
| *102 | 5 | 4½ | S-S-S-S | 43.132 | +.012 | 43.144 | 43.139 | 43.142 | 5018.67 | 5018.6 | 30 | 15 |
| 103 | 2 | 4½ | Note | 43.064 | +.012 | 43.076 | 43.056 | 43.066 | 5015.80 | 5015.9 | 30 | 10 |
| 104 | 1 | 1 | ? H-V³H | | | | 43.034 | 43.034 | 5014.59 | | | |
| 105 | 1 | 1 | S-S | 42.987 | +.012 | 42.999 | | 42.999 | 5013.28 | | | |
| 106 | 1 | 1 | S-S-H-S | 42.943 | +.012 | 42.955 | 42.968 | 42.962 | 5011.89 | | | |
| 107 | 1 | 1 | BH-BH-HG-BG | 42.796 | +.012 | 42.808 | 42.81 | 42.808 | 5006.12 | | | |
| 108 | 1 | 1 | BHG-BHG | 42.64 | | | 42.71 | 42.71 | 5002 | | | |
| 109 | 1 | 1 | | | | | 42.61 | 42.61 | 4999 | | | |
| 110 | 1 | 1 | H-VH-S-S | 42.462 | +.013 | 42.475 | 42.477 | 42.476 | 4993.78 | 4994.32 | 2 | 1 |
| 111 | 1 | 1 | S-VH-S-H | 42.378 | +.013 | 42.391 | 42.400 | 42.396 | 4990.83 | | | |
| 112 | 1 | 1 | Hazy group | 42.15 | +.01 | 42.16 | 42.18 | 42.17 | 4982 | | | |
| *113 | 3 | 3½ | S-S-S-S | 41.466 | +.16 | 41.482 | 41.480 | 41.481 | 4957.57 | 4957.48 | 1 | 2 |
| 114 | 1 | 1 | Hazy | | | | 41.165 | 41.165 | 4946.31 | | | |
| 115 | 1 | 1 | H-H-VH-H | 40.937 | +.017 | 40.954 | 40.958 | 40.956 | 4938.92 | | | |
| *116 | 5 | 4 | S-S-S-S | 40.806 | +.017 | 40.823 | 40.820 | 40.822 | 4934.21 | 4934.02 | 30 | 5 |
| *117 | 5 | 4 | S-S-S-S | 40.514 | +.018 | 40.532 | 40.534 | 40.533 | 4924.12 | 4924.11 | 40 | 10 |
| 118 | 1 | 1 | S-S with shading | | | | 40.432 | 40.432 | 4920.61 | 4919.18 | 20 | 5 |
| 119 | 1 | 4 | Note | | | | 40.390 | 40.390 | 4919.16 | 4919.1 | 20 | 3 |
| 120 | 1 | 1 | S-H | | | | 40.238 | 40.238 | 4913.91 | 4912.3 | 3 | 2 |
| 121 | 1 | 1 | HB-BG | 40.12 | +.019 | 40.14 | 40.140 | 40.140 | 4910.54 | | | |
| 122 | 1 | 1 | V²H-V²H-S-S | 39.921 | +.020 | 39.941 | 39.920 | 39.930 | 4903.35 | | | |
| 123 | 1 | 1 | S-S | | | | 39.960 | 39.960 | 4904.37 | | | |
| *124 | 2 | 1½ | S-S-S-S | 39.814 | +.020 | 39.834 | 39.832 | 39.833 | 4900.05 | 4900.31 | 30 | 6 |

TABLE I.—*Continued.*

| No. | I | Elev. | Character | M Plate 3 | Reduced to 6 | M Plate 3 Reduced to 6 | M Plate 6 | Mean | Computed Wave-length | Young's Chrom. Lines λ | Young's Chrom. Lines F | Young's Chrom. Lines B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 125 | 2 | 2 | S-S-H-S | 39.552 | +.020 | 39.572 | 39.574 | 39.573 | 4891.24 | | | |
| *126 | 2 | 2 | S-S-H-H | 39.328 | +.021 | 39.348 | 39.351 | 39.350 | 4883.74 | 4883.94 | 10 | 4 |
| 127 | 1 | 1 | HG-H-S-S | 39.142 | +.021 | 39.163 | 39.190 | 39.176 | 4877.92 | | | |
| 128 | 2 | 1 | SB-H-H-H | 38.968 | +.022 | 38.988 | 38.989 | 38.988 | 4871.67 | 4870.4 | 5 | 1 |
| 129 F | 20 | 5 | S-S-VBS-S | 38.668 | +.023 | 38.691 | 38.669 | 38.680 | 4861.51 | 4861.50 | 100 | 80 |
| 130 | 1 | 1 | G on 3-S-S | 38.45 | +.02 | 38.47 | 38.488 | 38.488 | 4855.22 | 4855.5 | 5 | 2 |
| 131 | 1 | 1 | S-H-BH-$V^2$H | 38.255 | +.024 | 38.279 | 38.276 | 38.278 | 4848.39 | 4848.7 | 3 | 2 |
| *132 | 2 | 3½ | S-BPD-BH-H | 37.489 | +.026 | 37.515 | 37.517 | 37.516 | 4823.95 | 4824.33 | 10 | 2 |
| 133 | 1 | 1 | H-H | | | | 37.102 | 37.102 | 4810.90 | | | |
| *134 | 2 | 3½ | H-B-S-S | 36.891 | +.027 | 36.918 | 36.918 | 36.918 | 4805.16 | 4805.25 | 3 | 1 |
| 135 | 1 | 1 | S-S | | | | 36.712 | 36.712 | 4798.76 | | | |
| 136 | 1 | 1 | S-S | | | | 36.412 | 36.412 | 4789.51 | | | |
| 137 | 1 | 1 | S-S | | | | 36.316 | 36.316 | 4786.52 | | | |
| 138 | 1 | 1 | H-H-S-S | 36.184 | +.029 | 36.213 | 36.216 | 36.214 | 4783.45 | | | |
| 139 | 1 | 1 | H-H-S-S | 36.078 | +.029 | 36.107 | 36.102 | 36.104 | 4780.10 | 4779.7 | 3 | 2 |
| 140 | 1 | 1 | S-?S | | | | 35.969 | 35.969 | 4776.00 | | | |
| 141 | 1 | 1 | Wide group | | | | 35.86 | 35.86 | 4773 | | | |
| 142 | 1 | 1 | H-H | | | | 35.646 | 35.646 | 4766.26 | | | |
| 143 | 2 | 1 | BG-HWG | 35.52 | | | 35.60 | 35.60 | 4765 | | | |
| 144 | | | | | | | 35.49 | 35.49 | 4762 | | | |
| 145 | 1 | 1 | S-$V^3$H-H-H | 35.206 | +.032 | 35.238 | 35.236 | 35.237 | 4754.06 | | | |
| 146 | 1 | 1 | S-H | | | | 34.952 | 34.952 | 4745.64 | | | |
| 147 | 1 | 1 | W-H | | | | 34.651 | 34.651 | 4736.82 | | | |
| 148 | 1 | 1 | H-H-S-S | 34.434 | +.034 | 34.468 | 34.477 | 34.472 | 4731.62 | 4731.7 | 1 | 1 |
| 149 | 1 | 1 | ?H-H | | | | 34.342 | 34.342 | 4727.85 | | | |
| 150 | 1 | 1 | VH-$V^2$H | | | | 34.147 | 34.147 | 4722.23 | | | |
| 151 | 4 | 5 | S-S-HW-SSR | 33.820 | +.036 | 33.856 | 33.840 | 33.848 | 4713.67 | 4713.4 | 2 | 2 |
| 152 | 1 | 1 | Wide group | 33.62 | +.04 | 33.66 | 33.73 | 33.73 | 4710 | | | |
| 153 | 1 | 1 | | | | | 33.60 | 33.60 | 4707 | | | |
| 154 | 1 | 1 | H-H-S-S | 33.442 | +.036 | 33.478 | 33.472 | 33.475 | 4703.09 | | | |
| 155 | 1 | 1 | VH—H | | | | 33.338 | 33.338 | 4699.23 | | | |
| 156 | 1 | 1 | S-H | | | | 33.064 | 33.064 | 4691.56 | | | |
| 157 | 1 | 4½ | Note | 32.840 | +.038 | 32.878 | 32.870 | 32.874 | 4686.28 | | | |
| 158 | 1 | 1 | S-H-S-S | 32.694 | +.038 | 32.732 | 32.725 | 32.728 | 4682.24 | | | |
| 159 | 2 | 1 | H-H-S-S | 32.260 | +.039 | 32.299 | 32.306 | 32.302 | 4670.54 | | | |
| 160 | 2 | 1 | H-H-H-H | 32.134 | +.039 | 32.173 | 32.180 | 32.176 | 4667.10 | 4667.5 | 3 | 1 |
| 161 | 1 | | Broad group | | | | 32.02 | 32.02 | 4663 | 4664.2 | 2 | 1 |
| 162 | 1 | 1 | H-S-H-VH | 31.768 | +.040 | 31.808 | 31.804 | 31.806 | 4657.09 | 4657.1 | 2 | 1 |
| 163 | 1 | 1 | H-$V^2$H | | | | 31.712 | 31.712 | 4654.56 | | | |
| 164 | 2 | 1 | H-H-S-BH | 31.575 | +.041 | 31.616 | 31.610 | 31.613 | 4651.90 | | | |
| 165 | 2 | 1 | H-$S^2$R-$S^2$R-$V^2$H | 31.359 | +.042 | 31.401 | 31.404 | 31.402 | 4646.27 | | | |
| 166 | 1 | 1 | ?H-?H | | | | 31.074 | 31.074 | 4637.57 | | | |
| 167 | 1 | 1 | H-$V^2$H-S-H | 30.910 | +.043 | 30.953 | 30.948 | 30.950 | 4634.30 | | | |
| *168 | 4 | 3 | S-S-S-S | 30.722 | +.043 | 30.765 | 30.767 | 30.766 | 4629.47 | 4629.52 | 15 | 18 |
| 169 | 1 | 1 | H-$V^2$H | | | | 30.652 | 30.652 | 4626.49 | | | |
| 170 | 1 | 1 | S-H | | | | 30.438 | 30.438 | 4620.92 | 4621.1 | 1 | 1 |
| 171 | 1 | 1 | H-$S^2$R-H-H | 30.318 | +.044 | 30.362 | 30.370 | 30.366 | 4619.05 | | | |
| 172 | 1 | 1 | H-H-BH-BVH | 30.219 | +.045 | 30.264 | 30.258 | 30.261 | 4616.34 | | | |
| 173 | 1 | 1 | BH-$V^2$H | | | | 30.152 | 30.152 | 4613.53 | | | |
| 174 | 1 | 1 | H-??$V^2$H | | | | 30.068 | 30.068 | 4611.37 | | | |
| 175 | 1 | 1 | H-VH | | | | 29.736 | 29.736 | 4602.88 | | | |
| 176 | 1 | 1 | H-H-BH-BH | 29.608 | +.046 | 29.654 | 29.644 | 29.649 | 4600.67 | | | |

TABLE I.—*Continued.*

| No. | I | Elev. | Character | M Plate 3 | Reduced to 6 | M Plate 3 Reduced to 6 | M Plate 6 | Mean | Computed Wave-length | Young's Chrom. Lines λ | F | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 177 | 1 | 2 | H-H-S-S | 29.192 | +.047 | 29.239 | 29.230 | 29.234 | 4590.18 | 4590.13 | 1 | 1 |
| 178 | 1 | 1 | H-H-S-H | 29.116 | +.047 | 29.163 | 29.160 | 29.162 | 4588.37 | 4588.38 | 2 | 2 |
| 179 | 4 | 4 | RSVH-S-S-S | 25.940 | +.048 | 28.988 | 28.982 | 28.985 | 4583.94 | 4584.1 | 15 | 6 |
| 180 | 1 | 1 | S-H | | | | 28.836 | 28.836 | 4580.23 | | | |
| 181 | 1 | 1 | S-S-H-H | 28.634 | +.048 | 28.682 | 28.686 | 28.684 | 4576.46 | 4576.6 | 4 | 2 |
| *182 | 5 | 4½ | S-S-S-S | 28.464 | +.049 | 28.513 | 28.505 | 28.509 | 4572.14 | 4572.16 | 10 | 4 |
| *183 | 4 | 4½ | S-S-S-S | 28.130 | +.050 | 28.180 | 28.174 | 28.176 | 4563.96 | 4563.94 | 10 | 5 |
| 184 | 1 | 3 | S-S-S-H | 27.918 | +.050 | 27.968 | 27.962 | 27.965 | 4558.82 | 4558.9 | 8 | 1 |
| 185 | 1 | 1 | S-S | 27.805 | +.051 | 27.856 | | 27.856 | 4556.18 | 4556.2 | 10 | 5 |
| 186 | 5 | 4½ | S-S-S-S | 27.726 | +.051 | 27.777 | 27.771 | 27.774 | 4554.19 | 4554.21 | 10 | 5 |
| *187 | 5 | 4½ | S-S-S-S | 27.541 | +.051 | 27.592 | 27.590 | 27.591 | 4549.78 | 4549.8 | 10 | 8 |
| 188 | | 1 | Broad Group | | | | 27.44 | 27.44 | 4546 | | | |
| 189 | | | | | | | 27.34 | 27.34 | 4544 | | | |
| 190 | | 1 | Broad Group | | | | 27.26 | 27.26 | 4542 | | | |
| 191 | | | | | | | indefinite | | | 4540 | 2 | 1 |
| *192 | 5 | 4½ | S-S-S-S | 26.887 | +.053 | 26.940 | 26.932 | 26.936 | 4534.15 | 4534.2 | 5 | 5 |
| 193 | 1 | 1 | VH-V$^2$H | | | | 26.806 | 26.806 | 4531.08 | | | |
| 194 | 1 | 1 | H-V$^2$H | | | | 26.712 | 26.712 | 4528.86 | | | |
| 195 | 2 | 2½ | S-S-S-H | 26.400 | +.054 | 26.454 | 26.452 | 26.453 | 4522.79 | 4522.9 | 3 | 3 |
| 196 | 1 | 1 | H-H-H-VH | 26.286 | +.054 | 26.340 | 26.348 | 26.344 | 4520.25 | | | |
| 197 | 2 | 1½ | H-H-H-V$^2$H | 26.080 | +.055 | 26.135 | 26.136 | 26.136 | 4515.41 | 4514.5 | 2 | 1 |
| 198 | 2 | 1½ | H-H-H-V$^2$H | 25.777 | +.056 | 25.833 | 25.831 | 25.832 | 4508.39 | 4506.9 | 2 | 1 |
| *199 | 5 | 4½ | S-S-S-H | 25.472 | +.057 | 25.529 | 25.525 | 25.527 | 4501.40 | 4501.44 | 15 | 6 |
| 200 | 1 | 1 | BH-V$^2$H | | | | 25.336 | 25.336 | 4497.05 | | | |
| 201 | 1 | 1 | H-V$^3$H | | | | 25.217 | 25.217 | 4494.35 | | | |
| 202 | 1 | 1 | H-V$^3$H | | | | 25.082 | 25.082 | 4491.30 | 4491.5 | 20 | 8 |
| 203 | 1 | 1 | Broad Group | 25.06 | +.06 | 25.12 | | 25.12 | 4492 | | | |
| 204 | 1 | 1 | | 24.88 | +.06 | 24.94 | | 24.94 | 4488 | | | |
| 205 | 1 | 1 | BH-V$^2$H | | | | 24.993 | 24.993 | 4489.29 | 4490.2 | 15 | 3 |
| 206 | 1 | 1 | V$^2$H-G-BH-V$^2$H | 24.608 | +.059 | 24.667 | 24.674 | 24.670 | 4482.05 | 4481.7 | 5 | 2 |
| 207 | 1 | 1 | H-V$^2$H | | | | 24.396 | 24.396 | 4475.95 | | | |
| 208 | 10 | 5 | S-S-H-H | 24.160 | +.060 | 24.220 | 24.200 | 24.210 | 4471.83 | 4471.8 | 100 | 25 |
| 209 | 5 | 4½ | S-S-H-H | 24.006 | +.060 | 24.066 | 24.060 | 24.063 | 4468.59 | 4469.5 | 20 | 5 |
| 210 | 1 | 1 | S-V$^2$H | | | | 23.878 | 23.878 | 4464.53 | | | |
| 211 | 1 | 1 | V$^2$H-V$^2$HG-H-V$^2$H | 23.682 | +.060 | 23.743 | 23.736 | 23.740 | 4461.51 | | | |
| 212 | 1 | 2 | V$^2$H-H-VH-V$^2$H | 23.369 | +.062 | 23.431 | 23.428 | 23.430 | 4454.77 | | | |
| 213 | 1 | 2 | V$^2$H-V$^3$H-VH-V$^2$H | 23.165 | +.062 | 23.227 | 23.238 | 23.234 | 4450.54 | | | |
| 214 | 5 | 4½ | H-VH-H-VH | 22.862 | +.063 | 22.925 | 22.924 | 22.924 | 4443.88 | 4443.5 | 10 | 2 |
| 215 | 1 | 1 | BH-V$^2$H-VH-V$^2$H | 22.450 | +.064 | 22.514 | 22.515 | 22.514 | 4435.15 | 4434.0 | 1 | 1 |
| 216 | 1 | 1 | V$^2$H-VH-VH-V$^2$H | 22.074 | +.065 | 22.139 | 22.138 | 22.138 | 5427.22 | 4426.6 | 2 | 3 |
| 217 | 1 | 1 | V$^2$H-V$^2$H-G | 21.834 | +.066 | 21.900 | 21.90 | 21.900 | 4422.24 | | | |
| 218 | 4 | 2½ | V$^2$H-V$^2$H-VH-V$^3$H | 21.617 | +.066 | 21.683 | 21.678 | 21.680 | 4417.66 | 4419.0 | 2 | 1 |
| 219 | 3 | 2½ | V$^2$H-V$^2$H-VH-V$^3$H | 21.488 | +.067 | 21.555 | 21.554 | 21.554 | 4415.05 | 4415.30 | 1 | 1 |
| 220 | | | Hazy Group | | | | 21.22 | 21.22 | 4408 | 4408.6 | 1 | 1 |
| 221 | 1 | 1 | V$^2$H-V$^2$H | 21.146 | +.068 | 21.214 | | 21.214 | 4408.04 | 4408.6 | 1 | 1 |
| 222 | 2 | 3½ | V$^2$H-V$^2$H-VH-V$^3$H | 20.975 | +.068 | 21.041 | 21.052 | 21.046 | 4404.60 | 4404.93 | 1 | 1 |
| 223 | 3 | 3½ | V$^3$H-V$^3$H-VH-V$^3$H | 20.737 | +.069 | 20.806 | 20.822 | 20.814 | 4399.87 | 4398.9 | 1 | 1 |
| 224 | 5 | 4½ | V$^2$H-V$^2$H-VH-V$^2$H | 20.495 | +.069 | 20.564 | 20.574 | 20.569 | 4394.90 | 4395.2 | 15 | 3 |
| 225 | 3 | 3½ | V$^2$H-V$^2$H-VH-V$^3$H | 19.924 | +.071 | 19.995 | 20.003 | 19.999 | 4383.46 | 4383.72 | 1 | 1 |
| 226 | 3 | 3½ | V$^3$H-V$^2$H-V$^2$H-V$^3$H | 19.480 | +.072 | 19.552 | 19.562 | 19.557 | 4374.69 | 4374.8 | 8 | 3 |
| 227 | 1 | 1 | V$^2$H-V$^4$H | | | | 18.74 | 18.74 | 4359 | 4359.78 | 1 | 1 |
| 228 | 2 | 1 | V$^2$H-V$^4$H | | | | 18.36 | 18.36 | 4351 | 4352.4 | 3 | 1 |
| 229*Hγ* | 15 | 5 | VBSH-V$^5$H | | | | 17.79 | 17.79 | 4340 | 4340.66 | 100 | 65 |

These plates were taken so nearly at the instant of contacts that the continuous spectrum is reduced to a few narrow streaks, which when examined under a low power are seen to be full of bright lines, and when examined under the measuring engine lose all appearance of a continuous spectrum and appear simply as streaks of maximum density of the bright lines. They supply, however, an easy means of adjusting the plate parallel to the scale, it being only necessary to make a spot of dust in the eye-piece follow the streak as the plate is moved rapidly from end to end. As the instrument at Barnesville was set for the mean position angle of the two contacts, the tangents to the solar crescents at the points where crossed by the streaks are not at right angles to the direction of the streaks, and it would be very difficult to make an accurate pointing were the micrometer wire set perpendicular to the direction of motion of the plate under the microscope. This difficulty was entirely avoided by rotating the plate micrometer until its wire was tangent to the curved lines where crossed by the streak, keeping the plate micrometer constantly at zero and making the bisections with the slow motion that moved simultaneously both plate and scale. With this precaution I think nearly if not quite as accurate pointings can be made as if the lines were straight.

In comparing the values of $M$ from plate 3 and plate 6 as given in Table I, it is at once evident that there is a progressive difference in the value $M_3 - M_6$. This can be due to two causes: first, in shifting the instrument at Barnesville during totality from its setting for second contact to that for third it was impossible to bring the image of the solar crescent to exactly the same point in the focal plane of the collimator. This would produce an effect exactly similar to a slight shift of the slit of the spectroscope parallel to itself, and would result in the lines on the two plates having a progressive shift. Secondly, should the long lines be at a greater elevation above the Sun's limb than the short ones, these would be shifted relatively to the short ones, too far to the violet on one plate and too far to the red on the other. In order to test if there were any such relative displacement as

this latter would indicate, 72 short lines common to both plates 3 and 6 were selected and the residuals $M_3-M_6$ plotted on cross section paper. It was at once seen that these residuals could be represented about as well by a straight line as by any other form of curve; accordingly a straight line was passed through them by the method of least squares and the equation

$$M_6 = M_3 + 0.0016\text{ mm} - 0.00256\text{ mm}\,(M - 47.0)$$

was found. The values of these corrections are given in Table I, column 6; column 7 gives the value of $M$ for plate 3 thus reduced, and column 9 the mean value of $M$ from plates 6 and 3 thus reduced. Since this correction was based solely upon the short lines, it is evident that, were the long lines at any considerable elevation, this would show in the difference $M_3 - M_6$ after $M_3$ had been reduced to plate 6 as above. Table II gives this difference for all lines having a length greater than 4.

Assuming that the differences in column $M_3-M_6$ correspond to a difference of elevation of the substance emitting the given line, and that the stratum was of uniform brilliancy, it is evident that the elevation of its upper limit would be given by the expression.

$$E = 206264 \times 450 \times \frac{f_1\,(M_6 - M_3)}{1000\,f_2 \times F} = 64\,(M_6 - M_3)$$

where $E$ = elevation in miles, $f_1$ = 383 mm = focal length of collimator, $f$ = 375 mm = focal length of camera, $F$ = 1486 mm = focal length of image lens, and $M_6 - M_3$ is expressed in 0.001 mm. This, of course, would be exact only for minimum deviation, but would be sufficiently accurate for the extent of spectrum covered by these plates. Upon this assumption I have computed the elevations given in Table II.

Of the two negative values found, one is so small as to be easily accounted for by accidental errors of observation, while for the third the line is near the limit of the plate and is so badly out of focus as to render the measurements very uncertain. The four lines whose elevation comes out greater than 1000 miles are F, 4713.67, 4471.83, and 5015.80. The behavior of this latter line

TABLE II.

| No. | λ | $M_4$ | $M_3$ | $M_3-M_4$ | $E$ in miles |
|---|---|---|---|---|---|
| 1 $D_1$ ................ | 5896.16 | 60.002 | 60.002 | ±0.000 | ±0 |
| 2 $D_2$ ................ | 5890.00 | 54.914 | 59.918 | +0.004 | +256 |
| 3 $D_3$ ................ | 5878.05 | 59.746 | 59.749 | +0.003 | +192 |
| 45.................. | 5328.33 | 50.375 | 50.375 | ±0.000 | ±0 |
| 47.................. | 5316.88 | 50.135 | 50.540 | +0.005 | +320 |
| 53.................. | 5276.16 | 49.276 | 49.280 | +0.004 | +256 |
| 54.................. | 5270.02 | 49.144 | 49.149 | +0.005 | +320 |
| 62.................. | 5226.95 | 48.199 | 48.205 | +0.006 | +384 |
| 64.................. | 5208.70 | 47.788 | 47.796 | +0.008 | +512 |
| 65.................. | 5205.43 | 47.714 | 47.723 | +0.009 | +576 |
| 70.................. | 5184.18 | 47.230 | 47.233 | +0.003 | +192 |
| 71.................. | 5173.22 | 46.974 | 46.981 | +0.007 | +448 |
| ½ (72+73)............ | ½ (63+64) | 46.862 | 46.868 | +0.006 | +384 |
| 102.................. | 5018.67 | 43.139 | 43.144 | +0.005 | +320 |
| 103.................. | 5015.80 | 43.056 | 43.076 | +0.020 | +1280 |
| 113.................. | 4957.57 | 41.480 | 41.482 | +0.002 | +128 |
| 116.................. | 4934.21 | 40.820 | 40.823 | +0.003 | +192 |
| 117.................. | 4924.12 | 40.534 | 40.532 | −0.002 | −128 |
| 129 F................ | 4861.51 | 38.669 | 38.691 | +0.022 | +1408 |
| 132.................. | 4823.95 | 37.517 | 37.515 | +0.002 | +128 |
| 134.................. | 4805.16 | 36.918 | 36.918 | ±0.000 | ±0 |
| 151.................. | 4713.67 | 33.840 | 33.856 | +0.016 | +1024 |
| 157.................. | 4686.28 | 32.870 | 32.876 | +0.008 | +512 |
| 179.................. | 4583.94 | 28.982 | 28.988 | +0.066 | +384 |
| 182.................. | 4572.14 | 28.505 | 28.513 | +0.008 | +512 |
| 183.................. | 4563.96 | 28.174 | 28.180 | +0.006 | +384 |
| 186.................. | 4554.19 | 27.771 | 27.777 | +0.006 | +384 |
| 187.................. | 4549.78 | 27.590 | 27.592 | +0.002 | +128 |
| 192.................. | 4534.15 | 26.932 | 26.940 | +0.008 | +512 |
| 199.................. | 4501.40 | 25.555 | 25.529 | +0.004 | +256 |
| 208.................. | 4471.83 | 24.200 | 24.220 | +0.020 | +1280 |
| 209.................. | 4468.59 | 24.060 | 24.066 | +0.006 | +384 |
| 214 ................. | 4443.88 | 22.924 | 22.925 | +0.001 | +64 |
| 224.................. | 4394.90 | 20.574 | 20.564 | −0.010 | −640 |

is, however, very peculiar, and is similar to that of the line at 4686.28. Both these lines on both plates show much more markedly at the horns of the crescent than at the vertex, where they seem to almost entirely disappear, so much so in fact that it was impossible to actually set the micrometer wire upon them at the vertex, and the measurements were made by estimating the distance at the horns from a close companion. This peculiarity is shared by no other lines found on either plate, and is clearly marked for both lines on each plate. I feel, therefore, that these two lines must be common to some substance, and

that they are not due to a substance which shows other lines on the photographs.

Since the above was written Professor Frost has called my attention to the close agreement of three of these lines with those found by Runge and Paschen[1] in the spectrum of Clèveite gas. The line 4713.67, with an intensity of 1, agrees with the mean of the two lines 4713.252, intensity 3, and 4713.475, intensity $<1$, within the limit of error of my wave-lengths; 4471.83, intensity 10, agrees with the mean of 4471.646 and 4471.858, intensities 6 and $<1$ respectively, and 5015.80, intensity 2, with 5015.732, intensity 6. The first two belong to the second and first subordinate series of helium proper, the last to the principal series of the lighter constituent. This explanation would be very satisfactory were it not for the line at 4686.28, which is not found by these observers in Clèveite gas, nor by Kayser in the spectrum of argon. Rowland gives a line at 4686.40 with an intensity 3 as due to *Ni*, but this line in its behavior is so exactly like that at 5015.80, and so radically different from all the others, that I cannot believe it can be due to *Ni*. In a paper entitled "The new Series in the Spectrum of Hydrogen,"[2] Rydberg deduces from his formulæ on page 236 a line at 4687.88, concerning which he says:

> These conclusions are confirmed in every respect, if we consider the spectra of stars of the fifth type. . . . . As we see, *all the known lines of hydrogen are surpassed in intensity by the line 4688, which corresponds almost exactly to the computed value 4687.88 and which we can, with full certainty, indicate as the first line of the hydrogen spectrum, being at once the first term of the principal and of the sharp series.*

The line 4686.28 may well be this hydrogen line; the difference in wave-length, 1.6 Ångström units, is somewhat large, but the peculiar character of the line made measurements rather uncertain. Its intensity is, however, much less than that of the other hydrogen lines, being certainly not over $\frac{1}{10}$ that of either $H\beta$ or $H\gamma$.

The low value of these elevations appears to me somewhat surprising. Even if the stratum which gives rise to these radiations

[1] ASTROPHYSICAL JOURNAL, 13, 4, 1896. [2] ASTROPHYSICAL JOURNAL, 6, 233.

at greater elevation is not of uniform intensity, but much brighter on the inner side, it would hardly seem that the center of density should come very much nearer the level of the shorter crescents. And in that case the crescent should appear sharp on the inner edge and hazy on the outer, a condition of affairs which is by no means markedly in evidence. Furthermore, the elevations are in a great many cases but a very small fraction of the width of the line, thus indicating that the high level (long) lines, while being shifted bodily by a small amount relatively to the low level (short) lines, have nevertheless been broadened on each side of this shifted position by a much greater amount. If this be true, the explanation is not at once apparent, at least to me. That the shift above is real, and an approximation at least to the true amount, is confirmed by plate 1, exposed several seconds before totality. Here both the dark and bright F lines are seen side by side, evidently overlapping but not superimposed. A number of other lines are found both bright and dark, but only in the case of F was the definition good enough to permit even an approximation to a measurement. The value found in this case was 0.033mm, or 2112 miles, as the distance between the centers of 4224 from the outer edge of the high level stratum. But these lines clearly overlapped, as both the bright and dark F lines narrowed down where they came together, thus tending to greatly increase the measured elevation. I have given this discussion thus fully, for I can see no instrumental cause that would cause this shift, and am at a loss to understand why it is not greater in amount. I can only say that I have given the facts as they have been observed, absolutely without bias, as I was ignorant until all the computations had been finished as to whether this shift corresponded to a low or high elevation.

DETERMINATION OF WAVE-LENGTHS.

For the determination of the wave-lengths the following plan was adopted. Three normal places were formed from the measurements given in column 9, Table I. For the first normal place $D_1$, $D_2$, and $D_3$, were used; for the second, $b_1$, $b_2$, and $\frac{b_3 + b_4}{2}$;

for the third, F. From these the following normal places were found:

$$\text{For } M = 59{,}900 \quad \lambda = 5888.35$$
$$M = 47{,}000 \quad \lambda = 5173.95$$
$$M = 38{,}680 \quad \lambda = 4861.50$$

From these a Cornu-Hartmann interpolation formula was computed, giving the following equation:

$$\lambda = 2697.89 - \frac{[5.1542630n]}{M - 104{,}610}\,.$$

With this formula the normal lines given in Table III were identified with lines in Young's list of chromospheric lines as revised by Frost in his *Astronomical Spectroscopy*. The first column gives the number, the second the intensity, the third the adopted value of $\lambda$, the sixth the value of $M$ computed by the above formula, the seventh the observed value of $M$, being the mean of plate 6 and plate 3 reduced as above to plate 6, the eighth C—O; the ninth, tenth, and eleventh columns are from Young's list of chromospheric lines. The remaining columns explain themselves.

I think this table explains the values of the adopted wave-lengths. In adopting these values, while adhering to no fixed rule, but trusting to my judgment as to what seemed the probable value from the material at my command, I was guided by the following considerations: Where two lines were found too close to be resolved on my plates, and of equal or nearly equal intensity as given by Kayser and Runge in the arc spectrum, their mean wave-length was adopted, while if their intensities were markedly different, that of the brighter one was used. In every case Rowland's value of $\lambda$ was employed. Though these adopted wave-lengths may be slightly in error, I do not believe they will be so changed as to appreciably alter the wave-lengths computed from them as normal lines, even though they may change quite appreciably the constants of the interpolation formula. $D_3$ and F were rejected from this second list of normal lines as being by far too wide for accurate measurement. From these 28 residuals, corrections to $\lambda_0$, $c$, and

TABLE III.

| Number | I | Adopted λ | Computed λ | C—O | Computed M | Observed M | C—O | Young's Chrom. lines λ | F | B |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 $D_1$ | 4 | 5896.16 | 5896.16 | ±0.00 | 60.009 | 60.002 | +0.007 | | 50 | 30 |
| 2 $D_2$ | 4 | 5890.19 | 5890.00 | −0.19 | 59.925 | 59.916 | +0.009 | | 50 | 30 |
| 18 | 2 | 5535.35 | 5535.30 | −0.05 | 54.337 | 54.330 | +0.007 | 5535.07 | 50 | 12 |
| 25 | 3 | 5455.83 | 5455.86 | +0.03 | 52.888 | 52.882 | +0.006 | 5455.83 | 10 | 4 |
| 26 | 2 | 5447.13 | 5447.03 | −0.05 | 52.724 | 52.717 | +0.007 | 5447.13 | 10 | 4 |
| 28 | 2 | 5429.91 | 5429.65 | −0.26 | 52.397 | 52.386 | +0.011 | 5429.9 | 8 | 3 |
| 45 | 4 | 5328.47 | 5328.33 | −0.14 | 50.383 | 50.375 | +0.008 | 5328.7<br>5328.2 | 3<br>3 | 2<br>2 |
| 53 | 4 | 5276.21 | 5276.16 | −0.05 | 49.284 | 49.278 | +0.006 | 5276.21 | 10 | 10 |
| 54 | 5 | 5270.14 | 5270.02 | −0.12 | 49.154 | 49.146 | +0.008 | 5270.5<br>5269.72 | 5<br>10 | 2<br>2 |
| 64 | 3 | 5208.69 | 5208.70 | +0.01 | 47.797 | 47.792 | +0.005 | 5208.8<br>5208.6 | 4 | 5 |
| 65 | 1 | 5205.50 | 5205.43 | −0.07 | 47.724 | 47.714 | +0.008 | 5205.9 | 4 | 5 |
| 70$b_1$ | 8 | 5183.79 | 5184.18 | +0.39 | 47.228 | 47.232 | −0.004 | | 50 | 35 |
| 71$b_2$ | 5 | 5172.86 | 5173.22 | +0.34 | 46.974 | 46.978 | −0.004 | | 50 | 30 |
| $\frac{72+73}{2}\ \frac{1}{2}(b_3+b_4)$ | 5 | 5168.36 | 5168.37 | +0.01 | 46.869 | 46.865 | +0.004 | | 40<br>20 | 25<br>10 |
| 102 | 5 | 5018.63 | 5018.67 | +0.04 | 43.144 | 43.142 | +0.002 | 5018.5 | 30 | 15 |
| 113 | 3 | 4959.63 | 4957.57 | −0.06 | 41.485 | 41.481 | +0.004 | 4957.48 | 2 | 1 |
| 116 | 5 | 4934.24 | 4934.21 | −0.03 | 40.824 | 40.822 | +0.002 | 4934.2 | 30 | 5 |
| 117 | 5 | 4924.11 | 4924.12 | +0.01 | 40.534 | 40.533 | +0.001 | 4924.11 | 40 | 10 |
| 124 | 2 | 4900.10 | 4900.05 | −0.05 | 39.335 | 39.833 | +0.002 | 4900.31 | 30 | 6 |
| 126 | 2 | 4883.87 | 4883.74 | −0.13 | 39.855 | 39.350 | +0.005 | 4883.9 | 10 | 4 |
| 132 | 2 | 4824.01 | 4823.95 | −0.06 | 37.517 | 37.516 | +0.001 | 4824.33 | 10 | 2 |
| 134 | 2 | 4805.28 | 4805.16 | −0.12 | 36.921 | 36.918 | +0.003 | 4805.25 | 3 | 1 |
| 168 | 4 | 4629.52 | 4629.47 | −0.05 | 30.762 | 30.766 | −0.004 | 4629.52 | 15 | 18 |
| 182 | 5 | 4572.16 | 4572.14 | −0.02 | 28.502 | 28.509 | −0.007 | 4572.16 | 10 | 4 |
| 183 | 4 | 4563.94 | 4563.96 | +0.02 | 28.167 | 28.176 | −0.009 | 4563.94 | 10 | 5 |
| 187 | 5 | 4549.72 | 4549.78 | +0.06 | 27.579 | 27.591 | −0.012 | 4549.8 | 10 | 8 |
| 192 | 5 | 4534.14 | 4534.15 | +0.01 | 26.926 | 26.936 | −0.010 | 4534.2 | 5 | 5 |
| 199 | 5 | 4501.44 | 4501.40 | −0.04 | 25.518 | 25.527 | −0.009 | 4501.44 | 15 | 6 |

$M$ of the interpolation formula, together with their weights and probable errors, were computed by the method of least squares, and the final values found were

$$\lambda = (2695.60 \pm 0.64) - \frac{(142857 \pm 79)}{M - (104.637 \pm 0.018)},$$

while the probable error of an observation of $M$, whose weight is unity, came out $\pm$ 0.0022 mm, corresponding to a probable error of λ at $m = 20$, of $\pm$ 0.04; at $m = 40$, of $\pm$ 0.07; and at $m =$ 60, of $\pm$ 0.16.

TABLE III.

| Rowland's intense lines | | | | | | | | | Kayser and Runge's $Fe_2$ lines | | | | | | Hasselberg's *Ti* lines | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| λ | I | Ele. | λ | I | Ele. | γ | I | Ele. | λ | I | λ | I | λ | I | λ | I | λ | I | λ | I |
| 6.16 | | | | | | | | | | | | | | | | | | | | |
| 0.19 | | | | | | | | | | | | | | | | | | | | |
| 5.06 | 2 | *Fe* | 5.64 | 2 | *Fe* | | | | 5.52 | 4 | 4.86 | 6 | | | | | | | | |
| 5.83 | 4 | *Fe* | 5.67 | 2 | *Fe?* | | | | 7.72 | 5 | 5.80 | 1 | 4.53 | 6 | | | | | | |
| 6.80 | 2 | *Ti* | 7.13 | *Fe* | *6d?* | | | | 7.05 | 1 | | | | | 6.80 | 2 | | | | |
| 9.91 | *6d?* | *Fe* | | | | | | | 9.81 | 1 | 9.11 | 6 | | | 9.37 | 2.3 | | | | |
| 8.24 | 8 | *Fe* | 8.70 | 2 | *Fe* | | | | 8.15 | 1 | 8.50 | 2 | 8.94 | 6 | | | | | | |
| 5.93 | 1 | *Cr* | 6.17 | 3 | *Fe?* | 6.24 | 2 | *Cr* | 6.19 | 6 | | | | | | | | | | |
| 9.72 | 8 | *Fe* | 0.56 | 4 | *Fe* | | | | 9.65 | 1 | 0.43 | 1 | | | | | | | | |
| 8.60 | 5 | *Cr* | 8.78 | 2 | *Fe* | | | | | | | | | | | | | | | |
| 4.77 | 3 | *Fe* | 4.68 | 5 | *Cr* | 6.22 | 5 | *Cr, Ti* | | | | | | | | | | | | |
| 3.79 | | | | | | | | | | | | | | | | | | | | |
| 2.86 | | | | | | | | | | | | | | | | | | | | |
| 8.36 | | | | | | | | | | | | | | | | | | | | |
| 8.63 | 4 | *Fe* | 8.46 | 1 | *Ni* | | | | 8.53 | 4 | | | | | 8.50 | 2 | | | | |
| 7.48 | 5 | *Fe* | 7.78 | 8 | *Fe* | | | | 7.80 | 2 | 7.43 | 3 | | | | | | | | |
| 4.21 | 3 | *Ba* | 4.28 | 4 | *Ba* | | | | | | | | | | | | | | | |
| 4.11 | 5 | *Fe* | 4.96 | 3 | | | | | 4.00 | 6 | 4.89 | 5 | | | | | | | | |
| 0.10 | 2 | *Ti, Ca* | 0.30 | 2 | *Yi* | | | | | | | | | | 0.08 | 3 | | | | |
| 3.87 | 2 | *Yt* | | | | | | | | | | | | | | | | | | |
| 3.70 | 5 | *Mn* | 4.32 | 3 | *Fe* | | | | 3.63 | 4 | 4.27 | 6 | | | | | | | | |
| 5.28 | 3 | | 5.61 | | *O, Ti* | | | | | | | | | | 5.25 | 1.2 | 5.56 | 2.3 | | |
| 9.52 | 6 | *Ti, Co* | | | | | | | | | | | | | | | | | | |
| 2.16 | 6 | *Ti* | | | | | | | | | | | | | 2.15 | 3 | | | | |
| 3.94 | 4 | *Ti* | | | | | | | | | | | | | 3.60 | 1.2 | 3.94 | 2.3 | | |
| 9.64 | 2 | *Fe* | 9.81 | 6 | *Ti, Co* | | | | | | | | | | | | | | | |
| 4.14 | 6 | *Ti, Co* | | | | | | | | | | | | | | | | | | |
| 1.44 | 5 | *Ti* | | | | | | | | | | | | | | | | | | |

With this equation the wave-lengths of the normal lines were computed and are given in column 4, Table III, and the residuals C – O in column 5. The average probable error of a single wave-length deduced from these residuals comes out ±0.09.

With this equation the final wave-lengths given in Table I were computed. These were first computed directly with seven-place logarithms and checked by computing a table for every millimeter of *M* from 20 to 60 and interpolating, using second differences; they are, I think, free from any errors of computation.

I have carried these wave-lengths out to the $\frac{1}{100}$ of an Ångström unit, not because I think the hundredths are of real significance, but, except in the case of very broad lines, such as $D_1$, F, and $H\gamma$, and in cases of lines classed as very hazy, I believe the tenths are, and it was as easy to carry out the computation to the $\frac{1}{100}$ as to be certain of having the tenths correct. The normal lines were not extended beyond $\lambda$4549.78, as above this point the want of definition due to the non-achromatic properties of the instrument employed became very marked. But I computed their wave-lengths, as one does not feel inclined to throw away any material secured at the time of an eclipse, even if not of very great value.

I had hoped to be able to determine the elevations of these lines by measuring the arc of their crescents. But it soon became evident that measurements of this character would be valueless, since the irregularities of the Moon's surface were so large in proportion to the elevations that lines frequently appeared, disappeared, and reappeared again several times. I decided therefore to divide the lines into five classes and denote their elevations on a scale of from 1 to 5, 1 being the shortest and 5 the longest.

To Table I, I have added three columns, giving all lines in Young's list of chromospheric lines which are certainly found on my plates and in addition those lines of Young's list which, though near the lines of my plate, yet seem so far away as to render the identification doubtful. The remaining 86 lines are certainly not present in Young's list, while a number of the lines given in Young's list are not found on my plates.

It was my original intention to make no effort at identifications of these lines, but at Professor Frost's suggestion I have made a careful comparison with the Fraunhofer lines given by Rowland in his table of solar spectrum wave-lengths and the results are so striking that I think they would be of general interest. In comparing spectra obtained with two instruments differing greatly in resolving power, great caution must be observed in order to avoid faulty identifications. It is evident that a group

of two or more lines shown by Rowland as widely separated and differing considerably in their relative intensities might all merge into one in my instrument; and the question to be answered is, at what point of the resulting superimposed diffraction patterns will the point of maximum density occur. The case of two lines of equal intensity is simple enough, but if the intensities differ widely we may find the point of maximum density anywhere between the mean of the two and the center of the strongest line. What seemed to me, therefore, a plausible plan to follow was to select all the lines of Rowland's list, having an intensity of 1 or greater, which may fairly be considered as possibly combining into one on my plates. In order to determine the point of maximum density of this group of lines I multiplied the wave-length of each by its intensity, added their products together, and divided the sum thus formed by the sum of the intensities. By this somewhat arbitrary method I have made my comparison and find that of the 200 lines given in my table, whose wave-lengths are carried out to $\frac{1}{100}$ of an Ångström unit, 163 or 81 per cent. have residuals of between 0 and 0.3 of an Ångström unit, 20 or 10 per cent. residuals of from 0.3 to 0.4, 6 or 3 per cent. of from 0.4 to 0.5, 6 or 3 per cent. of from 0.5 to 0.71 and 5 or $2\frac{1}{2}$ per cent. have no corresponding line in Rowland's list; of these, one is $D_3$, and three others have been shown to agree with lines in the spectrum of Clèveite gas. The remaining one at $\lambda$ 5661.96 is found on only one of my plates and was classed as "very large and broad" the first time and "doubtful and hazy" the second; its intensity is estimated as 1. If we regard the intensity of such a group of lines as equal to the sum of the intensities of its components the agreement of my estimated intensities with those computed from Rowland's table is far from satisfactory, the relative intensities being frequently reversed. The character of the lines, however, agrees better. In most cases a line classed as broad or hazy corresponds to a group of several lines, though this is not always so. In view of the above facts I feel safe in saying that so far as these two plates are concerned the lines may be completely explained by

a reversal of *some* of the Fraunhofer lines combined with a change in their relative intensities, plus those lines due to helium.

That the flash spectrum is the reversal of all the Fraunhofer lines is far from being borne out by my plates. Of the 1144 solar lines given by Rowland within the limits of my plates and having an intensity of 1 or over, a total of only 445, or 40 per cent., can by any fair assumption be considered as making up lines or groups on my plates, and among the 60 per cent. lacking are many of the brightest lines. Thus there are 73 lines with an intensity of 4 or greater certainly not found on my plates, over one half of which are due to iron. Why this should be so is not easy to see; possibly they are at such low levels as not to have been caught on my plates, in which case iron must give a different spectrum at high levels than at low. However, whatever may be the explanation, I simply offer the facts as they have been observed.

In conclusion I wish to express my deep obligation to Professor Brown and the other members of the Naval Observatory staff at Barnesville, whose cordial support and earnest coöperation made this work possible. It is also proper to state that the actual expense of this work was borne by the government, while the trustees of the Ohio State University granted me a month's leave of absence.

EMERSON MCMILLIN OBSERVATORY,
OHIO STATE UNIVERSITY,
December 10, 1900.

# *MINOR CONTRIBUTIONS AND NOTES*

## COÖPERATION IN OBSERVING VARIABLE STARS.[1]

THE number of known variable stars of long period is now so great, and is increasing so rapidly, that the observation of many of them has been greatly neglected. Observations by Argelander's method are so easily made that they are especially adapted to observers who, for various reasons, cannot use precise photometric methods. In the case of variables of small range, including those of short period and many of the *Algol* variables, the subjective errors greatly diminish the value of observations by Argelander's method. In these cases, also, the periods and light curves appear to be so regular that continuous observations are not needed. It appears to be better to observe such objects photometrically throughout their variation, if possible, and thus determine the light curves. Small variations in the period can then be determined by occasional observations at times when the light is varying most rapidly. Many of the variables of long period appear to change irregularly, and continuous observations are required until the nature of the changes are known. Moreover, the range is, in many cases, so great that the errors of observation are not sufficient to affect seriously the form of the curve. The method of observation for these stars, which has been in use here for the last twelve years, is as follows: A sequence of comparison stars is first selected as near the variable as possible, and each about half a magnitude brighter than the next in order, the brightest being somewhat brighter than the variable at maximum, and the faintest fainter than the variable at minimum. Care is taken not to include double stars or those near brighter stars. The stars brighter than the tenth magnitude are then measured with the meridian photometer. This has been done for nearly all of the comparison stars selected here. Magnitudes determined with the meridian photometer, for all stars of the seventh magnitude and brighter, can now be furnished upon a uniform system. For the fainter stars, measures have been made of large numbers of stars as

[1] *Harvard College Observatory Circular* No. 53.

faint as the thirteenth magnitude, and photometers are now in use with which the faintest stars visible in the largest telescopes can be measured. As the apparent magnitude may sometimes differ from the measured magnitude, it has been found best to estimate independently on several nights the interval between each of the adjacent stars in the sequences, and adopt magnitudes found by combining these estimates with the photometric magnitudes. Having thus provided standards of comparison on the same scale for stars in all parts of the sky, a variable may be compared on any night with the stars nearest it in brightness in its sequence, taking care to select one that is brighter and another fainter. From estimates of these intervals in grades, the light of the variable is readily reduced to the standard scale. When a variable is faint, it is impossible to observe it for several days every month near the time of full Moon. At least one observation should be obtained in the interval between successive times of full Moon. This can be done only for polar stars, owing to the proximity of the Sun at certain seasons. Since the periods of a large portion of the variables of long period exceed half a year, it is evident that monthly observations will, in general, give a good idea of the form of the light curve. Of course, additional observations should also be obtained, but failure to secure any observation during a long interval should be avoided, if possible. Since 1889 an attempt has been made to observe seventeen circumpolar variables north of declination $+50°$ at least once a month. These stars are always above the horizon at Cambridge, so that they can be observed at all seasons. The results for the years 1889–1899 will be found in Volume XXXVII, Part I, of the *Annals*, which is now printed and in process of distribution. Similar observations have been made of about sixty other variables, but less regularly. At Arequipa similar observations have been made of a large number of southern variables. It is much to be desired that all variables of long period should be observed in the same way, or at least so that all can be reduced to a uniform scale of magnitudes. Coöperation is necessary to attain success in this work. Variables near the ecliptic can be observed when near the Sun much better at tropical stations than at those near the pole. The reverse is true for polar variables. Northern variables can be observed for a longer portion of the year at northern observatories, and southern variables at southern observatories. When a variable can be observed only in the morning, it is much more likely to escape observation than at

other seasons. A computation has been made of the date on which the variables mentioned above are 20° above the horizon at Cambridge at midnight, and also in the morning and evening when the Sun is still 10° below the horizon. Thus these three dates for the star *T Andromedæ* are July 1, May 1, and March 12. Accordingly, from May 1 to July 1 this star can be observed only in the morning, from July 1 to March 12 it can be observed in the evening, while from March 12 to May 1 observations are difficult owing to twilight. When a variable is bright it is best observed with a small telescope, that is, one having an aperture of not more than 6 or 8 inches. Observations of great value could be obtained by an observer with a large telescope if he was notified when the star was too faint to be observed with smaller instruments. The excellent charts of Father Hagen are almost indispensable for observing the stars when fainter than the ninth magnitude. When the variables are bright, the need has been felt here for charts on a smaller scale and covering a larger region. After various experiments, photographic enlargements have been made of portions of the admirable charts of the Bonn *Durchmusterung*. A region 3° square surrounding each variable has been enlarged three times, thus giving a map on the standard scale of one minute of arc to one millimeter. The stars on these maps, while appearing coarse by daylight, are thus easily seen and identified at night without using a light bright enough to dazzle the eye. The designations of the stars in the seqüence are marked upon these enlargements, and copies will be furnished at cost. Charts will be furnished free of cost to experienced observers who are ready to coöperate in the above plan of work. Observations of nearly equal value can be obtained by those unaccustomed to estimating intervals in grades. It is only necessary to enter on the charts the standard magnitudes of the comparison stars, and from these to estimate directly the magnitude of the variable. Charts are now being prepared, and with the corresponding magnitudes can soon be furnished for the following stars:

*T Andromedæ, T Cassiopeiæ, R Andromedæ, S Ceti, S Cassiopeiæ, R Piscium, R Arietis, o Ceti, S Persei, R Ceti, U Ceti, R Tauri, R Aurigæ, U Orionis, R Lyncis, R Geminorum, S Canis Minoris, R Cancri, V Cancri, S Hydræ, T Hydræ, R Leonis Minoris, R Leonis, R Ursæ Majoris, X Virginis, R Comæ Berenices, T Virginis, R Corvi, Y Virginis, T Ursæ Majoris, R Virginis, S Ursæ Majoris, U Virginis, V Virginis, R Hydræ, S Virginis, R Canum Venaticorum, S Boötis, R Camelopardali, R Boötis, S Libræ, S*

*Serpentis, S Coronæ Borealis, R Herculis, R Scorpii, S Scorpii, U Herculis, V Herculis, R Ursæ Minoris, R Draconis, S Herculis, R Ophiuchi, T Herculis, R Scuti, R Sagittarii, R Cygni, χ Cygni, S Cygni, RS Cygni, R Delphini, U Cygni, V Cygni, T Aquarii, R Vulpeculæ, T Cephei, S Cephei, SS Cygni, S Aquarii, R Pegasi, S Pegasi, R Aquarii, and R Cassiopeiæ.*

If the above plan proves successful, it is hoped that it may be extended to the other variable stars of long period.

EDWARD C. PICKERING.

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## ANDERSON'S NEW STAR IN *PERSEUS*.[1]

THE cable message announcing the discovery of a new star in the constellation *Perseus* by the Rev. T. D. Anderson was received at the Observatory early in the evening of February 22, 1901. Owing to clouds, the new star was only occasionally visible, and twice it was necessary to cover the instruments on account of falling snow. During the intervals, however, various observations were made, which have a value owing to their early date. Numerous comparisons by Miss Cannon with α *Aurigae*, magnitude 0.21, α *Orionis*, magnitude 0.92, and α *Tauri*, magnitude 1.06, showed that the magnitude of the star was about 0.9. Photometric comparisons by Professor Wendell with the 15-inch telescope, of the *Nova* with the star $+43°732$, magnitude 7.25, at $14^h\ 0^m$ and at $17^h\ 25^m$, Greenwich Mean Time, gave the magnitudes 0.35 and 0.39, respectively.

Meanwhile an examination was being made by Mrs. Fleming of the photographs of the region obtained here earlier in the month, with the various instruments. Although photographs are taken with the transit photometer throughout every clear night, yet owing to twilight they cannot be taken as early in the evening as this star culminates. Fortunately, for some weeks the work of the transit photometer, which photographs objects only near the meridian, has been supplemented by photographs with Cooke and Ross-Zeiss Anastigmat lenses. With these instruments, an attempt is made to cover the entire sky, both east and west of the meridian, at short intervals. The completeness with which this has been done is shown by the fact that we have photographs of the region of the *Nova* with the Cooke lens on February 8, 18, and 19, and with the Ross-Zeiss lens on February 2, 6, 18, and 19. The photograph taken with the Cooke lens on February 19

[1] *Harvard College Observatory Circular* No. 56.

had an exposure of $66^m$, beginning at $11^h\ 18^m$ Greenwich Mean Time, While this photograph showed not only the faintest stars contained in the *Durchmusterung*, but also stars as faint as the eleventh magnitude, no trace of the *Nova* was seen. This result was confirmed by the other plates mentioned above. A general examination of the large number of earlier plates of this region did not seem to be necessary. Plates taken with the 8-inch Bache telescope as early as November 6, November 8, and December 12, 1887, fail to show the *Nova*, although the spectra of stars as faint as the eighth magnitude are clearly visible on all, and those of the ninth magnitude, on the plate taken on November 6. A photograph taken with the 24-inch Bruce telescope on October 18, 1894, with an exposure of $15^m$, shows no trace of this object, although stars as faint as the magnitude 12.5 are well seen.

On this same evening, February 22, eighteen photographs were taken with various instruments under the direction of Mr. Edward S. King. They showed that, photographically, the *Nova* was 0.3 fainter than *α Aurigae*. The general appearance of the photographic spectrum resembled that of the *Orion* type and was very unlike that of other new stars, in which the bright lines are the most conspicuous feature. This star had a strong continuous spectrum traversed by 33 dark lines. The approximate wave-lengths, as derived by Hartmann's formula, from the measures of $H\epsilon$, $H\gamma$, and $H\beta$, are given below. Each is followed by its relative intensity, and by the difference found by subtracting it from the wave-length of the corresponding line, if any, in the spectrum of *β Orionis*. As the lines having greater wave-length than 5000 have thus been determined by extrapolation, they may be subject to large systematic errors.

3894, 10, $H\zeta$, $-5$; 3970, 20, $H\epsilon$, 0; 4026, 3, 0; 4077, 2, $-1$; 4102, 30, $H\delta$, 0; 4126, 5, $+2$; 4151, 1, $-4$; 4266, 2, $+1$; 4341, 40, $H\gamma$, 0; 4366, 1, $+1$; 4388, 2, 0; 4415, 1; 4435, 1, $+3$; 4470, 2, $+2$; 4481, 20, 0; 4510, 2, $-2$; 4530, 2; 4552, 2; 4572, 1; 4616, 1; 4643, 1; 4665, 3; 4714, 3, $-1$; 4862, 40, $H\beta$, 0; 4885, 2; 4922, 2, 0; 5325, 1; 5399, 1; 5431, 1; 5677, 2; 5695, 7; 5719, 5; and 5761, 1. On careful examination the lines 3970, 4102, 4341, 4481, and 4862 were seen to be bright on the edge of greater wave-length. The line 4665 was bright on the edge of shorter wave-length, or there was a bright line whose approximate wave-length was 4660. The line 4026 was not measured, but identified from its position.

On February 23, the clouds were so dense that few observations

could be made. The star appeared to be brighter and bluer than $\alpha$ *Aurigae*, and to have the approximate magnitude 0.0. The spectrum was photographed faintly and showed no marked change except that the line K, which was absent on the previous evening, was present and nearly as intense as $H\epsilon$.

On February 24, it became clear soon after noon, and at one o'clock the *Nova* was seen with the 6-inch equatorial, and also with the 2-inch finder, in strong sunlight. In the evening, the magnitude according to visual comparisons, was 0.54, from measures with the 15-inch equatorial, 0.59, and with the meridian photometer, in strong daylight, 0.28. Photographically it was 0.4 or 0.5 fainter than $\alpha$ *Aurigae*. The spectrum showed a remarkable change. It was traversed by numerous bright and dark bands, and closely resembled that of *Nova Aurigae*. The principal lines were dark with accompanying bright lines of somewhat greater wave-length. The bright lines accompanying K and $H\epsilon$ were reversed, and traversed by narrow well defined dark lines. These last lines, and one of somewhat shorter wave-length than $H\beta$, are the only sharply defined lines in the spectrum, all of the others being broad and hazy, and difficult to measure with accuracy.

Clouds interfered with observations on February 25, but the *Nova* was evidently much fainter than on the previous evening. Its magnitude from visual comparisons was 1.4, from photometric measures, 1.07. The spectrum differed slightly from that on February 24. The lines $H\delta$, $H\gamma$, and $H\beta$ were also reversed and replaced by one or more narrow dark lines.

On February 26, the magnitude from visual comparisons was 1.3, from photometric measures 1.49. The changes in spectrum were slight.

Observations of the position of the *Nova* were made by Mr. J. A. Dunne, with the 8-inch meridian circle, on February 23, 24, and 25, with the result for 1900.0, R.A. $3^h\ 24^m\ 24^s.02$, Dec. $+43^\circ\ 33'\ 42''.4$.

It therefore appears that on and before February 19, 1901, the star was invisible, or at least fainter than the eleventh magnitude. On February 21, its magnitude was 2.7, according to Mr. Anderson. On February 22, its magnitude was 0.5, perhaps becoming a little brighter on February 23, and then diminishing, so that on February 25 its magnitude was 1.1. Its spectrum on February 22 and 23, was of the *Orion* type, nearly continuous, traversed by narrow dark lines. During the next 24 hours an extraordinary change took place, so that on February 24 the spectrum resembled that of the other *Novae*. It was

traversed by bright and dark bands, and the principal dark lines had accompanying bright lines of slightly greater wave-length.

During the last fourteen years, and since the general application of photography to astronomy, eight new stars are known to have appeared, *Nova Persei*, in 1887; *Nova Aurigae*, in 1891; *Nova Normae*, in 1893; *Nova Carinae*, in 1895; *Nova Centauri*, in 1895; *Nova Sagittarii*, in 1898; *Nova Aquilae*, in 1899; and *Nova Persei*, in 1901. The second and last of these, which were much brighter than the others, were found visually by Dr. Anderson. All of the others were found by Mrs. Fleming, from an examination of the Draper Memorial Photographs. *Nova Aquilae* was announced by telegraph, but has not been described in these circulars. Its position for 1900 is R.A. $19^h\ 15^m.3$, Dec. —$0^\circ\ 19'$. It was not seen on plates taken on November 1, 1898, and earlier, although stars of the thirteenth magnitude appeared on some of them. On April 21, 1899, it was seventh magnitude. It appears on eighteen photographs taken during that summer, and on October 27, 1899, it was tenth magnitude. In July 1900, when it was discovered, it was about twelfth magnitude. Seven bright lines $H\zeta$, $H\epsilon$, $H\delta$, $H\gamma$, 4693, $H\beta$, and the nebular line 5007, were seen in the spectrum photographed on July 3, 1899. On September 7, 1899, $H\gamma$ and a somewhat fainter line, which is probably 4959, were the only bright lines visible. On October 27, 1899, $H\gamma$ and 5007 were alone visible and bright, so that the spectrum had then become that of a gaseous nebula.

EDWARD C. PICKERING.

February 27, 1901.

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## THE YERKES OBSERVATORY OF THE UNIVERSITY OF CHICAGO.

### BULLETIN NO. 16.

### THE NEW STAR IN *PERSEUS*.

THE first news of Anderson's discovery of a new star in *Perseus* was received at this Observatory on February 24. An examination of the region near the star, made that evening with the 40-inch telescope, failed to show any evidence of nebulosity, but the bright moonlight would have rendered a faint nebula invisible. At that time the magnitude of the star appeared to be about 0.5. Its color was yellow, with a

decided reddish cast, very similar to that of $\alpha$ *Orionis*. Very little time was spent in examining the spectrum visually, as it was felt that photographs would be more valuable than drawings based on micrometer measures. We had fortunately just received a fresh supply of Erythro plates through the kindness of the International Color-Photo Company of Chicago, and it was therefore possible to photograph the entire spectrum from $H\alpha$ to $H\iota$. Beyond this point in the ultra-violet the absorption of the 40-inch objective greatly enfeebles the spectrum, which is still further weakened by the lack of perfect achromatism in this region.

Photographs of the spectrum have been obtained by Mr. Ellerman as follows:

| Date | No. of plates | Dispersion | Region |
|---|---|---|---|
| 1901, Feb. 24 | 5 | 3 prisms | $H\beta$ to $H\gamma$, Bruce spectrograph |
| 24 | 5 | 3 prisms | D to 4400 |
| 24 | 1 | 3 prisms | $H\alpha$ to 4500 |
| 25 | 3 | 3 prisms | $H\beta$ to $H\gamma$, Bruce spectrograph |
| 25 | 3 | 3 prisms | D to 4400 |
| 25 | 3 | 1 prism | 5700 to 3700 |
| 25 | 1 | 1 prism | $H\alpha$ to 3900 |
| 26 | 3 | 3 prisms | D to 4400 |
| 26 | 1 | 1 prism | $H\alpha$ to 3900 |
| 27 | 8 | 3 prisms | D to 4400 |
| 27 | 1 | 3 prisms | $H\alpha$ to 4500 |
| 27 | 6 | 1 prism | 5700 to 3700 |
| 28 | 5 | 1 prism | 5700 to 3700 |
| 28 | 2 | 1 prism | $H\alpha$ to 3900 |
| 28 | 3 | 3 prisms | D to 4400 |
| Mar. 4 | 3 | 1 prism | 5700 to 3700 |
| 4 | 1 | 1 prism | $H\alpha$ to 3900 |
| 6 | 2 | 3 prisms | D to 4400 |
| 6 | 1 | 3 prisms | $H\beta$ to $H\gamma$ |
| 6 | 1 | 1 prism | 5700 to 3700 |
| 11 | 3 | 3 prisms | D to 4400 |
| 11 | 1 | 1 prism | 5700 to 3700 |

The comparison spectra which appear on these plates are those of titanium, hydrogen, and sodium.

On February 24 and 25 Mr. Ritchey photographed the region of the *Nova* with the 40-inch telescope and color screen. In order to obtain a sufficient number of comparison stars the plates were given an exposure of one hour. The light of the *Nova* was intercepted by a small movable occulting disk, with which four (for the second plate, five) very brief exposures were given at intervals of about fifteen minutes.

PLATE III.

Fig. 1.—$D_3$ and the Sodium Lines.


Fig. 2.—Region *Ha* to D.

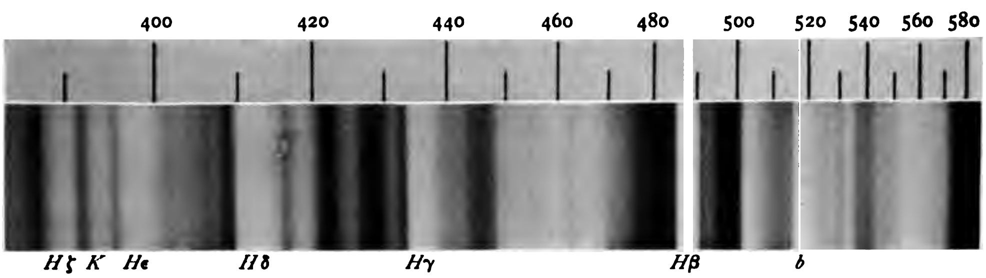



Fig. 3.—Spectrum of *Nova Persei*, Feb. 28, 1901.

The total exposure for the *Nova* was probably about half a second. In the resulting photographs, the images of the *Nova* and the neighboring stars (of which more than forty appear in a region 12′ square) are small and appear to be well adapted for measurement. Through the kindness of Director Rees, these plates will be measured at the Columbia College Observatory. The position of the *Nova* was measured micrometrically by Professor Burnham on March 3.

The wedge photometer used with the 40-inch telescope in the determination of standards of faint stellar magnitude has been employed by Mr. Parkhurst in measuring the brightness of the *Nova*. Hitherto objectives of one and two inches aperture have sufficed, but as the *Nova* decreases in brilliancy it will be followed with the 12-inch and 40-inch telescopes. A preliminary reduction gives the following magnitudes:

| Date | Mag. | Date | Mag. |
|---|---|---|---|
| 1901, Feb. 25 | 1.0 | 1901, Mar. 3 | 2.7 |
| 26 | 1.1 | 4 | 2.8 |
| 27 | 2.0 | 5 | 2.7 |
| 28 | 1.9 | 6 | 3.1 |

A photograph of the spectrum (G 440) taken with the one prism spectrograph on February 28 has been measured by the writer. The resulting wave-lengths of the lines and bands, computed by the aid of Cornu-Hartmann formulæ, furnished data for attaching a scale to an enlargement of the photograph reproduced in Fig. 3, Plate III.

Inspection of the photograph will show that the spectrum is very similar to the earlier spectrum of *Nova Aurigae*. The hydrogen lines, notably C (Fig. 2) and F, are bright and very broad. The dark lines superposed upon them (not shown in the cut) are probably reversals caused by the absorption of an outer layer of cooler gas at lower pressure.

On the more refrangible side the hydrogen lines are accompanied by dark lines, just as was the case with *Nova Aurigae*. As Wilsing has shown, this is doubtless due to the great pressure under which the radiation occurs. The bright sodium line has broadened into a band, on which appear the two dark D lines (Fig. 1). These appear on the photographs, and are clearly visible in visual observations with a three-prism spectroscope. As the titanium poles were moistened with a weak solution of sodium chloride, the comparison spectrum contains the bright sodium lines. Thus the motion of the star in the line of

sight can be measured. Some preliminary determinations indicate that the *Nova* is moving away from the Earth at a low velocity.

The helium line $D_3$ seems to be present as a dark line, lying close to the bright sodium band on the more refrangible side (Fig. 1). The bright calcium lines H and K are notable for their great breadth and for the narrow lines of reversal which traverse them. The chief nebular line seems to be present ($\lambda$ 5002 – 5041), and a fainter line or band ($\lambda$ 4911 – 4988) covers the region of the second nebular line. The *b* group of magnesium is doubtless represented by the very bright band $\lambda$ 5154 – 5204. The green coronal line ($\lambda$ 5303) would fall near the more refrangible edge of a bright band in the spectrum of the *Nova*.

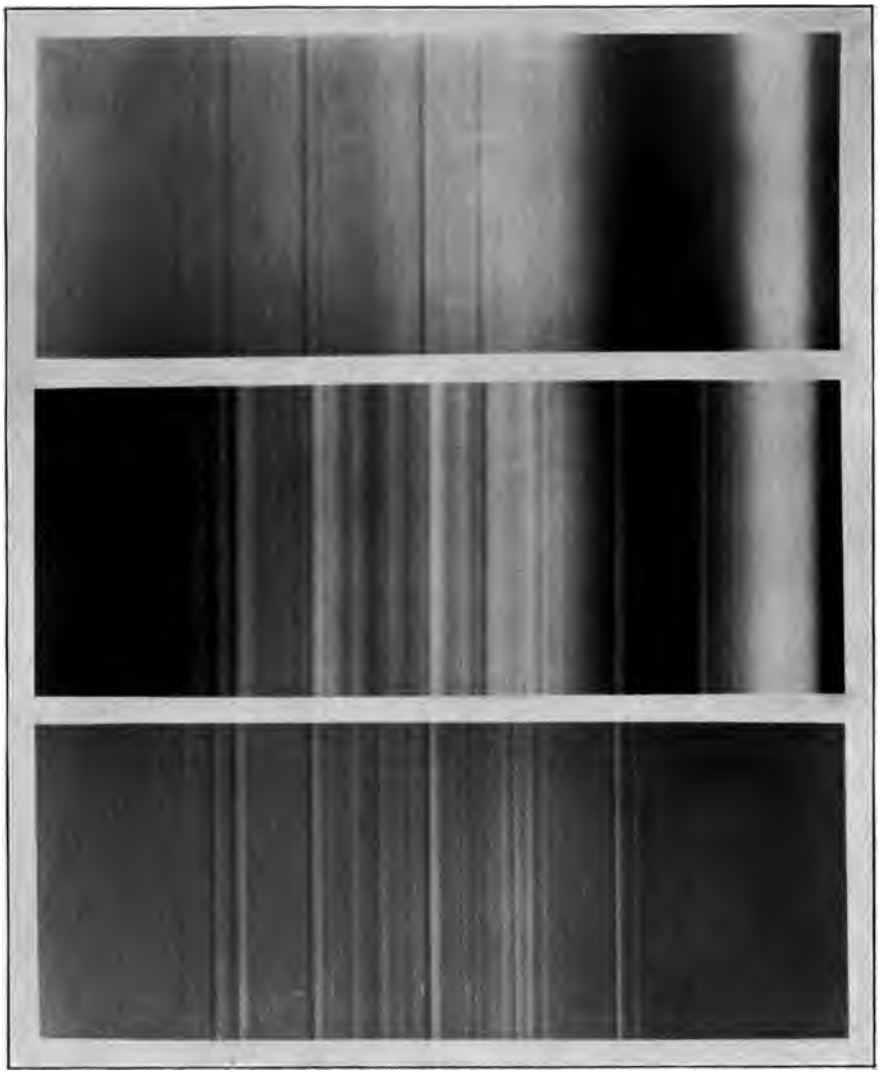
Further results, based upon measurements of photographs taken with the three-prism spectrograph, will be given in a subsequent paper.

*Note added March 18.*—A comparison of photographs taken on March 4 and March 15 shows that the dark lines on the more refrangible edge of the bright hydrogen lines continue to increase in sharpness. At first single and rather diffuse, they have become sharply defined double lines. K is much fainter than before, and *b* is apparently decreasing in intensity.

GEORGE E. HALE.

March 12, 1901.

PLATE IV



SPECTRA OF *NOVA PERSEI* AND *NOVA AURIGAE*

PHOTOGRAPHED AT HARVARD COLLEGE OBSERVATORY

1. *Nova Persei*, February 22, 1901. 2. *Nova Persei*, February 24, 1901. Isochromatic Plates.
3. *Nova Aurigae*, February 5, 1892. Ordinary Plate.

# THE ASTROPHYSICAL JOURNAL

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## ON THE ORBITS OF THE *ALGOL* VARIABLES *RR PUPPIS* AND *V PUPPIS*.


By ALEXANDER W. ROBERTS.


SUFFICIENT observations of the two recently discovered *Algol* variables, *RR Puppis* (*Ch.* 2781) and *V Puppis* (*Ch.* 2852), have been secured at Lovedale to yield, with some degree of exactness, the chief characteristics of their orbital movement.

Both stars are fine examples of the two outstanding types of *Algol* variation: of the type resulting from the revolution of two stars nearly equal in size and brightness, and of the type produced by the revolution of two stars considerably unequal both in size and in brightness.

There is also this added interest, that *V Puppis*, which belongs to the first type of variation, is a spectroscopic binary, and when observations sufficiently refined to yield certain measurements in the line of sight have been obtained, not only the relative, but the absolute dimensions of the system will be known. Any addition to our knowledge of this star is therefore of interest.

*RR Puppis* lies far beyond spectroscopic reach, at least with its present limitations.

*RR PUPPIS.* (*Ch.* 2781.)
R. A. $7^h\ 43^m\ 31^s$ (1900)
Dec. $-41°\ 7'.6$.

Observations of this star were begun as soon after the announcement of its variation as possible. The star has accordingly been under observation for the better part of a year, nearly 200 observations having been secured.

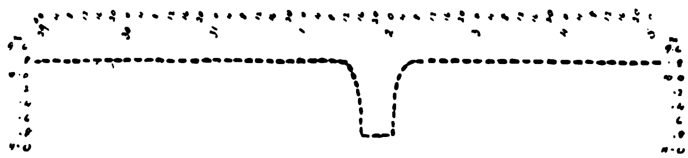


FIG. 1.

The period of variation cannot be far from

$$6^d\ 10^h\ 19^m.6.$$

With this period all the observations were reduced to the first light curve of 1900.

Figs. 1 and 2 indicate this mean curve. In Fig. 1 is given the whole light curve, and in Fig. 2 the portion of the curve near and at minimum.

The light curve of *RR Puppis* is almost identical in form

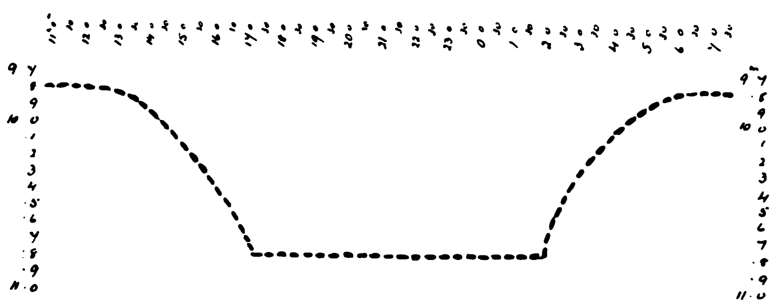


FIG. 2.

with that of *S Velorum*, and naturally the interpretation of the light changes of both stars run on parallel lines.

The data from which the form and relative dimensions of the orbit of *RR Puppis* are deduced may be summarized as follows:

(1) The full period is, as stated, $6^d\ 10^h\ 19^m.6$

(2) The limits of variation are $9^m.1$ and $10^m.8$

(3) The duration of increasing or decreasing phase is $4^h\ 15^m$
(4) The stationary period at minimum is $8^h\ 30^m$.

Statement 3 would indicate that the orbit of *RR Puppis* is practically circular. There is apparently a slight want of correspondence between the increasing and decreasing phases; but this apparent inequality may be due to errors of observation. A series of observations with the new prismatic telescope is being made, with the purpose of settling this difficulty, and also with the hope of obtaining some certain trace of a secondary minimum. As this minimum will not amount to much more than $0^m.05$, the probability of securing unmistakable evidence of its existence is remote.

The presumption of the evidence already obtained is that *RR Puppis* moves in a circular orbit.

Statements 1, 3, and 4 lead to a determination of the relative size of the component stars, assuming a circular orbit.

Thus, let

$$r\ ,\ r_{\imath} = \text{radii of the two stars}\ ;$$

then

$$r_1 + r_2 = \sin \Big\} 360^\circ \left(\frac{8^h\ 30^m}{6^d\ 10^h\ 19^m.6}\right)$$
$$= 0.34\ ,$$

the radius of the orbit being equal to unity.

The relation between $r_1$ and $r_2$ is given approximately by the ratio

$$\frac{r_1}{r_2} = \frac{4^h\ 15^m}{12^h\ 45^m} = \frac{0.08}{0.26}\ .$$

That is,

| | |
|---|---|
| Radius of orbit - - - - - | 1.00 |
| Radius of *comes* (1) - - - - | 0.26 |
| Radius of *comes* (2) - - - - | 0.08 |

The relation between the light intensity of the two stars is readily determined by a simple consideration of the magnitude at maximum and the magnitude at minimum.

At a maximum *RR Puppis* is 2.7 times brighter than it is at a minimum. It is evident that at a maximum we have the combined light of the two stars.

That is, $L_1 + L_2 = 2.7$ .

At a minimum either *comes* (2), the smaller star, passes *behind comes* (1), and is thus eclipsed by it for 8½ hours, or it transits across it during the same period.

In the latter case we would have

$$L_1 + L_2 = 2.7$$
$$\frac{8}{9} L_1 + L_2 = 1.0 \ ,$$

an inadmissible relation, as in that case $L_2$ would have a negative value.

In the former case, however, the equation stands

$$L_1 + L_2 = 2.7$$
$$L_1 \qquad = 1.0 \ ,$$

and consequently

$$L_1 = 1.0$$
$$L_2 = 1.7 \ .$$

That is, the smaller star is nearly twice as bright as its larger companion; or, surface for surface, fifteen times more luminous.

As already stated, a similar disparity between size and brightness holds good in the case of the *Algol* variable *S Velorum*.

The relative proximity of two stars so unequal in brightness is indeed remarkable. It is by no means singular, however. And that it is not singular makes the inequality in size and brightness of such systems one of the most interesting problems in stellar physics. Interesting, not only because of its relation to many other allied questions, but because the problem leads to the very threshold of the origin of binary stars.

The solution of the problem will be partially effected when we are able to determine the relative densities of the two component stars. The data at our disposal enables us to say that the density of *RR Puppis* cannot be greater than 0.16, the Sun's density being unity. This result is in complete accordance with what has been ascertained of other close binaries.

Gathering together what we have been able to deduce readily from an examination of the light curve of this star, we find that the system consists of two stars, one three times the diameter of

the other. The smaller star, as in the case of *S Velorum*, is nearly twice as bright as the larger star. The distance between the circumferences of the two stars is about two thirds of the radius of the orbit.

The density of the system cannot be greater than one sixth that of the Sun. It is impossible to say what the relative density of the two components is. It is also not possible from the measures now available to determine the exact form of the orbit or

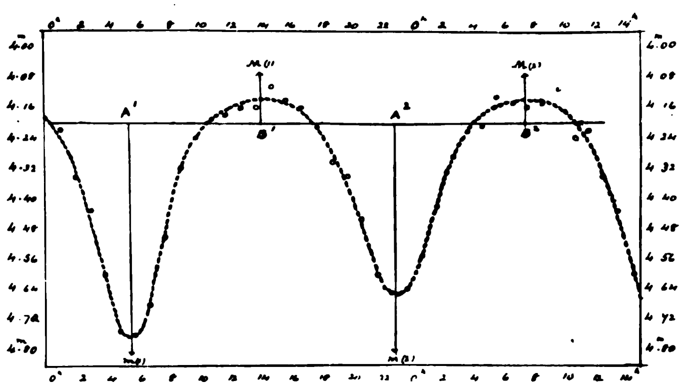


FIG. 3.

its inclination to the plane of sight; but a series of observations are being carried out with the hope of arriving at the value of both these elements.

*V PUPPIS.* (*Ch.* 2852.)

R. A. $7^h\ 55^m\ 22^s$ (1900)

Dec. $-48^\circ\ 58'.4$

The mean period of this star as determined from a comparison of all available observations is

$1^d\ 10^h\ 54^m\ 26^s.7.$

With this period the observations made at Lovedale during 1899 and the earlier months of 1900 were grouped into thirty-five hourly sets. The mean magnitudes derived from these groups are given in Table I, column 3. If these magnitudes be plotted down and the mean light curve be drawn we have the characteristic form of curve indicated in Fig. 3.

The salient features of this curve, its double and unequal minima, its double and equal maxima, its symmetry, and its close resemblance to the light curve of *U Pegasi* will at once be evident.

The following table gives the observed magnitudes, grouped in thirty-five hourly sets; the magnitude indicated by the mean light curve; the difference between the two, and the number of observations in each set:

TABLE I.

| No. | Date (G. M. T.) | Obs. mag. | Mean mag. | O-M | No. of obs'ns |
|---|---|---|---|---|---|
| 1 | 1900, Jan. $1^d$ $0^h$ $27^m$ | $4^m.22$ | $4^m.23$ | $-0^m.01$ | 9 |
| 2 | 1 1 26 | 4.33 | 4.33 | +0.00 | 12 |
| 3 | 1 2 30 | 4.43 | 4.44 | −0.01 | 13 |
| 4 | 1 3 31 | 4.60 | 4.58 | +0.02 | 21 |
| 5 | 1 4 32 | 4.75 | 4.76 | −0.01 | 13 |
| 6 | 1 5 25 | 4.76 | 4.76 | +0.00 | 19 |
| 7 | 1 6 30 | 4.68 | 4.60 | +0.08 | 10 |
| 8 | 1 7 26 | 4.50 | 4.46 | +0.04 | 10 |
| 9 | 1 8 30 | 4.32 | 4.34 | −0.02 | 12 |
| 10 | 1 9 32 | 4.24 | 4.25 | −0.01 | 8 |
| 11 | 1 10 33 | 4.19 | 4.19 | +0.00 | 9 |
| 12 | 1 11 28 | 4.18 | 4.16 | +0.02 | 13 |
| 13 | 1 12 39 | 4.16 | 4.15 | +0.01 | 11 |
| 14 | 1 13 32 | 4.16 | 4.14 | +0.02 | 9 |
| 15 | 1 14 34 | 4.10 | 4.14 | −0.04 | 11 |
| 16 | 1 15 26 | 4.14 | 4.15 | −0.01 | 10 |
| 17 | 1 16 34 | 4.16 | 4.16 | +0.00 | 8 |
| 18 | 1 17 28 | 4.21 | 4.20 | +0.01 | 8 |
| 19 | 1 18 25 | 4.30 | 4.25 | +0.05 | 7 |
| 20 | 1 19 34 | 4.34 | 4.33 | +0.01 | 8 |
| 21 | 1 20 28 | 4.45 | 4.43 | +0.02 | 10 |
| 22 | 1 21 30 | 4.60 | 4.57 | +0.03 | 9 |
| 23 | 1 22 27 | 4.65 | 4.65 | +0.00 | 13 |
| 24 | 1 23 35 | 4.64 | 4.62 | +0.02 | 13 |
| 25 | 2 0 28 | 4.51 | 4.52 | −0.01 | 15 |
| 26 | 2 1 26 | 4.42 | 4.39 | +0.03 | 11 |
| 27 | 2 2 20 | 4.29 | 4.29 | +0.00 | 7 |
| 28 | 2 3 24 | 4.22 | 4.23 | −0.01 | 17 |
| 29 | 2 4 30 | 4.21 | 4.18 | +0.03 | 7 |
| 30 | 2 5 32 | 4.13 | 4.15 | −0.02 | 10 |
| 31 | 2 6 30 | 4.15 | 4.14 | +0.01 | 9 |
| 32 | 2 7 33 | 4.16 | 4.14 | +0.02 | 7 |
| 33 | 2 8 23 | 4.15 | 4.15 | +0.00 | 8 |
| 34 | 2 9 29 | 4.11 | 4.16 | −0.05 | 9 |
| 35 | 2 10 28 | 4.24 | 4.18 | +0.06 | 5 |

The average departure of any set from the mean light curve is only —$0^m.019$, sufficient testimony to the accordance between the observed and mean magnitudes.

In seeking to trace the light variations of this star to the conditions of orbital movement which produce them, we are met at the outset by a grave difficulty. In 1895 Professor Pickering discovered that *V Puppis* is a binary star, the announcement being made in the *Harvard College Observatory Circular* No. 14 as follows:

An examination of the Draper Memorial photographs taken at the Arequipa station shows that the star *Lacaille* 3105, *Argentine General Catalogue* 10534, is a spectroscopic binary.

The lines in its spectrum were first noticed to be double, and its binary character discovered by the writer (Professor Pickering) in February 1895.

Professor Bailey was notified, and accordingly secured additional photographs and confirmed the binary character of this star.

As in the case of $\mu'$ *Scorpii* one component is brighter than the other. A discussion of all the photographs of its spectrum here and at Arequipa gives the mean period $3^d\ 2^h\ 46^m$.

The time of inferior conjunction can be represented by the formula

$$J.\ D.\ 241277.16 + 3^d.115\,E\ .$$

At these times the lines are single; for the next thirty-seven hours the lines are double, the fainter component of each having a greater wave-length than the brighter component and being therefore toward the red end of the spectrum. The lines then again become single and after that for the remainder of the period are again double, the fainter component having shorter wave-lengths and being therefore toward the violet end of the spectrum.

Now the theory which relates light variation to orbital movement requires that the two entirely different lines of investigation, a discussion of the spectroscopic observations, and a consideration of the light changes, should yield precisely the same period, otherwise the relation between variation and revolution is neither intimate nor dependent.

The spectroscopic observations yield a period of $3^d\ 2^h\ 46^m$ and the light observations a period of $1^d\ 10^h\ 54^m.4$. The only connection between these two results is that seven of the former periods are performed in $21^d\ 19^h\ 20^m$ and fifteen of the latter in $21^d\ 19^h\ 36^m$. It is evident therefore that the period obtained from an examination of the variation of the star's light will not satisfy the spectroscopic observations. And if we take the period obtained from the spectroscopic observations and reduce the

magnitude estimates by it to a mean period, the result represents no possible light curve.

I am most reluctant, therefore, to follow out an investigation which apparently has no unequivocal array of facts to justify it. But I am still more reluctant to let a consideration of the variation of this star lie over indefinitely, and, as the only reasonable line along which the investigation can be pursued is that the variation of *V Puppis* is due to orbital motion, and that the period of orbital motion must be, from the nature of things, the period in which a complete cycle of light-changes is performed, this assumption is taken as the starting point of the investigation.

From an examination of the light curve, Fig. 3, we obtain the following phases:

| | | | | | |
|---|---|---|---|---|---|
| First minimum $(m_1)$ | 1900 January | $1^{d}$ | $5^{h}$ | $5^{m}$ | (G. M. T.) |
| First maximum $(M_1)$ | | 1 | 13 | 50 | |
| Second minimum $(m_2)$ | | 1 | 22 | 40 | |
| Second maximum $(M_2)$ | 1900 January | 2 | 7 | 15 | |

From these dates we have

| | | | | |
|---|---|---|---|---|
| (1) | $M_1 - m_1 =$ | $8^{h}$ | $45^{m}$ | Diff. from mean $+ 1^{m}$ |
| | $m_2 - M_1$ | 8 | 50 | $+ 6^{m}$ |
| | $M_2 - m_2$ | 8 | 35 | $- 9^{m}$ |
| | $m_1 - M_2$ | 8 | 45 | $+ 1^{m}$ |
| | Mean | 8 | 44 | |

These four approximately equal periods indicate that *V Puppis* passes through each of the four quadrants of its orbit in equal times; that is, the eccentricity must be very small, probably zero.

The excess or defect of the four values given in (1), over or from the mean value $8^{h}$ $44^{m}$ yield data for a determination both of the eccentricity of the orbit and the position of the line of apsides. The four residuals, however, are well within the margin of probable error and thus it is useless to base a determination of the eccentricity of the system upon values which may have no objective reality.

A graphical indication of the form of the orbit is concurrent with the numerical determination.

Let an abscissa $A^1\ B^1\ A^2\ B^2$ be drawn through the light curve, the direction of the abscissa being along any given magnitude. Join the two minima points, $m_1$ and $m_2$, and the two maxima points, $M_1$ and $M_2$, with the points on the abscissa midway between the intersections of the abscissa and the light curve. If all the angles at $A^1$, $B^1$, $A^2$, $B^2$, are right angles, the orbit is circular. If not, then the amount of departure from a right angle is a function both of the eccentricity and apsidal angle. Of course this graphical consideration is only the previous numerical investigation expressed otherwise.

It is evident from an examination of the direction of the ordinates at $A^1$, $B^1$, $A^2$, and $B^2$, that the orbit of *V Puppis* must be practically circular. Indeed, the nearness of the two component stars, the investigation of which follows later, makes an elliptical orbit an impossibility.

The magnitudes of *V Puppis* at its four principal phases are:

| | |
|---|---|
| $M_1 = 4^{\mathrm{m}}.14$ | light value $= 1.00$ |
| $m_1 = 4.78$ | $= 0.56$ |
| $M_2 = 4.14$ | $= 1.00$ |
| $m_2 = 4.66$ | $= 0.62$ |

At $M_1$ and $M_2$ we have the combined light of both components. At $m_1$ we have the whole of the light of one component, which we may call $V_2$, and a portion, probably, of the light of its companion $V_1$.

At $m_2$, $V_1$ is now unobscured and its neighbor $V_2$ is either wholly or partially eclipsed.

Let $L_1 =$ light of $V_1$,

$L_2 =$ light of $V_2$,

$m =$ portion of $V_2$ obscured by $V_1$, when the stars are at $m_2$,

$k =$ ratio of surface of $V_1$ to surface of $V_2$,

$\frac{m}{k} =$ portion of $V_1$ eclipsed by $V_2$ when the stars are at $m_1$.

Then we have the following equations:

$$\begin{aligned} L_1 + L_2 \qquad &= 1.00 \\ L_1 + L_2 - mL_2 &= 0.62 \\ L_1 + L_2 - \frac{m}{k} L_1 &= 0.56\ , \end{aligned} \qquad (2)$$

which yield the relation:

$$\frac{m}{k} - \frac{0.38}{k} - 0.44 = 0 \,. \tag{3}$$

Solutions of this indeterminate equation are conditioned by the fact that $\frac{m}{k}$ and $m$ must both be positive and not greater than unity.

Giving therefore to $m$ such values as will satisfy these conditions we may arrange the dependent values

$$k\,,\ \sqrt{k}\,,\ \frac{m}{k}\,,\ L_1 \text{ and } L_2$$

in the following table:

TABLE II.

| $m$ | $k$ | $\sqrt{k}$ | $\frac{m}{k}$ | $L_1$ | $L_2$ |
|---|---|---|---|---|---|
| 0.70 | 0.73 | 0.85 | 0.96 | 0.46 | 0.54 |
| 0.80 | 0.95 | 0.98 | 0.84 | 0.53 | 0.47 |
| 0.90 | 1.18 | 1.09 | 0.76 | 0.58 | 0.42 |
| 1.00 | 1.41 | 1.19 | 0.71 | 0.62 | 0.38 |

These values indicate the extreme limits of inequality, both in size and brightness, between the two components $V_1$ and $V_2$.

The third column, $\sqrt{k}$, gives the possible values for the ratio of the two diameters. The lowest ratio is 0.85 and the highest, 1.19. That is, the diameter of $V_1$ is not a fifth less, and cannot be a fifth greater, than the diameter of $V_2$. We may reasonably infer, therefore, that the two stars which make up *V Puppis* are not markedly unequal in size.

The relative brightness of $V_1$ and $V_2$ is also, within certain limits, indeterminate, but these limits are narrow. Under no condition can $V_2$ be more than one fifth as bright again as $V_1$, and it cannot be less than two thirds of the brightness of $V_1$. Whatever possible values we assign to $m$ in equation (3), it appears that surface for surface $V_1$ is always slightly brighter than $V_2$.

From the data furnished in Table II we can readily determine the greatest possible inclination, or rather the limits of the inclination. The maximum value of this element is obtained if we consider the two stars to revolve in contact. With this

assumption the greatest possible inclination is 8°; under any circumstance the inclination cannot be less than 5°.

The important bearing of this last conclusion upon spectroscopic observations will be evident when we consider that these yield no data for a determination of the inclination.

An independent solution, therefore, is necessary in order to reduce the spectroscopic measures in the line of sight to movement in the real orbit. With regard to the system *V Puppis* the narrow range of possible inclination enables this reduction to be readily made; indeed, there will be no need for any reduction, as the reducing factor, cos $i$, can in no case be less than 0.99.

We come now to the relative size of the component stars as compared with the orbit they move in.

In the case of two stars nearly spherical in form, and revolving at some distance apart, the beginning of constant phase will usually be distinctly marked. When the two stars are near enough for their mutual attractions to produce considerable departure from the spherical form, both stars will be more or less pear-shaped, and the exact time when transit begins or ends will be difficult to determine.

Up to this point we have assumed a spherical form for both $V_1$ and $V_2$.

If, however, the stars revolve in close contiguity there must be distortion to such an extent as to modify slightly the form of the light curve. But whatever be the amount of distortion, unless actual contact takes place, there will be a stationary period.

On the other hand, if the stars are near enough for their mutual attractions to form a nexus between them, then there will be no stationary period, but the light curve at maximum will be rounded, the sharpness of the curve depending on the oblateness of the stars.

A simple examination of the light curve of *V Puppis* indicates that there is no stationary period at either maximum, and accordingly we must infer that the two component stars revolve around one another *in actual contact.*

In this case there must be considerable distortion in the form of the two stars, especially at the point where the two bodies meet.

It is not possible, although an attempt has been made, to determine the amount of this distortion; the conditions of the problem are too complex, the nature of the action of the forces to be considered too indefinite, and the data at our disposal too meager to enable us to come to a satisfactory conclusion.

We can best appreciate the nature of the difficulty by considering how near we come to a complete explanation of the variation of *V Puppis* by supposing that it is caused by the revolution of two stars, equal in size, but slightly unequal in brightness.

The orbit of the system is circular, slightly inclined to the plane of sight; and the two components stars revolve round one another in contact, yet preserve their spherical form.

Equations (2) and (3) yield the following numerical values:

by assumption, $k$ = unity,

therefore

$$\left.\begin{aligned} m &= 0.82 \\ \frac{m}{k} &= 0.82 \\ L_1 &= 0.54 \\ L_2 &= 0.46 \end{aligned}\right. \tag{4}$$

and,

Radius of orbit $= 1.00$
Radius of $V_1$ $= 0.50$
Radius of $V_2$ $= 0.50$
Inclination of orbit $= 8^\circ$
Period of *V Puppis* $= 1^d\ 10^h\ 54^m\ 27^s$
Epoch of prin. min. $=$ 1900 January $1^d\ 5^h\ 5^m$.

With these values we can compute what the variation of *V Puppis* would be.

In the following table (III) is given for comparison the observed magnitudes as set down in Table I, and the theoretical magnitudes computed from the foregoing elements.

TABLE III.

| No. | Date | Obs. mags. | Theoretical mags. | O.—T. | No. of obs'ns |
|---|---|---|---|---|---|
| 1 | 1900, Jan. 1$^d$ 0$^h$ 27$^m$ | 4$^m$.22 | 4$^m$.22 | +0$^m$.00 | 9 |
| 2 | 1 1 26 | 4.33 | 4.32 | +0.01 | 12 |
| 3 | 1 2 30 | 4.43 | 4.43 | +0.00 | 13 |
| 4 | 1 3 31 | 4.60 | 4.58 | +0.02 | 21 |
| 5 | 1 4 32 | 4.75 | 4.75 | +0.00 | 13 |
| 6 | 1 5 25 | 4.76 | 4.76 | +0.00 | 19 |
| 7 | 1 6 30 | 4.68 | 4.60 | +0.08 | 10 |
| 8 | 1 7 36 | 4.50 | 4.46 | +0.04 | 10 |
| 9 | 1 8 30 | 4.32 | 4.34 | −0.02 | 12 |
| 10 | 1 9 32 | 4.24 | 4.25 | −0.01 | 8 |
| 11 | 1 10 33 | 4.19 | 4.19 | +0.00 | 9 |
| 12 | 1 11 28 | 4.18 | 4.16 | +0.02 | 13 |
| 13 | 1 12 39 | 4.16 | 4.15 | +0.01 | 11 |
| 14 | 1 13 32 | 4.16 | 4.14 | +0.02 | 9 |
| 15 | 1 14 34 | 4.10 | 4.14 | −0.04 | 11 |
| 16 | 1 15 26 | 4.14 | 4.15 | −0.01 | 10 |
| 17 | 1 16 34 | 4.16 | 4.16 | +0.00 | 8 |
| 18 | 1 17 28 | 4.21 | 4.20 | +0.01 | 8 |
| 19 | 1 18 25 | 4.30 | 4.25 | +0.05 | 7 |
| 20 | 1 19 34 | 4.34 | 4.33 | +0.01 | 8 |
| 21 | 1 20 28 | 4.45 | 4.43 | +0.02 | 10 |
| 22 | 1 21 30 | 4.60 | 4.57 | +0.03 | 9 |
| 23 | 1 22 27 | 4.65 | 4.66 | −0.01 | 13 |
| 24 | 1 23 35 | 4.64 | 4.58 | +0.06 | 13 |
| 25 | 2 0 28 | 4.51 | 4.45 | +0.06 | 15 |
| 26 | 2 1 26 | 4.42 | 4.34 | +0.08 | 11 |
| 27 | 2 2 20 | 4.29 | 4.25 | +0.04 | 7 |
| 28 | 2 3 24 | 4.22 | 4.20 | +0.02 | 17 |
| 29 | 2 4 36 | 4.21 | 4.16 | +0.05 | 7 |
| 30 | 2 5 32 | 4.13 | 4.15 | −0.02 | 10 |
| 31 | 2 6 30 | 4.15 | 4.14 | +0.01 | 9 |
| 32 | 2 7 33 | 4.16 | 4.14 | +0.02 | 7 |
| 33 | 2 8 23 | 4.15 | 4.15 | +0.00 | 8 |
| 34 | 2 9 29 | 4.11 | 4.16 | −0.05 | 9 |
| 35 | 2 10 28 | 4.24 | 4.18 | +0.06 | 5 |

To illustrate graphically the agreement between the observed and computed magnitudes Fig. 4 has been drawn representing the actual observed light curve and the theoretical light curve, computed from the orbital elements given in (4).

The agreement between the two curves is very remarkable; indeed there is only one portion of the light curve where the agreement is not complete.

Now it may be urged that this very agreement militates against the correctness of the results obtained, for the theoretical light curve is based upon the assumption that both stars are

spherical, and this assumption is, of course, incorrect. If we consider $V_1$ and $V_2$ to be distorted to the amount indicated by the dotted lines in Fig. 5, then the theoretical curve would not be altered at any point as much as $0^m.02$.

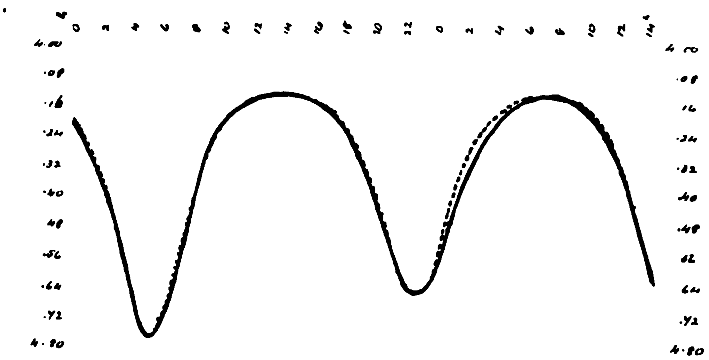


FIG. 4.

It is evident, therefore, that the solution we have obtained does not preclude the possibility of the shape of the two stars being considerably disturbed by their mutual attractions.

Indeed, to introduce this consideration would bring the two curves, the observed and theoretical, into closer agreement. As we have seen, the average departure of a single set of observations from the mean value is $0^m.019$; but the average departure from the theoretical value is $0^m.025$. That is, the theoretical curve can be made to conform still more closely with the observed places.

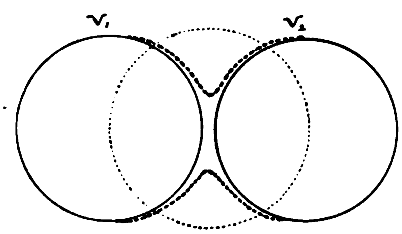



FIG. 5.

In November 1900 a series of observations of *V Puppis* has been begun with the purpose of more thoroughly dealing with the orbit of this star.

Each observation is the mean of four taken with a prism which rotates the field. One observation is taken in each quadrant,

and the mean error of each combined observation will be less than $0^m.03$.

It only remains to consider the density of the system. Applying to the results already obtained any of the formulæ for the determination of the density of close binary stars, we arrive at the result that the density of *V Puppis* cannot be greater than 0.02 of the Sun's density.

The results of the present investigation may be thus summarized:

| | | | | | |
|---|---|---|---|---|---|
| Period of *V Puppis* - - - - | $1^d$ | $10^h$ | $54^m$ | $26^s.7$ | |
| Principal minimum - - 1900 Jan. | 1 | 5 | 5 | | (G. M. T.) |
| First maximum - - - - - - | 1 | 13 | 50 | | |
| Second minimum - - - - - | 1 | 22 | 40 | | |
| Second maximum - - - - - | 2 | 7 | 15 | | |
| Magnitude at max. - - - - | | $4^m.14$ | | | |
| Magnitude at prin. min. - - - | | 4.78 | | | |
| Magnitude at sec. min. - - - | | 4.66 | | | |
| Diameter either $V_1$ or $V_2$ - - - | | 1.00 | | | |
| (Radius of orbit being unity) | | | | | |
| Eccentricity of orbit - - - | | 0.00 | | | |
| Inclination of orbit - - - - | | 8° | | | |
| Density of system - - - - | | 0.02 | | | |

LOVEDALE,
December 1900.

# ON DOPPLER'S PRINCIPLE.

By W. MICHELSON.

IF $n$ be the number of vibrations per second produced by the source, $a$, the velocity of the source along a straight line uniting it to the observer, $N$, the number of vibrations perceived by the observer, and $b$, the rate at which he moves along the same line, then, according to Doppler's assertion,

$$N = n\frac{v \pm a}{v \pm b}\,, \tag{1}$$

where $v$ is the velocity of propagation of the wave-motion in the given medium.

Apart from the assumptions by which it is supported, Doppler's principle has by itself a purely kinematic meaning, and therefore cannot be called in question.

But some of the assumptions on which its application is based are in great measure arbitrary, and can hardly be proved except *a posteriori*, by experimental verification. I shall mention but two of these assumptions: (1) that the period of vibration of the source is not influenced by its motion along the line of sight; (2) that the medium carrying on the waves is at rest as a whole, and that its properties are not changing.

When we have to deal with sound, the source of the waves as well as the medium in all its parts are within our reach. Therefore it is generally easy to test the above mentioned assumptions.

It is quite different when we are observing the displacement of lines in the spectra of celestial bodies. In this case we can neither verify immediately nor prove indirectly either of the assumptions referred to. It is very likely that these displacements are actually due to those motions by which they are usually explained in astrophysics, but, from a strictly logical point of view, it cannot be asserted as yet that no other explanation is possible.

Nor do I mean to remove from Doppler's principle its hypothetical part, which probably belongs to it by the very nature of the question.

All I want is to give it a somewhat different expression in order to comprise under one law also those cases where a change of the frequency is caused not only by the motion of the source or that of the observer, but also by a rapid alteration in the density of the medium crossed by the ray.

It is hardly possible to produce or observe, on a large scale, such rapid changes of density on the Earth, but they not only are conceivable, but very probably take place in the Sun's atmosphere.

Let us return to Doppler's formula (1). As the velocity $a$ and $b$ are generally small compared to the velocity of light, the ratios $\frac{a}{v}$ and $\frac{b}{v}$ are small fractions, whose squares and higher powers can be neglected. Hence the equation (1) can be represented as

$$N = n\frac{1+\frac{a}{v}}{1+\frac{b}{v}} = n\left(1 + \frac{a-b}{v} + \ldots\right), \tag{2}$$

reckoning $a$ and $b$ as positive in the direction from the source of vibrations to the observer.

If $l$ be the variable distance of the source from the observer it is evident that

$$b - a = \frac{dl}{dt}$$

is the derivative of the distance to time.

Doppler's formula can then be written as follows:

$$N = n\left(1 - \frac{1}{v}\cdot\frac{dl}{dt}\right). \tag{3}$$

This equation holds also for rays which do not travel in a straight line from the source to the observer, but undergo any number of reflections or refractions on their way. In this case, however, the distance $l$ should be replaced by the optical length of the path of the ray $L$ from the source to the observer.

If the geometrical length of the single parts of the ray's path through different consecutive media be $l_1, l_2, l_3, \ldots l_n$, and the corresponding refractive indices of the media referred to one of them (ether) be $\mu_1, \mu_2, \mu_3, \ldots \mu_n$, it is obvious that

$$L = l_1\mu_1 + l_2\mu_2 + l_3\mu_3 + \ldots l_n\mu_n = \sum l\mu ,$$

and Doppler's principle will be expressed by the equation

$$N = n\left[1 - \frac{1}{v}\sum\left(l\frac{d\mu}{dt} + \mu\frac{dl}{dt}\right)\right] , \tag{4}$$

where $v$ represents the velocity of light in the medium to which the indices of refraction are referred.

In this equation the additional members of the type

$$\frac{1}{v}\sum \mu\frac{dl}{dt}$$

represent the change of frequency which alone is usually considered in Doppler's principle. This involves also the cases where the length of the ray's path is altered by a rapid displacement of a mirror reflecting it. Mr. W. Wien[1] has made a successful application of a similar change in the period of vibrations produced by reflection from a moving mirror, to the thermodynamics of radiant energy.

These additional members may represent as well the cases where several media of unvarying properties are moving in the path of the rays so as to change rapidly the distance crossed in each of them.

Let us examine two special examples illustrating the case. Suppose that the monochromatic light issuing from the source $S$ passes first through the ether ($\mu_1 = 1$) over the distance $l_1$, then in a liquid with the index $\mu_2$ over the distance $l_2$ and then reaches the observer.

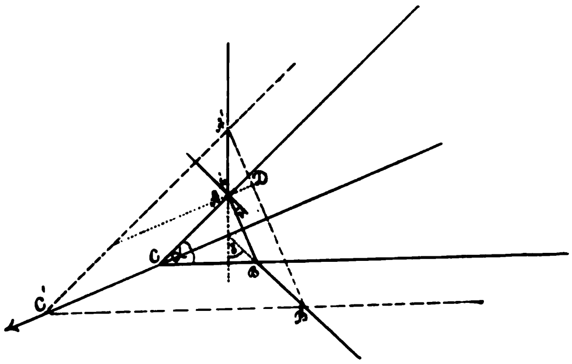
Let $l_1$ and $l_2$ change in such a manner that the limiting surface of the ether and the liquid remaining parallel to itself should be displaced rapidly (at the rate $c$) in the direction of the source.

[1] WILLY WIEN, *Sitz-ber. d. Berl. Acad.*, **6**, 1893, and *Wiedemann's Annalen*, **52**, p. 156, 1894.

In other words, the thickness of the liquid $l_2$ increases at the rate $c$. Then

$$\frac{dl_1}{dt} = -c \; ; \; \frac{dl_2}{dt} = c \; ; \; N = n\left[1 - \frac{c}{v}(\mu_2 - 1)\right] . \tag{5}$$

If $c$ is commensurable with the velocity of light and $\mu_2$ differs considerably from 1, the displacement of the line can be of the same order as in the case of a rapid motion of the source or the observer. As a second example let us examine the case of a prism moving rapidly across the path of the ray in such a way that the length of the path inside the prism continually increases.



Suppose that the ray passes through the prism at the angle of minimum deviation, which we shall denote by $\delta$. The prism is moving in the direction of $CC_1$ at the rate $c$, and its side faces are supposed to be unlimited in their length.

| | | |
|---|---|---|
| Then let | the refracting angle of the prism be | $SCB = \alpha$ |
| | the angle of incidence of the ray, | $i$ |
| | the angle of refraction, | $r$ |
| | the refractive index of the prism, | $\mu$ |
| | the length of the ray's path inside the prism, | $l_2$ |
| | the length of the ray's path outside the prism, | $l_1$ ; |

then, according to the previous notation,

$$N = n\left[1 - \frac{1}{v}\left(\frac{dl_1}{dt} + \mu\frac{dl_2}{dt}\right)\right] . \tag{6}$$

If the diagram shows the displacement of the prism in unit of time, then evidently

$$\frac{dl_1}{dt} = -2AA_1 = -2c\frac{\sin\frac{\alpha}{2}}{\cos i} \; ;$$

$$\frac{dl_2}{dt} = +2A'D = 2c\frac{\sin\frac{\alpha}{2}\cos\frac{\delta}{2}}{\cos i} .$$

Putting these formulæ into equation (6) and remembering that

$$r = \frac{a}{2}\;;\quad i = \frac{a+\delta}{2}\;;\quad \mu = \frac{\sin i}{\sin r}\;;$$

we get

$$N = n\left[1 - \frac{2c}{v}\sin\frac{\delta}{2}\right]. \tag{7}$$

The displacement of the spectral line depends only upon the movement of the prism and the angle of deviation of ray $\delta$. If this angle equals 60°, the displacement is equal to that produced by the receding of the source of light at the rate $c$. If the prism moved in the opposite direction the displacement of the lines should take place in the direction of the violet end of the spectrum.

In spite of the artificiality of these suppositions, some more or less distant analogies must occur in nature. Immediately before a total eclipse of the Moon the Fraunhofer lines in the Moon's spectrum must be displaced toward the red in consequence of the rapid drift of the Earth's atmosphere across the path of the rays. A reverse displacement ought to be observed at the end of totality. A far more considerable displacement must occur immediately before some star is covered by Jupiter, in whose atmosphere the optical path of the rays is considerably lengthened. But one must confess that it will hardly be possible to witness these phenomena by means of observation on account of the strong light of Jupiter himself, as well as on account of the refraction of the rays passing through his atmosphere.

It is not so with the Sun. There phenomena of this kind not only are continually going on, but they have probably been the object of manifold observation, although, to my knowledge, no attempts have as yet been made to explain them in the direction I have indicated. If a velocity of 500 kilometers per second is attributed to the luminous gases of the photosphere and the chromosphere in order to account for the displacement and the distortion of the spectral lines, there is no reason to suppose that higher and non-luminous gases should be less movable. The thinness and sharpness of the Fraunhofer lines show that the gases of the

"reversing layer" are already in a considerably rarefied condition, and are therefore capable of a very swift motion. On the other hand, it can hardly be doubted at present that Schmidt's theory concerning the bearing of the refraction in the Sun's atmosphere is true, at least to a certain extent. Accordingly the rays of the Sun describe probably a very long curvilinear path in the atmosphere before reaching us. This especially concerns the rays issuing from the edges of the solar disk. In this case even comparatively small fluctuations of the solar atmosphere may bring layers of different density into the path of the ray emerging from a certain point of the photosphere or the chromosphere. As "the optical length" of the ray's path may change thereby very rapidly and irregularly, these motions of the non-luminous gases may at least in part account for those extremely rapid and irregular distortions of the spectral lines belonging to Sun-spots and protuberances, which have been observed by Sir N. Lockyer, Professor C. A. Young, and others.

The difficulty of explaining these rapid deviations of the spectral lines in the ordinary way lies less in the assumption of enormous velocities for luminous elements than in the necessity of admitting the existence of inconceivable forces and accelerations which are hardly compatible with the rarefied condition of matter in the Sun's atmosphere. Whereas, according to the explanation I am proposing, a given displacement may be accounted for, in certain cases, also by much smaller velocities; here are acting not only the velocities in the direction of the ray, but also those perpendicular to it. More than this, constant (stationary) velocities may be the cause of most irregular and even opposite distortions of the lines in accordance with the changes of density which take place in the matter carried before the luminous point.

In the case of increasing density in the layers traversed by the ray, the lines are displaced towards the red end of the spectrum, whilst a decrease of density produces a shifting towards the violet end.

Until now it has apparently been considered as an incontrovertible statement that the displacement or distortion of single spectral lines indicates a motion precisely of that matter to which those lines belong. Whereas it becomes evident from the above statements that the motion of hydrogen or helium may have an influence on the displacement of the lines belonging to calcium, iron, etc. It is generally said that if the displacement of the lines depends upon any processes taking place in the path of the rays, and not in the source itself, it would affect not only some of the lines, but all the spectral lines equally. That is true if by "the source of light" one means the whole Sun with the whole of its atmosphere. But since it must be admitted that different elements of this atmosphere may be endowed with velocities of different magnitude and directions—in other words, that they are not mixed,—one must necessarily admit also that the optical paths of the rays issuing from different elements may be different and may be altered almost independently of one another. This may be sufficient to explain in certain cases how the lines belonging to one element may be displaced, while other lines show no disturbances at all.

A strictly scientific solution of the questions, considered from an elementary point of view in this brief account, is hardly possible at present, since it involves the difficult problems of the connection between the ether and ponderable matter.

Moscow,
February 22, 1901.

# DIFFRACTION BY LIGHT OF VARIABLE INTENSITY.

By H. M. Reese.

The existence of many spectroscopes in which the absorption of the prisms is considerable makes it of interest to know whether the diffraction pattern due to plane waves passing through an opening of any form is materially altered when the amplitude varies much from one edge of the aperture to the other. The resolving power obtained with such instruments as the Mills spectrograph, in which the light is quite strongly absorbed, would indicate that any such alteration as there may be is not great; but at the same time an analytical investigation seems worth while.

The method employed is merely an extension of that given in the ordinary text-books, such as Preston's *Theory of Light*, for the simpler case of a beam of uniform intensity.

It has been pointed out to the writer by Professor Ames, of Johns Hopkins University, who has kindly criticised this work, that the principal difficulty lies in proving that the waves remain plane after passing through the prisms, or indeed that a wave-front in the ordinary sense of the word exists at all. Consequently, it was thought best to introduce this as a pure assumption. The excellent definition obtained when very dense prisms are used in an ordinary spectroscope seems to show that in such cases at least the ordinary laws of refraction are followed quite closely. In cases such as that of a cyanine prism traversed by light of wave-length nearly that of the absorption-band, the state of affairs may be somewhat different; but it will be shown that the broad, ill-defined image of the slit obtained under these conditions can be explained purely on the basis of diffraction from a plane wave which has a very great range of amplitude, without supposing the wave-surface to have been disturbed at all.

We shall also assume that the absorption by the prisms is proportional to the logarithm of the thickness traversed by the

light. The amplitude of the emergent light at any point will then be proportional to $e^{-nl}$, where $l$ is the distance of the point from that edge of the beam which passes through the vertex of the prisms. This proportionality will not be changed by reflections from the faces of the prisms, since each reflection merely reduces the amplitude by a constant factor over the whole width of the beam.

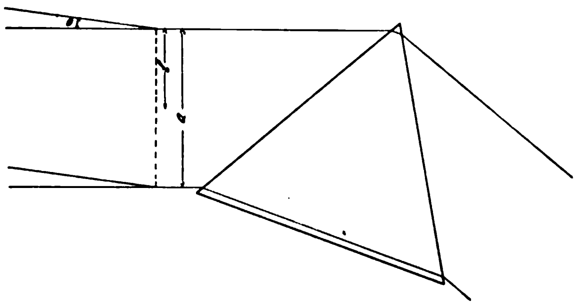


FIG. 1.

Let us suppose the light to be monochromatic, and the aperture to be rectangular, of width $a$. Fig 2. is the "Cornu spiral" which represents the combined effect, at an angle $\theta$ with the wave-normal, of all the elements of the wave-front. Let $ds$ be that element of the spiral which corresponds to an element of wave-front of width $dl$ and of length equal to the dimensions of the aperture in a perpendicular direction. Then $ds = Ae^{-nl}dl$. If $\phi$ is the phase of this element, $\phi = \frac{2\pi}{\lambda} l \sin\theta$. If $dx$ and $dy$ are the components of $ds$, we have

$$dx = \cos\phi\, ds = Ae^{-nl}\cos\left(\frac{2\pi}{\lambda} l \sin\theta\right) dl\,,$$

$$dy = \sin\phi\, ds = Ae^{-nl}\sin\left(\frac{2\pi}{\lambda} l \sin\theta\right) dl\,.$$

The resulting intensity at the angle $\theta$ is represented by $\overline{OM}^2 = x^2 + y^2$.

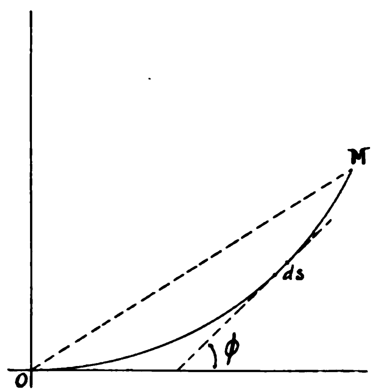



FIG. 2.

To integrate $dx$ and $dy$ let $\frac{2\pi}{\lambda}\sin\theta = p$ for convenience. Then, integrating by parts, we get

$$x = A\int_0^a e^{-nl}\cos pl\, dl = \frac{A}{n^2+p^2}\left\{ e^{-na}(p\sin pa - n\cos pa) + n \right\}.$$

$$y = A\int_0^a e^{-nl}\sin pl\,dl = \frac{-A}{n^2+p^2}\left\{ e^{-na}(n\sin pa + p\cos pa) - p \right\}.$$

$$\begin{aligned} I = x^2 + y^2 = \frac{A^2}{(n^2+p^2)^2}\Big\{ & e^{-2na}(p^2\sin^2 pa + n^2\cos^2 pa - 2np\cos pa\sin pa) \\ & + e^{-2na}(n^2\sin^2 pa + p^2\cos^2 pa + 2np\cos pa\sin pa) \\ & + 2e^{-na}(np\sin pa - n^2\cos pa) \\ & - 2e^{-na}(np\sin pa + p^2\cos^2 pa) + n^2 + p^2 \Big\}. \end{aligned}$$

$$\begin{aligned} I &= \frac{A^2}{(n^2+p^2)^2}\Big\{ e^{-2na}(n^2+p^2)(\sin^2 pa + \cos^2 pa) - 2e^{-na}(n^2+p^2)\cos pa \\ &\qquad + n^2 + p^2 \Big\} \\ &= \frac{A^2}{n^2+p^2}(e^{-2na} - 2e^{-na}\cos pa + 1) \\ &= \frac{A^2\left\{ e^{-2na} - 2e^{-na}\cos\left(\frac{2\pi a}{\lambda}\sin\theta\right) + 1 \right\}}{n^2 + \frac{4\pi^2}{\lambda^2}\sin^2\theta}. \end{aligned}$$

If we put $n = 0$, this reduces to

$$I = \frac{2A^2\left(1 - \cos\frac{2\pi a}{\lambda}\sin\theta\right)}{\left(\frac{2\pi}{\lambda}\sin\theta\right)^2} = \frac{A^2a^2\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2},$$

which is the same expression arrived at independently. (See Preston, first edition, p. 209.)

For convenience, let us put $\frac{2\pi}{\lambda}\sin\theta = p$, as before, and $e^{-na} = k$. Then we have

$$I = A^2\frac{1 + k^2 - 2k\cos ap}{n^2 + p^2}.$$

Since $\cos ap$ and $p^2$ are both even functions of $\theta$, $I$ is symmetrical about the center of the field. It seems at first sight rather strange that a beam of varying intensity should give a symmetrical diffraction pattern, but it can be shown that this will be the case no matter how the amplitude varies, so long we have plane waves to deal with. For the phase difference between the secondary waves coming from any two points in the wave-front is the same in amount for $-\theta$ as for $+\theta$, although opposite in sign, so that the total effect at the angle

$-\theta$ is the same in intensity as that at $+\theta$, but of opposite phase. Another aspect of the same thing is that the Cornu spirals for the two cases are symmetrical to one another with respect to the $x$-axis.

The expression for $I$ also shows that it is never zero. For in order that it may vanish we must have

$$1 + k^2 - 2k \cos ap = 0$$

or

$$\cos ap = \frac{1 + k^2}{2k} .$$

But $1 + k^2$ must be greater than $2k$ unless $k = 1$, which means that there is no absorption. Therefore $p$ cannot in general have such a value that $I$ can vanish. It will be shown later, however, that in ordinary cases $I$ does become very small for values of $\theta$ which are quite close to those which would make $I = 0$ if there were no absorption.

### MAXIMA AND MINIMA OF I.

In the first place, if the absorption of the prisms or the width of the beam is so great that $k = e^{-an}$ may be neglected (that is, the intensity of the plane wave at the base of the prism is practically zero), the formula reduces to

$$I = \frac{1}{n^2 + p^2} ,$$

which has no minima, but only a maximum at $p = 0$, the intensity becoming less and less toward the edges of the field. This case is perhaps of no practical interest in spectroscopy, but comes close to realization under conditions mentioned above, namely, when a prism showing anomalous dispersion is traversed by light close to its absorption-band. In this case we neglect $k$ not because of the great width of the beam, but because the coefficient of absorption, $n$, is very great. It is easy to see that under these circumstances the intensity would be quite low at the maximum and would diminish very slowly toward the edges of the field, giving the appearance of a spectral line of extraordinary width and nebulosity

The general condition for a maximum or minimum, $\frac{\delta I}{\delta p} = 0$, gives

$$ak(n^2 + p^2) \sin ap - p(1 + k^2 - 2k \cos ap) = 0 .$$

This equation is satisfied for $p = 0$. It is possible, although somewhat difficult, to determine analytically whether this gives a maximum or a minimum; but it is evidently a maximum from the physical consideration that at this angle all the secondary waves are in the same phase.

To find the secondary maxima and the minima, we might resort to plotting the curves

$$y = \frac{n^2 + x^2}{x} \text{ and } y = \frac{1 + k^2 - 2k \cos ax}{ak \sin ax}$$

and determine their intersections, provided we know the numerical values of $a$ and $n$.

It can be shown analytically, however, that minima exist at $p = \frac{2t\pi}{a} + e$ and maxima at $p = (2t + 1)\frac{\pi}{a} + e'$, where $t$ is a positive integer and $e$ and $e'$ are small quantities, the former positive, the latter negative. Of course symmetrical cases occur on the opposite side of the central fringe.

Let $p = s + e$, where $s = \frac{2t\pi}{a}$. Then $ap = 2\pi t + ae$. Therefore, if we let $\psi$ represent the function

$$(n^2 + p^2)\, ak \sin ap - p(1 + k^2 - 2k \cos ap)$$

we will have

$$\psi = (n^2 + s^2 + 2se + e^2)\, ak \sin ae - (s + e)(1 + k^2 - 2k \cos ae) .$$

Since we have supposed $e$ small we may write

$$\sin ae = ae , \ \cos ae = 1 - \frac{a^2e^2}{2}$$

and drop terms in $e^3$. Therefore, approximately

$$\psi = (n^2 + s^2 + 2se)\, a^2ke - (s + e)(1 + k^2 - 2k + a^2ke^2)$$
$$= a^2kse^2 + e\left[a^2k(n^2 + s^2) - (1 - k)^2\right] - s(1 - k)^2 .$$

In order that this vanish we must have

$$e = \frac{-P \pm \sqrt{P^2 + 4a^2ks^2(1 - k)^2}}{2a^2ks} ,$$

where $P = a^2k(n^2 + s^2) - (1 - k)^2$.

$k$ is always positive and less than unity, but is not very small unless the absorption is very great indeed. We may say, then, that in most cases $P^2$ is considerably larger than $4a^2ks^2\,(1-k)^2$, so that we can develop the radical into a power series.

$$\sqrt{P^2+4a^2ks^2(1-k)^2}=P+\frac{4a^2ks^2(1-k)^2}{2P}-\frac{[4a^2ks^2(1-k)^2]^2}{2\,.\,4P^3}$$
$$+\frac{1\,.\,3\,[4a^2ks^2(1-k)^2]^3}{2\,.\,4\,.\,6P^5}-\frac{1\,.\,3\,.\,5\,[4a^2ks^2(1-k)^2]^4}{2\,.\,4\,.\,6\,.\,8P^7}$$
$$+\frac{1\,.\,3\,.\,5\,.\,7\,[4a^2ks^2(1-k)^2]^5}{2\,.\,4\,.\,6\,.\,8\,.\,10P^9}-\,.\;.\;.\;.$$

Our hypothesis that $e$ is small precludes the use of the negative sign. Therefore

$$e=\frac{1}{2a^2ks}\left\{\frac{4a^2ks^2(1-k)^2}{2P}-\frac{[4a^2ks^2(1-k)^2]^2}{2\,.\,4\,.\,P^3}+\,.\;.\;.\;.\right\}$$
$$=\frac{s(1-k)^2}{P}\left\{1-\frac{a^2ks^2(1-k)^2}{P^2}+\frac{1\,.\,3\,[2a^2ks^2(1-k)^2]^2}{2\,.\,3\,.\,P^4}-\,.\;.\;.\;.\right\}$$
$$=\frac{2\pi t(1-k)^2}{aP}\left\{1-\frac{Q}{\underline{2|}}+\frac{1\,.\,3\,Q^2}{\underline{3|}}-\frac{1\,.\,3\,.\,5\,Q^3}{\underline{4|}}+\frac{1\,.\,3\,.\,5\,.\,7\,Q^4}{\underline{5|}}+\,.\;.\;.\;.\right\},$$

where

$$Q=\frac{2a^2ks^2(1-k)^2}{P^2}=\frac{8\pi^2t^2k(1-k)^2}{P^2}\,.$$

Whenever this series is convergent, and gives a value of $e$ so small that $a^3e^3$ may be neglected in computing the sine of $ae$, then $\frac{2\pi t}{a}+e$ gives a critical value of $p$. It will be shown later that it corresponds to a minimum.

Now, let $p=s'+e'$, where $s'=(2t+1)\frac{\pi}{a}$. Then $ap=(2t+1)\pi+ae'$, $\sin ap=-\sin ae'$, and $\cos ap=-\cos ae'$. Therefore

$$\psi=-(n^2+s'^2+2s'e')a^2ke'-(s'+e')(1+k^2+2k-a^2ke'^2)$$
$$=-a^2ks'e'^2-e'\left[a^2k(n^2+s'^2)+(1+k)^2\right]-s'(1+k)^2\,.$$

In order that this may vanish we must have

$$e=\frac{-P'\pm\sqrt{P'^2-4a^2ks'^2(1+k)^2}}{2a^2ks'}\,,$$

where $P'=a^2k(n^2+s'^2)+(1+k)^2$. For the Mills spectrograph

$P'^2$ is much greater than $4a^2ks'^2(1+k)^2$, therefore we can develop into a power series as in the former case; and we finally get

$$e' = -\frac{\pi(2t+1)(1+k)^2}{aP'}\left(1 + \frac{Q'}{\underline{2|}} + \frac{1\cdot 3\,Q'^2}{\underline{3|}} + \frac{1\cdot 3\cdot 5\,Q'^3}{\underline{4|}} + \dots\right),$$

where

$$Q' = \frac{2\pi^2 k(2t+1)^2(1+k)^2}{P'^2}.$$

It will be shown that $\frac{(2t+1)\pi}{a} + e'$ corresponds to a maximum.

The expression for $\frac{\delta^2 I}{\delta p^2}$ reduces to

$$\frac{A^2}{(n^2+p^2)^3}\Big[2a^2k\cos ap\,(n^2+p^2)^2 - (n^2+p^2)\big\{2(1+k^2-2k\cos ap) + 8apk\sin ap\big\} + 8p^2(1+k^2-2k\cos ap)\Big].$$

Since $\frac{A^2}{(n^2+p^2)^3}$ is always positive we are only concerned with the sign of the other factor. If $p = \frac{2t\pi}{a} + e$, this factor becomes approximately

$$2a^2k(n^2+p^2)^2 - 2(n^2+p^2)\big\{(1-k)^2 + 4a^2pke\big\} + 8p^2(1-k)^2,$$

which is always positive, since $k$ is positive and $p$ is large. Therefore we have a minimum in this case.

On the other hand, if $p = (2t+1)\frac{\pi}{a} + e'$, we have approximately

$$-2a^2k(n^2+p^2)^2 - 2(n^2+p^2)\big\{(1+k)^2 - 4a^2pke\big\} + 8p^2(1+k)^2,$$

which is negative, corresponding to a maximum.

It is seen that the effect of the absorption is to remove the minima farther from the center of the field, and so widen the central fringe. A short computation shows that in the particular case in hand this widening is very small. A rough experiment gave for $n$ the value 0.12, and the width of the beam, $a$, is 3.74 cm. This gives $k = .6384$. Putting $s = \frac{2\pi}{a}$, we get

$$P = 25.48$$
$$Q = 0.01015$$
$$\frac{2\pi(1-k)^2}{aP} = 0.00862.$$

This makes $e = 0.00858$. The increase in width of the central fringe is twice this amount, or one half of 1 per cent. of the whole width. A calculation of the intensity at the minimum shows that it is about one half of 1 per cent. of that at the principal maximum.

To determine the resolving-power of any combination of prisms we may plot the intensity-curve of its diffraction pattern

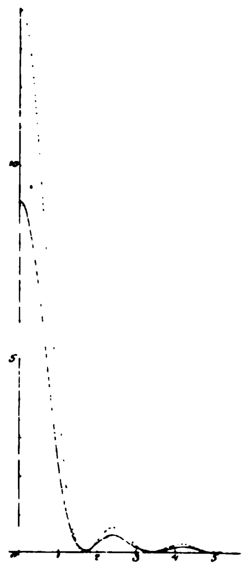


FIG. 3.

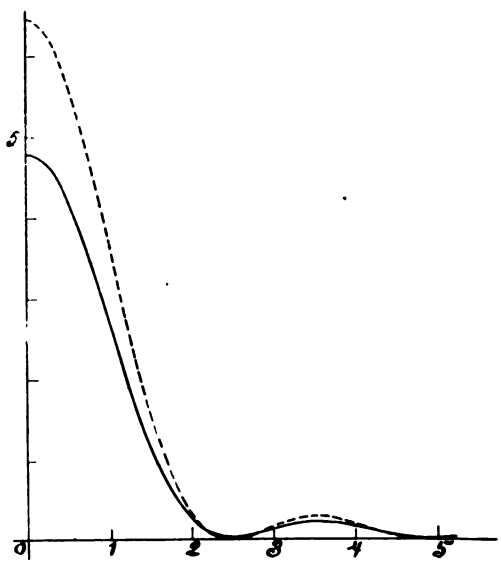


FIG. 4.

and superpose two such curves with the least lateral displacement that will give a combined effect of two maxima distinguishable from one another, as in the ordinary case.

The following considerations will show that the resolving-power of the Mills spectrograph is diminished by less than one half of 1 per cent. by the absorption of the prisms. Let us suppose the diffraction-patterns for $\lambda$ and $\lambda + \delta\lambda$ to be superposed, $\delta\lambda$ having such a value that the maximum of one falls on the minimum of the other, that is, so that the lateral displacement of the two curves is about one half of 1 per cent. greater

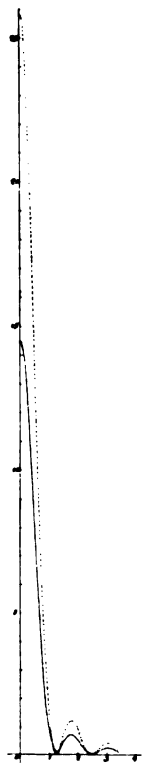


FIG. 5.

than for two resolvable lines in the case of no absorption. We then find that the combined intensity midway between the two principal maxima is $\frac{8097}{10000}$ that at either of the latter, instead of $\frac{8106}{10000}$, as in the corresponding case of no absorption. Therefore the two lines are more than resolved according to the usual definition of resolution.

It was not thought worth while to investigate the secondary maxima any further, but they are well shown in Figs. 3, 4, and 5, representing intensity curves under different conditions. Fig. 3 is for the case already considered where $n = 0.12$ and $a = 3.74$ cm (about 1½ inches), while Fig. 4 is for $n = 0.12$ and $a = 2.54$ cm (1 inch), and Fig. 5 is for $n = 0.12$ and $a = 5.08$ cm (2 inches). In each case the abscissa is $p$, the ordinate $I$. The dotted curves show the corresponding intensities when there is no absorption.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
February 1901.

# A GRAPHICAL STUDY OF REFRACTION AND DISPERSION.

By A. de Gramont.

The purpose of the following paper is to represent graphically the deviation and dispersion produced by a prism, and to study the properties of the curves thus obtained.

For any one wave-length, *angles of incidence* are used as ordinates, and *deviations* as abscissas. These coördinates have been determined both by experiment and by computation.

EXPERIMENTAL.

Of the several 60° flint prisms which were employed, one which we shall designate as "No. 3 W" had been ground and polished with great care by M. Werlein, so that the interference fringes which it gave by reflection were perfectly straight. This prism illustrates the close agreement between computation and observation.

The measures were made on a circle of 28 centimeters diameter, reading to minutes by means of verniers. The refracting edge of the prism coincided with the vertical axis of the circle. Refractive indices were measured by the minimum-deviation method. The wave-lengths employed were distributed throughout the entire visible spectrum, and were obtained from the following sources: sodium in the flame, hydrogen in Plücker-tube, aluminium, zinc, silver, tin, and lead in the condensed spark.

These results are summarized in the following table, and in the accompanying curve:

TABLE OF REFRACTIVE INDICES.

| Wave-lengths | Element | Prism No. 3 W | Prism No. 1 |
|---|---|---|---|
| 6563 | *H* | 1.6447 | 1.6272 |
| 5893 | *Na* | 1.6497 | 1.6322 |
| 5466 | *Ag* | 1.6543 | 1.6361 |
| 5209 | *Ag* | 1.6576 | 1.6395 |
| 4861 | *H* | 1.6630 | 1.6411 |
| 4811 | *Zn* | 1.6637 | 1.6461 |
| 4680 | *Zn* | 1.6663 | 1.6486 |
| 4525 | *Sn* | 1.6697 | 1.6515 |
| 4387 | *Pb* | 1.6733 | 1.6550 |
| 4341 | *H* | 1.6745 | 1.6560 |
| 4247 | *Pb* | 1.6770 | 1.6586 |
| 4058 | *Pb* | 1.6829 | 1.6645 |
| 3962 | *Al* | 1.6865 | 1.6673 |
| 3944 | *Al* | 1.6872 | 1.6682 |
| Density of prism | .. | 3.91 | 3.80 |
| Refracting angle | .. | 59° 59′ | 59° 58′ |

*Angles of incidence and deviations.*— In order to obtain the deviations of different rays corresponding to a series of different angles of incidence, it is necessary to know the exact reading of the collimator on the divided circle. For this purpose I employed the method of Cornu, which is applicable to both prisms[1] and gratings.[2] An unsilvered mirror is used to throw a beam of light through the slit of the collimator while the circle and the rigidly attached prism are rotated until the image of the slit, reflected

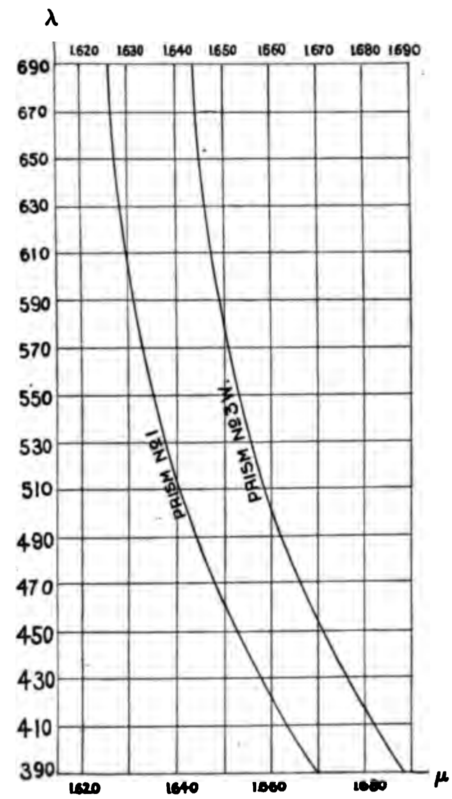



FIG. 1.—Variation of Index with Wave-length.

[1] *Annales de l'École normal supérieure*, 2e série, t. IX.

[2] Etudes sur les bandes telluriques du spectre solaire" (*Annales de Chimie et de Physique*, 6e série, t. VII, p. 48. 1886.

| Angles of incidence measured from normal | | Deviations | | | | | | Dispersion | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Partial | | Total |
| Measured | Calculated | For $H\alpha$ measured | $\lambda = 6563$. $n = 1.6447$ calculated | For D measured | $\lambda = 5893$. $n = 1.6497$ calculated | For $Al_2$ measured | $\lambda = 3944$. $n = 1.6872$ calculated | $S_{Na}-S_H$ | $S_{Al}-S_{Na}$ | $S_{Al}-S_{Na}$ |
| Limiting .... | 39° 04′ 41″ | .......... | 69° 05′ 41″ | .......... | .......... | .......... | .......... | .......... | .......... | .......... |
| Limiting .... | 39 28 54 | .......... | .......... | .......... | 69° 29′ 54″ | .......... | .......... | .......... | .......... | .......... |
| 40°........ | .......... | 61° 36′ | 61 36 30 | 63° 43′ | 63 43 44 | .......... | .......... | 2° 07′ 14″ | .......... | .......... |
| Limiting .... | 42 33 48 | .......... | .......... | .......... | .......... | .......... | 72° 34′ 48″ | .......... | .......... | .......... |
| 45°........ | .......... | 53 45 | 53 45 58 | 54 32 | 54 32 36 | 61° 35′ | 61 35 44 | 0 46 38 | 7° 03′ 08″ | 7° 49′ 46″ |
| 49° 26′..... | .......... | 51 28 | 51 27 44 | 52 05 | 52 04 07 | 57 02 | 57 02 05 | 0 36 23 | 4 57 58 | 5 34 21 |
| Minimum ... | 55 17 45 | 50 36 30″ | .......... | .......... | 51 07 19 | .......... | 55 08 15 | 0 30 49 | 4 00 56 | 4 31 45 |
| Minimum ... | 53 33 00 | .......... | 50 36 49 | 51 07 | .......... | .......... | 55 06 37 | 0 30 11 | 3 59 37 | 4 29 48 |
| 55° 34′..... | .......... | 50 37 | 50 36 49 | 51 07 | 51 06 56 | 55 06 | 55 06 22 | 0 30 07 | 3 59 26 | 4 29 33 |
| Minimum ... | 57 30 00 | .......... | 50 42 39 | .......... | 51 11 59 | 55 02 | .......... | 0 29 20 | 3 50 01 | 4 19 21 |
| 60°......... | .......... | 51 03 | 51 02 31 | 51 30 | 51 30 41 | 55 09 | 55 09 22 | 0 28 10 | 3 38 41 | 4 06 51 |
| 70°......... | .......... | 54 19 | 54 20 10 | 54 46 | 54 45 44 | 58 01 | 58 01 39 | 0 25 34 | 3 15 55 | 3 41 29 |
| 80°......... | .......... | 60 24 | 60 24 00 | 60 48 | 60 48 37 | 63 55 | 63 55 48 | 0 24 37 | 3 07 11 | 3 31 48 |
| 88°......... | .......... | 67 08 | 67 08 39 | 67 34 | 67 33 03 | 70 38 | 70 38 01 | 0 24 24 | 3 04 58 | 3 29 22 |
| Grazing..... | 90 | .......... | 69 05 41 | .......... | 69 29 54 | .......... | 72 34 48 | 0 24 13 | 3 04 54 | 3 29 13 |

at the face of the prism, coincides with the slit itself. The angle of incidence is limited, on the one hand, by "grazing incidence" and, on the other hand, by what may be called the "limiting angle," beyond which the incident ray no longer penetrates the second face of the prism. Within 2° of "grazing incidence" the spectrum is bright enough to permit of settings on the lines.

As a zero for measuring deviations the direct image of the slit was employed.

My study is limited to the following three lines:

The red line of hydrogen, $H\alpha$, - - - - - - - - - $\lambda = 6563$.
The middle of the D lines, - - - - - - - - - $\lambda = 5893$.
The more refrangible of the aluminium double in the violet, - $\lambda = 3944$.

The agreement between observed and computed values is complete to within the limits of errors of observation, that is, approximately one minute.

### COMPUTATION OF DEVIATIONS.[1]

As suggested by M. Cornu, deviations are rapidly computed as follows:

Let $A =$ the refracting angle of the prism,
$e$ and $e' =$ the angles of incidence and emergence, respectively,
$r$ and $r' =$ the interior angles at the first and second faces, respectively,
$D =$ the angle of deviation required,
$n =$ the refractive index for the ray considered.

For example, take prism No. 3 W, in which $A = 59° \; 59'$, $e = 45°$, and for sodium light, $n = 1.6497$.

$$\sin r = \frac{\sin e}{n} \left\{ \begin{array}{l} \log \sin e = \bar{1}.8494850 \\ \text{colog}\, n = \underline{\bar{1}.7825967} \\ \log \sin r = \bar{1}.6320817 \end{array} \right. \tag{1}$$

$$r' = A - r \left\{ \begin{array}{r} A = 59° \; 59' \\ -r = \underline{25 \;\; 22 \;\; 50''} \\ r' = 34° \; 36' \; 10'' \end{array} \right. \tag{2}$$

$$\sin e' = n \sin r \left\{ \begin{array}{l} \log \sin r' = \bar{1}.7542594 \\ \log n = \underline{0.2174033} \\ \log \sin e' = \bar{1}.9716627 \end{array} \right. \tag{3}$$

[1] A résumé of what follows was published in *Comptes Rendus*, February 12, 1900.

$$D = e + e' - A \left\{ \begin{array}{r} e' = 69^\circ\ 31'\ 36'' \\ + e = 45^\circ \\ \hline A + D = 114\ 31\ 36 \\ - A = 59\ 59 \\ \hline D = 54^\circ\ 32'\ 36'' \end{array} \right. \tag{4}$$

*Special cases.*—For minimum deviation $D_m$ we have $e' = e = e_m$, or

$$e_m = \frac{A + D_m}{2} . \tag{4a}$$

The case of grazing incidence, or what is geometrically the same,

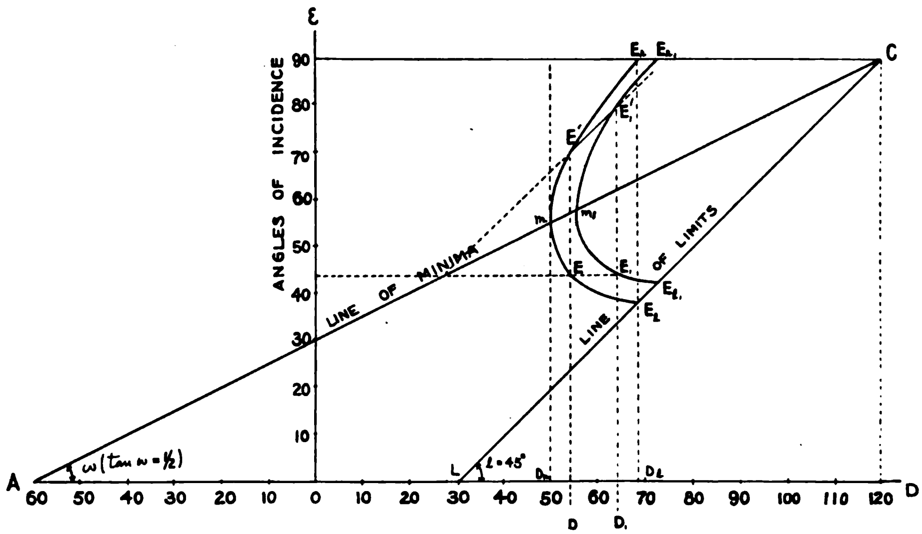



FIG. 2.—Deviations.

the case of grazing emergence, gives us a maximum deviation, viz.:

$$D = 90^\circ + e' - A . \tag{4β}$$

The values thus obtained for prism No. 3 W. have been plotted in a curve, Fig. 2, in which deviations are employed as abscissae and angles of incidence as ordinates. The two curves here given are for the red hydrogen line and the violet aluminium line.

Since eq. (4) is symmetrical with respect to $e$ and $e'$, it is evident from the principle of reversibility that to each value of $D$

there will correspond two values of $e$, $DE$ and $DE'$ in the figure. The curve thus also gives the values of the angles of emergence.

If now we write eq. (4) as follows,

$$\frac{e+e'}{2} = \frac{1}{2}(A+D)$$

and consider the half sum of $e$ and $e'$ as ordinates, it becomes evident that the locus of the middle points of the chords parallel to the axis of $Y$ lie on a straight line, which we shall call the "line of minima."

This equation shows also that this line makes with the axis of $X$ an angle $\tan^{-1}\frac{1}{2}$, viz., 26° 33′ and that its intercept on the axis of $X$ is numerically equal to the angle of the prism.

Differentiating eq. (4*a*) we have for any given prism

$$\frac{de_m}{dD_m} = \frac{1}{2},$$

or the minimum deviation varies at a rate which is double that of the angle of incidence.

But from eq. (4) we have, when $e =$ constant,

$$\frac{de'}{dD} = 1 .$$

Hence if, on the curves corresponding to two different wave-lengths, we select two points, $E$ and $E_1$, for which the angle of incidence is the same, we shall find that the points of emergence corresponding to these, $E'$ and $E_1'$ will always lie on a straight line inclined 45° to the axis of $X$.

For any two given angles of incidence, the ratio of the dispersions will be the ratio of the intercepts between the curves, on the straight lines, $e_1' =$ constant and $e_2' =$ constant, where $e_1'$ and $e_2'$ are angles of emergence corresponding to any given angle of incidence $e$. This follows at once from the fact that the straight line $E'\ E_1'$, Fig. 2, is inclined 45° to the axis of $X$.

It is evident also from Fig. 2 that we have a maximum deviation, $D_e$, determined twice, viz., first by *grazing incidence* and in

the second place, by *limiting incidence*, which gives grazing emergence.

It may also be easily demonstrated, by geometry, that the straight line of minima, the straight line of limits, and the line of grazing incidence, $e = 90°$, all three intersect in one point, $C$.

*Variation of refractive index.*— Since in the case of minimum deviation we have

$$\sin e_m = n \sin \frac{A}{2} ,$$

it is evident that the maximum permissible value of $n$ is

$$n_e = \frac{1}{\sin \frac{A}{2}} = \operatorname{cosec} \frac{A}{2} .$$

In this case maximum and minimum incidence coincide, each having a value of 90°. In other words, our curve of deviations reduces to a point. In the case of a 60° prism,

$$n_e = \operatorname{cosec} 30° = 2 .$$

Such a prism can therefore be used only when its refractive index is less than 2.

Let us next consider the curve whose foot is situated at the point $L$ where the line of limits intersects the axis of $X$. This evidently means grazing incidence and zero angle of emergence; and hence

$$r = A ,$$

and

$$n = \frac{1}{\sin A} = \operatorname{cosec} A .$$

For a 60° prism, the value of the index giving such a curve is 1.1547. This is a limiting value, for $A = 60°$, of all curves which lie entirely above the axis of $X$ and have positive angles of emergence.

*Curves which are partly negative.*—We are thus led to consider the case in which $r > A$ and hence $r' < 0$, giving us negative values for the angle of emergence.

The condition that a part of the curve should lie below the axis of $X$ is $n <$ cosec. $A$. For under this condition the angle of incidence may not only lie anywhere between 0° and 90° but

may even have a negative value and yet give an emergent ray. Total reflection occurs only when these negative values of the angle of incidence exceed a certain limit depending upon the refractive index.

In practice this case is realized only with prisms of very small refracting angle (since the index for liquids does not fall below about 1.33), or with prisms filled with gases optically denser than air.

*Variation of refracting angle.*—The expression for the maximum permissible index in any prism

$$n = \operatorname{cosec} \frac{A}{2}$$

shows us that $n$ varies inversely as $A$.

In the case of the extraordinary ray in cinnabar, which yields the highest known index for the red ray, viz., 3.201, the maximum refracting angle is $36° \ 24' \ 30''$. Indeed simple inspection of Fig. 2 shows us that, as the refracting angle $A$ of the prism is diminished, the rigidly connected system made up of the straight line of limits and the straight line of minima is displaced farther and farther from the axis of ordinates.

As has been indicated by Cornu, the equation of the curve connecting $e$ and $D$ is given by elimination of $e'$ from the following two equations:

$$e + e' = A + D$$

$$\tan \frac{e + e'}{2} = \frac{\dfrac{\sin^2 \frac{A + D}{2}}{\sin^2 \frac{A}{2}} - n^2}{n^2 - \dfrac{\cos^2 \frac{A + D}{2}}{\cos^2 \frac{A}{2}}}.$$

For this latter equation, see Cornu's memoir "De la réfraction à travers un prisme suivant une loi quelconque" (*An. Ec. Norm.* (2), I (1872), p. 240).

*Curve of total dispersion.*—In Fig. 3, I have plotted, from the data contained in the last column of the table for prism 3 W, a

curve which gives the total dispersion as a function of the angles of incidence, employing as abscissae the differences of deviation between the rays λ 3944 and λ 6563, and using as ordinates the angles of incidence.

This dispersion is a minimum for grazing incidence, increases

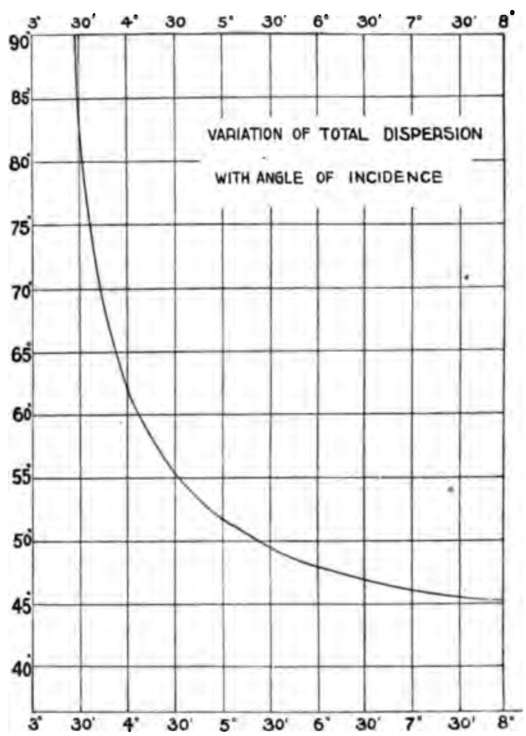



FIG. 3.—Dispersion, $D_{Al_2} - D_{H\alpha}$.

slowly until the minimum of deviation is reached; it then increases rapidly until the limiting angle of incidence is reached. But, as has been indicated above, these limiting angles of incidence depend not only upon the refractive index, but also upon the angle of the prism. Thus we are able to observe the aluminium ray with an index of 1.6872 even up to normal incidence, $e = 0$, provided the angle of the prism is less than 36°. One may thus increase the dispersion in a spectroscope by reducing the angle of incidence.

# ON THE SPECTRUM OF *NOVA PERSEI*.[1]

By H. C. VOGEL.

THE news that a "new star" of considerable brightness had appeared in the constellation Perseus was everywhere received with the highest expectations; for the new stars have ever been among the most puzzling of celestial bodies, and nothing beyond a temporary satisfaction has been furnished by the great number of hypotheses as to the nature of these stars, based upon more or less reliable scientific foundations.

Much obscurity has been cleared up by the application of spectroscopy, and interesting results have ensued in the development of the methods based on Doppler's principle. The perfected apparatus of the most recent date, particularly since the introduction of the spectrograph, has enabled us to recognize in the spectrum of new stars pairs of bright and dark, much broadened lines, which suggested that we are dealing with two bodies instead of one, of which one gives chiefly an emission spectrum and the other an absorption spectrum. The relative motion of the two bodies, deduced from the separation of the centers of the adjacent dark and bright lines, led, however, to velocities so great as to be quite improbable.

The observations made by myself and by others were communicated in detail in my paper "Ueber den neuen Stern im Fuhrmann,"[2] together with the consequences following from them, according to the point of view of that time, and with a discussion of the most important hypotheses.

Our knowledge of the spectra of the different elements has meanwhile been decidedly increased, and the investigations of Humphreys and Mohler, Eder and Valenta, and Wilsing have

[1] From advance sheets of a paper presented to the *Kgl. Akademie der Wissenschaften zu Berlin*, 1901.

[2] *Abhandlungen der Kgl. Akademie zu Berlin.* Translated in *Astronomy and Astro-Physics*, 12, 896, 1893; 13, 48, 136, 1894.

taught us that not all displacements of lines are to be regarded as consequences of Doppler's principle. On May 4, 1899, I was able to lay before the Royal Academy a paper by Professor Wilsing of Potsdam entitled "Ueber die Deutung des typischen Spectrums der neuen Sterne,"[1] which gives a very natural explanation of the double spectra of the new stars, based upon his own experiments, and hence of the physical processes effective in the atmosphere of a *Nova*.

We might reasonably expect a further confirmation of the validity of this hypothesis from *Nova Persei*, which we were first able to observe with the spectrograph in Potsdam on February 23, 1901, when it was the brightest star in the northern heavens. Our astonishment was not small to find almost no details visible on the photographs of the spectrum, which with a simple ocular spectroscope was very brilliant.

Measurements by Dr. Hartmann and myself on the spectrograms indicated the presence of the hydrogen lines; on the plates of small dispersion taken by Dr. Hartmann with the 80 cm refractor he could see and measure the nine lines from $H\beta$ to $H\kappa$; while on the plates of high dispersion taken by Dr. Ludendorff with the 32 cm refractor, on which only a small part of the spectrum (from $\lambda$ 4040 to 4520) was impressed, only the two lines $H\gamma$ and $H\delta$ were to be seen in my examination. The hydrogen lines appeared as broad absorption bands, very weak, diffuse, and only recognizable with difficulty, with an increased tendency to diffuseness of the less refrangible side. Other faint absorption bands of some other element could also be seen, but there was not any suggestion of emission lines or bands. Two quite sharp and narrow absorption lines were, however, very striking on Hartmann's plates, and were identified with the calcium lines at $\lambda$ 3934 and $\lambda$ 3969. They indicated a slight displacement toward the red, which according to the preliminary measures would correspond to a motion of the star of about +45 km per second relative to the Earth, or about +18 km per

[1] "On the Interpretation of the Typical Spectrum of the New Stars." The ASTROPHYSICAL JOURNAL, 10, 113, 1899.

second relative to the Sun. I would remark here that the position of these lines has remained unchanged on the later plates, and the velocity given may probably be regarded as that of the star. On the plates of February 23, which I measured, there were no sharp lines, but there was a somewhat better visible, diffuse absorption band for which I deduced a wave-length of $\lambda$ 4473. If I identify this band with the helium line $\lambda$ 4471.6, a velocity of the star of from +10 km to +20 km, relative to the Sun, would also follow. The case is quite different, however, with the hydrogen bands. Our measures agreed well among themselves and with each other, and yielded a very large displacement toward the direction of shorter wave-lengths, from which could be deduced a velocity of the hydrogen gas of — 700 km per second, in round numbers, relative to the Sun.

Everything that we could observe on February 23 — the strikingly sharp calcium lines, the absence of emission lines, the large displacement of the absorption lines toward the more refrangible side of the spectrum — was contradictory to what was to be expected from the above mentioned theory.

The plates of the spectrum made by Drs. Hartmann, Eberhard, and Ludendorff on February 26 and 27, and on March 2, 3, and 4, with the two instruments mentioned, show a marked change in the spectrum, for the absorption lines had become much more distinct, and were accompanied by intense and very broad emission bands, which could be readily seen as bright lines with the small ocular spectroscope. These emission lines are very broad, gradually becoming diffuse on the less refrangible side; their intensity-maxima are slightly, but their centers greatly, displaced toward red. The absorption lines are, however, shifted still further in the opposite direction than on the spectrograms of February 23. In a word, the spectrum has become that typical of new stars, and shows on a large scale the changes which Wilsing's[2] observations have shown to occur in the spectra of metals and of hydrogen under high pressure.

An attempt to explain the behavior of the calcium lines H

[2] The ASTROPHYSICAL JOURNAL, 10, 118, 1899.

and K has led me to interpret the large displacements of the hydrogen lines according to Wilsing's hypothesis.

The rapid increase in the brightness of the new star (which according to Pickering was certainly not of the eleventh magnitude on February 19, and at $10^h$ on February 23 was of mag. 0.24 according to the Potsdam observations) permits us to assume enormous disturbances in the atmosphere of the star accompanied by heavy increase of pressure—conditions under which we can hardly think of the easily broadened calcium lines as narrow and sharp cut absorption lines; whence we may regard their extreme sharpness as merely an effect of reversal. But if we make this assumption it further follows that a narrow absorption line can exist as a reversal only in a broader emission line, and I assume that layers have developed in the star's atmosphere, as premised in Wilsing's theory, from one of which has resulted broad absorption lines and from the other broad emission bands with reversal effects. If now the superposition of the layers causes absorption and emission to nearly cancel each other, there will remain only the sharp absorption line of the reversal. This is shown in the annexed figure. The upper curve of Fig. 1 *A* represents the intensity curve of the spectrum of emission, with reversal; the lower curve is that of the absorption spectrum. The intensity curve resulting from the superposition of the layers is given in Fig. 1 *B*. The dotted line indicates the position the calcium lines would have if there were no displacement; the narrow absorption line is somewhat displaced toward the red, as observed. In all the diagrams the red end of the spectrum is to be understood as toward the right.

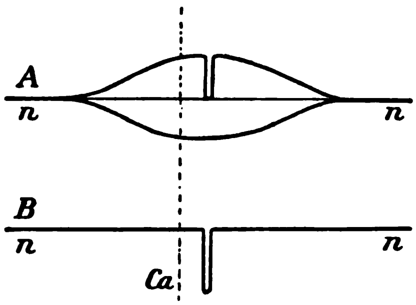


Fig. 1.

Similar considerations remove the difficulty of explaining why the absorption lines of hydrogen only, strongly displaced

toward violet, appeared on February 23. If several layers of different pressure are again superposed, it may easily happen that the emission line, displaced toward red by the higher pressure, is so faint that it does not brighten up the absorption line above the level of continuous spectrum, *n n* in the figures. In Fig. 2 $A_1$ let the upper curve be that of the emission spectrum, the lower that of absorption; while Fig. 2 $B_1$ represents the curve resulting from the superposition of the layers, corresponding to the intensity curve observed on February 23. The large displacement of the intensity maximum toward the more refrangible side is therefore only apparent: the center of the absorption band actually needs to be shifted only slightly toward red, as in Fig. 2 $A_1$.

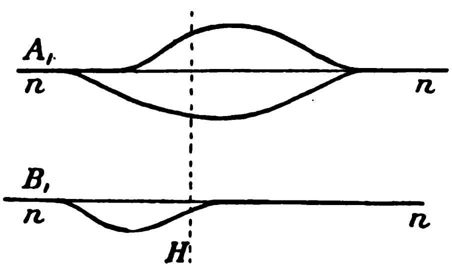



FIG. 2.

As already stated, emission lines are already present on the spectrograms of February 26, and they became increasingly distinct on the later plates. Fig. 3 roughly represents the intensity curve of the $H\gamma$ pair on the plates of March 3 and 4. The peculiar form of the curve is explained as above without difficulty.

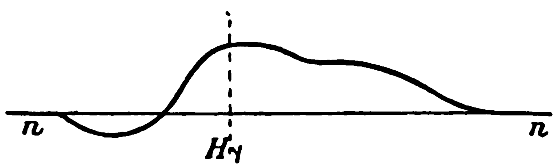



FIG. 3.

I would not fail to mention that these last considerations are only an extension, with inclusion of the phenomena observed in *Nova Persei*, of what Wilsing stated in his paper on the typical spectrum of new stars.

After what has been said I believe I may now express the opinion that the observations of the spectrum of *Nova Persei* have so far only confirmed the view of Wilsing, and that there

is no occasion for regarding the large displacements of the absorption lines of hydrogen as results of motions on Doppler's principle, even if only of motions of hydrogen gas.

The important rôle played by the hydrogen lines in respect to breadth and displacement in case of *Nova Aurigae* seems so far to have been even exceeded in case of *Nova Persei*. There are in this spectrum only a few bands, and no such number of single lines and pairs as in *Nova Aurigae*, and the blue and violet regions are particularly lacking in lines of other elements.

The extremely rapid increase in the star's brightness and now the quick decline in the intensity of the more refrangible parts of the spectrum arouses the suspicion that the hydrogen is chiefly responsible for the bright continuous spectrum in this part, and it is known that this may easily occur under certain conditions of pressure and temperature. The explanation of the rapid decrease in brightness would otherwise be difficult. We must await the further developments of the phenomena, however, before we can reach conclusions with certainty. Perhaps they will lead us to a final discrimination among the many hypotheses as to new stars.

# *Minor Contributions and Notes*

## WAVE-LENGTH DETERMINATIONS AND GENERAL RESULTS OBTAINED FROM A DETAILED EXAMINATION OF SPECTRA PHOTOGRAPHED AT THE SOLAR ECLIPSE OF JANUARY 22, 1898.[1]

In this paper the results are given of a detailed study and measurement of a series of spectra photographed at the eclipse of 1898, with a glass prismatic camera of 2¼ inches aperture. Ten exposures were made, all yielding good negatives, in which the great extension in the ultra-violet is a marked feature.

The first two photographs of the series were exposed at 20 seconds and 10 seconds before totality respectively, and are images of the cusp spectrum. They show the Fraunhofer lines with great distinctness, although the latter are much less dark than in the ordinary solar spectrum. The lines were measured and identified for the purpose of facilitating the reduction of the bright line spectra obtained during totality.

*Spectrum No. 3* was exposed for four seconds, beginning two seconds before second contact. In this the flash spectrum is fully developed, and extends from $\lambda$ 3340 to $\lambda$ 6000. The majority of the bright arcs, including those due to the upper chromosphere, extend over 40° of the limb, implying a depth of 1″.3 for the gases composing this layer. The total depth of the chromosphere deduced from the hydrogen arcs is 8″.2, and from the calcium arcs 11″.6. There are 313 measurable lines in this negative, and the wave-lengths and identifications of these are given in Table I.

*Spectrum No. 4*, exposed for half a second shortly after second contact, gives the spectrum of the upper chromosphere and prominences. Seven of the latter are shown. The images are about equally dense in calcium radiations, although in hydrogen there is a marked variation of intensity between the different prominences.

[1] Abstract. Read before the Royal Society on January 17, 1901.

A conspicuous feature in the spectrum of two of the prominences is a band of continuous spectrum, beginning at $\lambda$ 3668 near the end of the hydrogen series, and extending indefinitely in the ultra violet.

Good measures were obtained of the images of a small prominence at the center of the plate, the wave-lengths being given in Table II.

*Spectrum No. 5.*—This plate had a long exposure near mid-totality. The continuons spectrum of the corona is strongly marked, and the green corona line is well shown at position angles 60° to 78°, and 95° to 105°. A new corona line is faintly impressed at $\lambda$ 3388 $\pm$, the maxima of intensity being at the same position angles as those of the green line.

*Spectrum No. 7* shows the reappearing arcs of the flash spectrum, the exposure ending about four seconds before third contact. The green corona line is shown on both east and west limbs, and there is a faint corona line near H. The wave-length values of the lines measured on this plate are given in Table I.

*Spectrum No. 8.*—This was exposed almost at the instant of third contact, the reappearing photosphere showing as four narrow bands of continuous spectrum due to Baily's beads. The flash spectrum arcs extend between and across the bands, and can be traced over an arc of 55°, the depth of the layer, in this case exceeding 2".

The focus in this negative is poor, and no measures were made; but as far as can be judged, comparing this plate and No. 3, the spectra of the east and west limbs of the Sun are identical.

*Spectra Nos. 9 and 10.*—These are cusp spectra, very similar to Nos. 1 and 2.

GENERAL RESULTS AND CONCLUSIONS.

*The flash spectrum.*—Comparing the wave-length values of the flash spectra given in Table I with Rowland's wave-lengths of the solar lines, it is at once evident that practically all the strong dark solar lines are present in the flash as bright lines; and all the bright lines in the flash, excepting hydrogen and helium, coincide with dark lines having an intensity greater than three on Rowland's scale.

The relative intensities of the lines in the two spectra are, however, widely different, many conspicuous flash lines coinciding with weak solar lines, and some of the strong solar lines being represented by weak lines in the flash spectrum.

This, however, applies only to the spectrum taken as a whole. Selecting the lines of any one element, it is found that the relative intensities in the flash spectrum agree closely with those of the same element in the solar spectrum. This is particularly well shown in the case of the elements iron and titanium.

The want of agreement in the relative intensities of the lines of different elements in the bright line and dark line spectra is probably due to the unequal heights to which the various elements ascend in the chromosphere, a low-lying gas of great density giving strong absorption lines, but weak emission lines, on account of the excessively small angular width of the radiating area.

The more extensively diffused gases of small density, on the other hand, give strong emission lines in the flash spectrum, and weak absorption lines.

The spectrum arcs obtained with a prismatic camera are not true images of the strata producing them, but *diffraction* images more or less enlarged by photographic irradiation. Monochromatic radiations from a layer 2″ in depth will produce arcs or "lines" which are as narrow as can be defined by instruments of ordinary resolving power.

The intensities of these images do not represent the intrinsic intensities of the bright lines of the different elements; the apparent intensity of the radiation from an element depending on the extent of diffusion of that element above the photosphere.

But in the dark line spectrum the intensities depend on the total quantity of each absorbing gas above the photosphere irrespective of the state of diffusion of the different elements.

The flash spectrum as a whole appears from these results to represent the upper, more extensively diffused portion of a stratum of gas, which, by its absorption, gives the Fraunhofer spectrum.

Fifteen elements are recognized with certainty in the flash spectrum (No. 3), and five are doubtfully present. The atomic weights of these elements in no case exceed 91. All the known metals having atomic weights between 20 and 60 seem to be present in the lower chromosphere, but among these there does not seem to be any relation between the atomic weights and the elevations to which the gases ascend in the chromosphere.

The only non-metals found are *H*, *He*, *C*, and possibly *Si*.

Of the 225 lines measured in the ultra-violet region of the spectrum only 29 remain unidentified.

*The hydrogen spectrum.*—Twenty-eight hydrogen lines are shown in spectrum No. 3. The wave-lengths obtained are compared in Table III with the theoretical values derived from Balmer's formula. With the exception of $H\delta$, which seems to be unaccountably displaced towards the red, the wave-lengths of the ultra-violet lines are found to agree closely with the formula. A slight deviation occurs in the most refrangible lines, the positions of which seem to be distinctly more refrangible than those assigned by theory.

The continuous spectrum given by the prominences in the ultra-violet, beginning at the end of the hydrogen series, seems analogous to a feature noticed by Sir William Huggins in the absorption spectra of first type stars, and is possibly due to hydrogen.

*Hydrogen and helium in the lower chromosphere.*—From the character of some of the helium lines it is inferred that this element is probably absent from the lowest strata, whilst parhelium appears to be separated from helium, and to exist at a lower level.

Unlike helium, hydrogen gives very intense lines in the flash layer. These lines are well defined and narrow, even in the very lowest strata.

Reasons are given to show that the absence of hydrogen absorption in the ultra-violet, and of helium absorption in the visible spectrum, may be due to insufficient quantity of these elements above the photosphere, not to equality of temperature between the radiating gas and photospheric background.

*The corona spectrum.*—The wave-length of the green line deduced from measures of No. 3 and No. 7 spectra confirms the value obtained by Sir Norman Lockyer at the same eclipse. The only other lines shown on these photographs are at $\lambda$ 3388 and near H.

J. EVERSHED.

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## SIXTY-FOUR NEW VARIABLE STARS.[1]

THE photographs of the Henry Draper Memorial continue to furnish great numbers of new variable stars. A large part of those enumerated below were found from the presence of bright hydrogen

[1] *Harvard College Observatory Circular* No. 54.

lines in their spectra. Many stars whose spectra are of the fourth type also prove to be variable. These variables have been divided into two classes. First, those in which the variation is so great that it is obvious to the most inexperienced observer. Secondly, those in which the variation so far detected is small, about half a magnitude to a magnitude. In each of these cases, two or more experienced observers, who are accustomed to accurate measures of photographic brightness, are satisfied that the change is real. We have here a case like the confirmation visually by a second observer, since so many plates of each variable are examined, generally a dozen or more, that on several the star is bright, and on several, faint. There seems to be no way in which these changes can be rendered more evident, and owing to the redness of many of the stars it is doubtful if visual observations would be more conclusive. Perhaps photometric measures, which appear to be less influenced by color, or photographs taken with a reflector might be employed to advantage. Owing to the accidental errors, additional measures add but little to the certainty of variation, which is best shown by comparing two plates, on one of which the variable is bright, on the other, faint. It seems best therefore to publish the positions of these stars, hoping that by further observations the laws governing their changes may be learned. In both tables, the name of the constellation is given in the first column. For northern stars, the boundary of the constellations is taken from the *Atlas Cœlestis Novus* of Heis, and for southern stars from the *Uranometria Argentina*. The catalogue designation, if any, is given in the second column. The approximate right ascension and declination for 1900 are given in the third and fourth columns. The class of spectrum is given in the fifth column. Following the notation of the *Draper Catalogue*, Mc is used to denote a spectrum of the third type like that of *o Ceti* at minimum. Md denotes a similar spectrum in which, however, the hydrogen lines are bright as in *o Ceti* at maximum. Intermediate spectra are indicated by Mc 5 d. N denotes a spectrum of the fourth type, and Pec. that the spectrum is peculiar. The name of the discoverer is given in the sixth column. A few remarks on individual stars follow Table II. Each is preceded by the right ascension for 1900.

TABLE I.

VARIABLES HAVING LARGE RANGE.

| Constellation | Designation | R. A. 1900 | Dec. 1900 | Class | Discoverer |
|---|---|---|---|---|---|
| *Chamæleon* | *Z. C.* $8^h$ 2054 | $8^h$ $24^m.1$ | −76° 2′ | Md | W. P. Fleming |
| *Carina* | ............. | 9 18.2 | −68 20 | Md | W. P. Fleming |
| *Vela* | *A. G. C.* 13539 | 9 51.3 | −41 7 | N | L. D. Wells |
| *Antlia* | *A. G. C.* 14440 | 10 30.8 | −39 3 | N | L. D. Wells |
| *Carina* | ............. | 10 33.2 | −61 48 | .... | E. C. Pickering |
| *Centaurus* | *Z. C.* $11^h$ 129 | 11 2.9 | −54 35 | N | L. D. Wells |
| *Virgo* | −18° 3660 | 13 36.3 | −18 38 | .... | L. D. Wells |
| *Lupus* | *Z. C.* $14^h$ 3225 | 14 52.6 | −53 0 | N | W. P. Fleming |
| *Lupus* | ............. | 15 8.5 | −50 25 | .... | E. C. Pickering |
| *Circinus* | ............. | 15 20.0 | −57 22 | Md ? | W. P. Fleming |
| *Norma* | *Z. C.* $16^h$ 59 | 16 2.6 | −48 58 | Md ? | W. P. Fleming |
| *Norma* | *A. G. C.* 21999 | 16 9.0 | −52 21 | Md ? | W. P. Fleming |
| *Norma* | −51° 10147 | 16 17.7 | −51 42 | N | W. P. Fleming |
| *Triang. Austr.* | *Gilliss* 12037 | 16 39.8 | −67 36 | N | W. P. Fleming |
| *Scorpius* | −43° 11672 | 17 18.1 | −43 44 | Md | W. P. Fleming |
| *Scorpius* | −35° 11923 | 17 40.8 | −35 40 | N | W. P. Fleming |
| *Ophiuchus* | − 6° 4661 | 17 44.8 | − 6 40 | .... | W. P. Fleming |
| *Ara* | −48° 12145 | 17 47.3 | −48 17 | Md | W. P. Fleming |
| *Ara* | −49° 11810 | 17 49.2 | −49 46 | Mc | W. P. Fleming |
| *Corona Austr.* | −39° 12196 | 17 58.2 | −39 20 | N | W. P. Fleming |
| *Corona Austr.* | ............. | 18 2.6 | −45 26 | Mc | W. P. Fleming |
| *Corona Austr.* | ............. | 18 7.2 | −42 53 | Md | W. P. Fleming |
| *Telescopium* | ............. | 18 19.0 | −49 42 | Md | W. P. Fleming |
| *Sagittarius* | −16° 4904 | 18 24.6 | −16 59 | N | W. P. Fleming |
| *Scutum* | − 8° 4726 | 18 44.9 | − 8 1 | N | L. D. Wells |
| *Scutum* | − 8° 4764 | 18 50.0 | − 8 19 | N | W. P. Fleming |
| *Sagittarius* | ............. | 18 55.9 | −12 54 | Md | W. P. Fleming |
| *Sagittarius* | −22° 4958 | 18 57.7 | −22 51 | Mc | W. P. Fleming |
| *Telescopium* | ............. | 19 0.5 | −49 4 | Md | W. P. Fleming |
| *Telescopium* | *C. P. D.* −50° 11027 | 19 10.5 | −50 38 | Md ? | W. P. Fleming |
| *Lyra* | +42° 3338 | 19 22.2 | +42 36 | .... | W. P. Fleming |
| *Telescopium* | ............. | 19 43.1 | −50 15 | Md | W. P. Fleming |
| *Telescopium* | .......... .. | 20 11.2 | −52 56 | Md | W. P. Fleming |
| *Telescopium* | −51° 12487 | 20 12.9 | −51 1 | Mc 5 d | W. P. Fleming |
| *Cygnus* | ............. | 21 35.7 | +42 45 | .... | H. R. Colson |
| *Aquarius* | −22° 5901 | 22 17.7 | −22 35 | Md ? | W. P. Fleming |
| *Piscis Austr.* | *A. G. C.* 30605 | 22 20.5 | −29 35 | ... | W. P. Fleming |
| *Andromeda* | +48° 4093 | 23 28.8 | +48 16 | Md ? | W. P. Fleming |
| *Pegasus* | +25° 5054 | 23 55.0 | +25 21 | Md ? | W. P. Fleming |

TABLE II.

VARIABLES HAVING SMALL RANGE.

| Constellation | Designation | R. A. 1900 | Dec. 1900 | Class | Discoverer |
|---|---|---|---|---|---|
| *Hydrus* | ............ | $2^h\ 10^m.4$ | $-71°\ 57'$ | Mc 5 d | W. P. Fleming |
| *Hydrus* | *A. G. C.* 2634 | 2 26.3 | −69 58 | Mc 5 d | W. P. Fleming |
| *Cetus* | *A. G. C.* 2859 | 2 37.4 | −23 2 | Mc 5 d | W. P. Fleming |
| *Horologium* | *Z. C.* $2^h$ 1104 | 2 41.2 | −54 44 | Mc | W. P. Fleming |
| *Eridanus* | −1° 546 | 3 46.4 | − 1 41 | Mc 5 d | W. P. Fleming |
| *Puppis* | *A. G. C.* 8954 | 7 1.7 | −35 47 | Mc 5 d | W. P. Fleming |
| *Canis Major* | −11° 1805 | 7 3.4 | −11 46 | N | W. P. Fleming |
| *Lynx* | +46° 1271 | 7 20.9 | +46 10 | Mc | W. P. Fleming |
| *Hydra* | − 8° 2343 | 8 19.6 | − 8 11 | Mc 5 d | W. P. Fleming |
| *Hydra* | − 9° 2612 | 8 34.9 | − 9 14 | Mc 5 d | W. P. Fleming |
| *Virgo* | − 8° 3329 | 12 15.2 | − 8 27 | Md ? | W. P. Fleming |
| *Centaurus* | *A. G. C.* 17944 | 13 6.3 | −56 28 | .... | L. D. Wells |
| *Virgo* | − 2° 3653 | 13 8.9 | − 2 16 | Mc 5 d | W. P. Fleming |
| *Chamæleon* | *A. G. C.* 18352 | 13 24.6 | −77 3 | .... | W. P. Fleming |
| *Lupus* | *Z. C.* $14^h$ 970 | 14 16.9 | −47 4 | N | W. P. Fleming |
| *Norma* | −50° 10442 | 16 14.6 | −50 14 | Md | W. P. Fleming |
| *Serpens* | −15° 4923 | 18 13.6 | −15 39 | N | W. P. Fleming |
| *Corona Austr.* | ............ | 18 23.7 | −45 2 | Md | W. P. Fleming |
| *Telescopium* | −48° 12910 | 19 0.1 | −48 44 | Mc | W. P. Fleming |
| *Sagittarius* | −16° 5360 | 19 28.6 | −16 35 | N | L. D. Wells |
| *Sagittarius* | *C. P. D.* −41° 9189 | 19 40.6 | −41 26 | Mc 5 d | W. P. Fleming |
| *Sagittarius* | *A. G. C.* 27520 | 20 0.8 | −27 31 | Mc 5 d | W. P. Fleming |
| *Octans* | *Gilliss* 15580 | 22 5.7 | −85 10 | Mc 5 d | W. P. Fleming |
| *Aquarius* | −18° 6299 | 23 19.2 | −17 52 | Pec. | W. P. Fleming |
| *Cassiopeiæ* | +56° 3111 | 23 49.4 | +56 56 | Pec. | L. D. Wells |

$7^h\ 20^m.9$ The variation, although small, has been confirmed by two other observers, and is indicated by observations with the meridian photometer.

10 33.2 Found by superposing an original negative on a contact print from another negative taken on a different date.

15 8.5 Found by superposing an original negative on a contact print from another negative taken on a different date.

7 40.8 This star is *C. P. D.* −35°7243. Innes has announced the variability of −35°7270, which follows $51^s.9$, south $0'.1$. *A. J.*, **20**, 59, 95.

18 44.9 "Probably a variable of the *19 Piscium* type" in Espin's Catalogue of Red Stars, *Cunningham Memoirs*, No. V, 75. Discovered also independently by Mrs. Fleming.

19 22.2 Found by inspection of a photograph taken as described in *Circular* No. 29. Thirteen exposures of $29^m\ 40^s$ each were made on July 13, 1899, stopping the clock automatically for $20^s$ after each exposure. This variable is mentioned in the Fifty-fourth Annual Report. Photometric measures show that its maxima are represented by the formula, *J. D.* $2{,}414{,}856^d.500 + 0^d.5668\ E$. Range 0.83 magnitudes.

21 35.7 Discovered visually during observations of *SS Cygni.*

23 49.4 This star is ρ *Cassiopeiæ.* The variation, although small has been confirmed by four other observers. The spectrum closely resembles that of the second type.

Measures have been made of a number of the stars in the above tables and also of those announced without magnitudes in previous circulars. The right ascension and declination for 1900, the number of plates examined, and the brightest and faintest photographic magnitudes, are given in the successive columns of Table III. The last column gives the authority for the variability.

TABLE III.

PHOTOGRAPHIC MAGNITUDES.

| R. A. | Dec. | No. | Br. | Ft. | Authority | R. A. | Dec. | No. | Br. | Ft. | Authority |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $2^h$ $10^m.4$ | −71° 57′ | 75 | 9.6 | 10.5 | Table II | $16^h$ $54^m.3$ | −54° 55′ | 50 | 9.9 | 11.0 | *Circular* 24 |
| 2 37.4 | −23 2 | 68 | 7.7 | 8.6 | Table II | 17 34.7 | −57 40 | 43 | 8.3 | 9.8 | *Circular* 24 |
| 6 28.1 | − 8 48 | 54 | 9.0 | 10.1 | *Circular* 32 | 17 45.7 | −51 40 | 56 | 8.9 | 12.4 | *Circular* 24 |
| 8 1.7 | −38 29 | 47 | 0.3 | 10.2 | *Circular* 24 | 17 47.3 | −48 17 | 134 | 9.7 | <12.3 | Table I |
| 8 3.1 | −22 38 | 23 | 9.4 | 11.6 | *Circular* 24 | 18 13.6 | −15 39 | ... | 8.5 | 9.1 | Table II |
| 8 24.7 | − 5 59 | 48 | 8.0 | 9.6 | *Circular* 24 | 18 19.0 | −49 42 | 129 | 11.3 | <12.7 | Table I |
| 8 34.9 | − 9 14 | 70 | 7.7 | 9.0 | Table II | 18 23.7 | −45 2 | 107 | 11.9 | 12.6 | Table II |
| 9 13.5 | −65 49 | 116 | 0.9 | 12.1 | *Circular* 32 | 19 10.5 | −50 38 | 113 | 9.2 | 10.6 | Table I |
| 10 8.3 | +60 31 | 76 | 7.0 | 8.3 | *Circular* 32 | 19 22.2 | +42 36 | 191 | 7.2 | 8.1 | Table I |
| 10 33.2 | −61 48 | 210 | 0.1 | <12.5 | Table I | 19 37.1 | +32 23 | 58 | 8.7 | 10.3 | *Circular* 24 |
| 11 59.6 | − 5 13 | 77 | 7.2 | 8.8 | *Circular* 32 | 20 3.3 | −60 14 | 75 | 9.0 | 10.2 | *Circular* 24 |
| 12 2.1 | − 6 12 | 80 | 7.1 | 8.3 | *Circular* 24 | 20 11.2 | −52 56 | 91 | 10.5 | 12.9 | Table I |
| 13 15.1 | −61 3 | 55 | 0.5 | 11.3 | *Circular* 24 | 20 12.9 | −51 1 | 124 | 8.1 | 9.7 | Table I |
| 14 1.7 | +13 59 | 30 | 0.0 | 13.0 | *Circular* 24 | 21 13.6 | −45 27 | 83 | 7.2 | 8.9 | *Circular* 24 |
| 16 17.7 | −51 42 | 102 | 0 | <12.3 | Table I | 23 19.2 | −17 52 | 73 | 8.3 | < 9.4 | Table II |

EDWARD C. PICKERING.

January 24, 1901.

## THE SPECTRUM OF ζ *PUPPIS.*[1]

THE presence of a second series of hydrogen lines, in addition to the ordinary series, in the spectrum of ζ *Puppis,* was announced in *Circulars* Nos. 12, 16, and 18. Accurate wave-lengths could not then be determined for the less refrangible lines. Since then, measures have been made of six photographs of spectra of ζ *Puppis,* and two of spectra of δ *Orionis.* The measurements have been made by Miss F. Cushman, and the conversion into wave-lengths, by Mr. Edward S. King, with the assistance of Miss Cannon. This work will be pub-

[1] *Harvard College Observatory Circular* No. 55.

lished in full in a later volume of the *Annals*. Following the notation proposed by Vogel for the ordinary series of hydrogen lines, the new series may be designated, $H\alpha'$, $H\beta'$, $H\gamma'$, etc. The great difficulty in determining the wave-length of $H\beta'$ which is approximately 5414, is that no known lines of greater wave-length have been found in the spectrum of $\zeta$ *Puppis*. Accordingly, extrapolation, which is always uncertain, must be employed. The star, $\delta$ *Orionis*, besides the line $H\beta'$, contains also the known line $D_3$, wave-length 5876, which thus permits the wave-length of $H\beta'$ to be determined by interpolation. These lines also occur in $\epsilon$ *Orionis*, and probably in other stars of the so-called *Orion* type. It is greatly to be desired that the spectra of these stars should be photographed with a slit spectroscope attached to a large telescope, as the wave-lengths of these lines could then be determined with far greater accuracy by the help of a comparison spectrum. A comparison of the mean of the wave-lengths given in *Circular* No. 16, with those recently determined, and with two computed values, is given in the following table. The first of these is given in *Circular* No. 16, and is a slight modification of Balmer's formula. It is $3646.1 \frac{n^2}{n^2 - 16}$, in which $n$ is an even number for the ordinary series of hydrogen lines, and an odd number for the additional series. The second formula is $\frac{1}{\lambda} = A + B\frac{1}{m^2} + C\frac{1}{m^4}$. This form was proposed by Kayser, immediately after the announcement of the discovery of this series of lines. If we accept this formula, it would appear to be the true law connecting their wave-lengths, and would render them comparable with those of other elements. The designation of the lines is given in the first column of the following table, the mean of the observed wave-lengths given in *Circular* No. 16 is contained in the second column, and the wave-lengths derived by Mr. King in the third column. The next four columns give the value of $n$, taken from the sixth column of the table in *Circular* No. 16, and computed wave-lengths taken from the seventh column of the same table, and the residuals found by subtracting the computed values from the observed values given in the second and third columns. A similar comparison with the formula $\frac{1}{\lambda} = 27461 - 121790\frac{1}{m^2} - 352010\frac{1}{m^4}$, is contained in the last four columns of the table.

| Des. | Cir. 16 | Obs. | $n$ | Comp. | O–C | O–C | $m$ | Comp. | O–C | O–C |
|---|---|---|---|---|---|---|---|---|---|---|
| $H\alpha'$ | ...... | ...... | 5 | 10128.1 | ..... | ..... | 3 | 10435.2 | ..... | ..... |
| $H\beta'$ | ...... | 5413.6 | 7 | 5413.9 | ..... | −0.3 | 4 | 5413.0 | ..... | +0.6 |
| $H\gamma'$ | ...... | 4542.4 | 9 | 4543.6 | ..... | −1.2 | 5 | 4540.1 | ..... | +2.3 |
| $H\delta'$ | 4200.6 | 4200.7 | 11 | 4201.7 | −1.1 | −1.0 | 6 | 4200.6 | 0.0 | +0.1 |
| $H\epsilon'$ | 4026.3 | 4026.0 | 13 | 4027.4 | −1.1 | −1.4 | 7 | 4027.5 | −1.2 | −1.5 |
| $H\zeta'$ | 3924.8 | 3924.0 | 15 | 3925.2 | −0.4 | −1.2 | 8 | 3925.8 | −1.0 | −1.8 |
| $H\eta'$ | 3858.6 | 3860.8 | 17 | 3859.8 | −1.2 | +1.0 | 9 | 3860.6 | −2.0 | +0.2 |
| $H\theta'$ | 3816.0 | 3815.7 | 19 | 3815.2 | +0.8 | +0.5 | 10 | 3815.4 | +0.6 | +0.3 |
| $H\iota'$ | 3783.4 | ...... | 21 | 3783.4 | 0.0 | ..... | 11 | 3782.1 | +1.3 | ..... |
| | ...... | ...... | ∞ | 3646.1 | ..... | ..... | ∞ | 3641.5 | ..... | ..... |

It will be noticed that the two computed values do not differ very greatly for any of the observed lines. The difference is greatest for $H\gamma'$, and here the observed value is nearly midway between the two computed values. On the whole, the observed values agree more nearly with the first formula, than with the second. This is remarkable, if it does not represent the true law, since this formula contains no arbitrary constants. There is only one constant, and that is determined with great accuracy from the ordinary series of hydrogen lines. The second formula contains three arbitrary constants which are selected so as to represent the observed values as nearly as possible. A least square determination was not considered necessary, since the outstanding differences from observation were evidently systematic, and not accidental. The wave-length when $m$ or $n$ is infinite, could be accurately measured, but unfortunately these lines, like those of the ordinary series, do not appear to be present in the stars. The wave-length of the line $H\alpha'$ differs greatly according to the two formulæ, but no means as yet exist for determining radiations of such great wave-length in a star.

EDWARD C. PICKERING.

CAMBRIDGE, U. S.
February 11, 1901.

## *NOVA PERSEI*, NO. 2.[1]

THE early observations made here, of the new star in *Perseus*, are described in *Circular* No. 56. This star may be designated *Nova Persei*, No. 2, to distinguish it from the star in R. A. $1^h\ 55^m\!.1$, Dec. $+56°\ 15'$, which appeared in this constellation in 1887. A photograph

[1] *Harvard College Observatory Circular* No. 57.

of the vicinity of the *Nova*, taken with the Cooke lens on February 19, 1901, with an exposure of $66^m$, beginning at $11^h$ $18^m$ G. M. T., is shown in Fig. 1. For comparison, a similar photograph taken on February 26, 1901, with an exposure of $56^m$, beginning at $14^h$ $32^m$ G. M. T., and showing the *Nova*, is given in Fig. 2.[1]

The accompanying plate gives enlargements, made with a moving plate, of three photographs of spectra taken with the 11-inch Draper telescope. The first represents the new star in *Perseus* taken on February 22, 1901, with an exposure of $40^m$, beginning at $16^h$ $08^m$ G. M. T. The second represents the same star taken on February 24, 1901, with an exposure of $66^m$, beginning at $14^h$ $43^m$ G. M. T. The change from the first form of spectrum to the second must have been very sudden. A plate taken through dense clouds on February 23, with an exposure of $29^m$, beginning at $11^h$ $37^m$ G. M. T., showed but little change, while Professor Vogel has announced that a photograph taken the same night shows a spectrum traversed by several broad, hazy bands. The third spectrum represents *Nova Aurigæ*, taken on February 5, 1892, with an exposure of $123^m$, beginning at $11^h$ $09^m$, G. M. T. It will be seen that the second and third spectra closely resemble each other, but that the lines in *Nova Aurigæ* are much narrower and more sharply defined. The later photographs of the spectrum of the new star in *Perseus* show numerous changes, the dark lines and the edges of the bright lines being, in many cases, well defined.

EDWARD C. PICKERING.

March 15, 1901.

## SECOND CHART AND CATALOGUE FOR OBSERVING *NOVA PERSEI*.[2]

THIS second chart and catalogue have been prepared for telescopic observation of the *Nova*. They were both made on the plan of the *Atlas Stellarum Variabilium* (Series III), except that the new star is placed 15′ south of the center of the chart, on account of some bright stars north of the *Nova*, which will be needed for comparison.

An *auxiliary* chart was added to the principal one, to enable those observers who have no circles attached to their telescopes to find the *Nova*. They will experience little difficulty in first setting on δ *Persei*,

[1] These figures are here omitted.— EDS.

[2] The first chart prepared by the Georgetown College Observatory was intended for naked-eye observations of the *Nova*.— EDS.

which is of third magnitude, then passing to $\psi$ and $\sigma$, and from there sweeping directly south along the stars indicated on the auxiliary chart, until they reach the stars Nos. 2 and 3 of the principal chart.

On this chart three divisions or squares may be distinguished as regards the *density* of the stars. The large square measures one degree in each coördinate, and contains all the *B. D.* stars (except $+43^{\circ}.745$, which is of 10.5 magnitude). In a smaller square, which is 30′ wide and whose center is the *Nova*, all stars have been entered which are visible in our twelve-inch refractor with a power of about fifty diameters, while in the central square of only 10′ width fainter stars have been added by means of a magnifying power of about 100 diameters. Great difficulty was experienced in seeing and measuring these objects on account of the brightness of the new star.

The *positions* of all the stars within the square 30′ wide have been determined by means of a semicircular glass scale, divided into parts 3′ wide, and of a chronograph. These positions are differential with regard to the *Nova* as zero point. From the preface to the *Atlas*, in which the details of this method are described, we only recall the statement that the Right Ascensions are supposed to be correct within $1^s$, while the Declinations may be erroneous by $0'.3$ or even $0'.6$. The positions of the stars outside this square have been computed from the *B. D.* or the *A.G.C.*, assuming the position of the *Nova*, given in the title of the chart, to be correct for the beginning of the year 1901.

The *magnitudes* of all the stars on the chart were determined by sequences of steps based on at least two independent estimates. The formula for transforming these steps to the *B. D.* scale is of the same character as those in the *Atlas*. It will be an easy matter to replace it by any other process of transformation, graphical or arithmetical, when a scale of standard magnitudes is determined by photometric means. The faintest stars near the *Nova* could not be properly estimated on account of the brightness of the latter.

It is well to remark that only three good nights (March 7, 14, 16) were available for making this chart, and that then the *Nova* was brighter than fourth magnitude. Yet it has seemed better to distribute chart and catalogue as they are without delay, in order to facilitate observation. A more accurate scale of magnitudes, for the final reduction of the observations, may be determined at any future time, when the brightness of the new star shall have faded away.

## CHART II.

*NOVA PERSEI.*

$3^h\ 24^m\ 28^s; +43°\ 33'.9.$

| No. | Steps | Mag. | *B. D.* | | Δα | Δδ | Notes |
|---|---|---|---|---|---|---|---|
| 1 | 0 | $7^m.1$ | $7^m.0$ | +43° 730 | $-2^m\ 55^s$ | +27'.9 | *H. P.* $6^m.9$; red. |
| 2 | 10 | 7.4 | 6.5 | 44 734 | +1 22 | +57.3 | 6.5 |
| 3 | 17 | 7.5 | 7.5 | 44 732 | +1 0 | +56.3 | 7.3 |
| 4 | 21 | 7.6 | 7.3 | 44 742 | +3 25 | +54.4 | 7.8 |
| 5 | 23 | 7.7 | 7.5 | 43 732 | −2 37 | − 9.4 | 7.3 |
| 6 | 25 | 7.8 | 7.6 | 43 720 | −4 15 | −15.5 | |
| 7 | 31 | 7.9 | 8.0 | 43 766 | +4 7 | − 2.8 | |
| 8 | 37 | 8.1 | 8.4 | 43 728 | −3 10 | +28.7 | |
| 9 | 42 | 8.2 | 8.5 | 44 717 | −0 41 | +55.6 | |
| 10 | 46 | 8.3 | 8.9 | 43 726 | −3 29 | +15.8 | |
| 11 | 49 | 8.4 | 8.6 | 43 744 | +0 50 | −13.8 | |
| 12 | 52 | 8.5 | 8.7 | 43 729 | −3 10 | +25.7 | |
| 13 | 55 | 8.6 | 8.9 | 44 712 | −2 32 | +44.3 | |
| 14 | 59 | 8.7 | 9.1 | 44 741 | +3 20 | +41.3 | |
| 15 | 59 | 8.7 | 9.1 | 43 723 | −3 51 | −11.4 | |
| 16 | 61 | 8.7 | 8.9 | 44 721 | +0 4 | +45.0 | |
| 17 | 63 | 8.8 | 8.8 | 43 740 | −0 7 | +18.3 | |
| 18 | 65 | 8.8 | 9.1 | 43 746 | +1 10 | −22.8 | |
| 19 | 65 | 8.8 | 9.0 | 44 724 | +0 23 | +37.8 | |
| 20 | 67 | 8.9 | 9.0 | 43 739 | −0 21 | + 4.8 | |
| 21 | 70 | 9.0 | 9.0 | 43 749 | +1 41 | − 8.4 | |
| 22 | 70 | 9.0 | 9.0 | 43 751 | +1 50 | −17.9 | |
| 23 | 72 | 9.0 | 9.1 | 43 733 | −2 29 | − 3.9 | |
| 24 | 73 | 9.0 | 9.0 | 43 748 | +1 34 | +20.1 | |
| 25 | 73 | 9.0 | 9.1 | 43 731 | −2 49 | + 4.9 | |
| 26 | 75 | 9.1 | 9.0 | 43 758 | +3 1 | − 9.3 | |
| 27 | 75 | 9.1 | 9.0 | 43 742 | +0 37 | +29.9 | |
| 28 | 79 | 9.2 | 9.0 | 43 757 | +2 39 | − 2.9 | |
| 29 | 79 | 9.2 | 9.1 | 43 741 | +0 28 | +19.4 | |
| 30 | 80 | 9.2 | 9.2 | 43 759 | +3 4 | +22.5 | |
| 31 | 82 | 9.3 | 9.1 | 43 752 | +1 58 | +15.0 | |
| 32 | 84 | 9.4 | 9.2 | 43 743 | +0 43 | + 8.7 | |
| 33 | 89 | 9.5 | 9.4 | 43 760 | +3 20 | + 3.2 | |
| 34 | 92 | 9.6 | 9.5 | 43 735 | −1 5 | + 3.4 | |
| 35 | 93 | 9.6 | 9.5 | 43 755 | +2 27 | + 4.4 | |
| 36 | 94 | 9.6 | 9.5 | 43 734 | −2 9 | +16.6 | |
| 37 | 96 | 9.7 | 9.5 | 43 738 | −0 28 | −12.3 | |
| 38 | 102 | 9.8 | 9.5 | 43 737 | −0 46 | − 9.6 | |
| 39 | 105 | 9.9 | 9.5 | 43 756 | +2 36 | − 2.7 | |
| 40 | 107 | 10.0 | 9.5 | 43 753 | +2 21 | +20.1 | |
| 41 | 108 | 10.0 | 9.5 | 43 750 | +1 47 | −10.2 | |
| 42 | 111 | 10.1 | | | +0 35 | + 0.2 | |
| 43 | 117 | 10.2 | | | −1 32 | + 3.0 | |
| 44 | 120 | 10.3 | | | 0 0 | −10.9 | |
| 45 | 124 | 10.4 | | | −1 24 | + 9.3 | |

$$\text{Mag.} = 9.0 + 0.027\ (\text{St.} - 71.1).$$

CHART II. — *Continued.*

*NOVA PERSEI.*

$3^h\ 24^m\ 28^s$; $+43°\ 33'.9$.

| No. | Steps | Mag. | BD. | $\Delta\alpha$ | $\Delta\delta$ | Notes |
|---|---|---|---|---|---|---|
| 46 | 133 | $10^m.7$ | | −1 22 | + 6.3 | |
| 47 | 134 | 10.7 | | +0 54 | +11.9 | |
| 48 | 138 | 10.8 | | +1 1 | − 5.0 | |
| 49 | 145 | 11.0 | | −0 24 | − 1.5 | |
| 50 | 148 | 11.1 | | +0 31 | +11.4 | |
| 51 | 153 | 11.2 | | −1 17 | + 3.0 | |
| 52 | 153 | 11.2 | | −1 21 | + 9.4 | |
| 53 | 158 | 11.3 | | −0 32 | +14.4 | |
| 54 | 159 | 11.4 | | −0 21 | − 3.9 | |
| 55 | 160 | 11.4 | | +0 36 | −14.4 | |
| 56 | 161 | 11.4 | | −0 17 | + 2.4 | |
| 57 | 167 | 11.6 | | −0 6 | +11.5 | |
| 58 | 168 | 11.6 | | −0 45 | − 0.8 | |
| 59 | 170 | 11.6 | | +1 13 | −12.0 | |
| 60 | 171 | 11.7 | | −0 52 | +14.1 | |
| 61 | 172 | 11.7 | | −1 1 | − 9.0 | |
| 62 | 175 | 11.8 | | +0 3 | +12.6 | |
| 63 | 177 | 11.9 | | −0 49 | +12.3 | |
| 64 | 177 | 11.9 | | +0 45 | − 3.7 | |
| 65 | 178 | 11.9 | | +0 42 | + 4.8 | |
| 66 | 179 | 11.9 | | +0 33 | − 4.1 | |
| 67 | 182 | 12.0 | | −0 58 | − 8.3 | |
| 68 | 185 | 12.1 | | +0 34 | + 1.5 | |
| 69 | | (12) | | +0 3 | − 0.5 | elongated.[1] |
| 70 | 186 | 12.1 | | +1 11 | −10.0 | |
| 71 | 193 | 12.3 | | +0 26 | − 8.1 | |
| 72 | 196 | 12.4 | | +1 9 | − 9.7 | |
| 73 | 196 | 12.4 | | −0 7 | − 6.6 | |
| 74 | 199 | 12.4 | | +0 21 | + 1.6 | |
| 75 | 200 | 12.5 | | +0 17 | −12.0 | |
| 76 | 205 | 12.6 | | +0 20 | −11.2 | |
| 77 | 209 | 12.7 | | +0 13 | + 1.0 | |
| 78 | 212 | 12.8 | | −0 14 | + 1.6 | |
| 79 | 212 | 12.8 | | +0 13 | − 0.7 | |
| 80 | | (13) | | −0 3 | − 0.5 | |
| 81 | | (13-14) | | +0 9 | − 0.4 | |

Mag. = 9.0 + 0.027 (St. − 71.1).

JOHN G. HAGEN, S. J.

WASHINGTON, D. C.,
March 19, 1901.

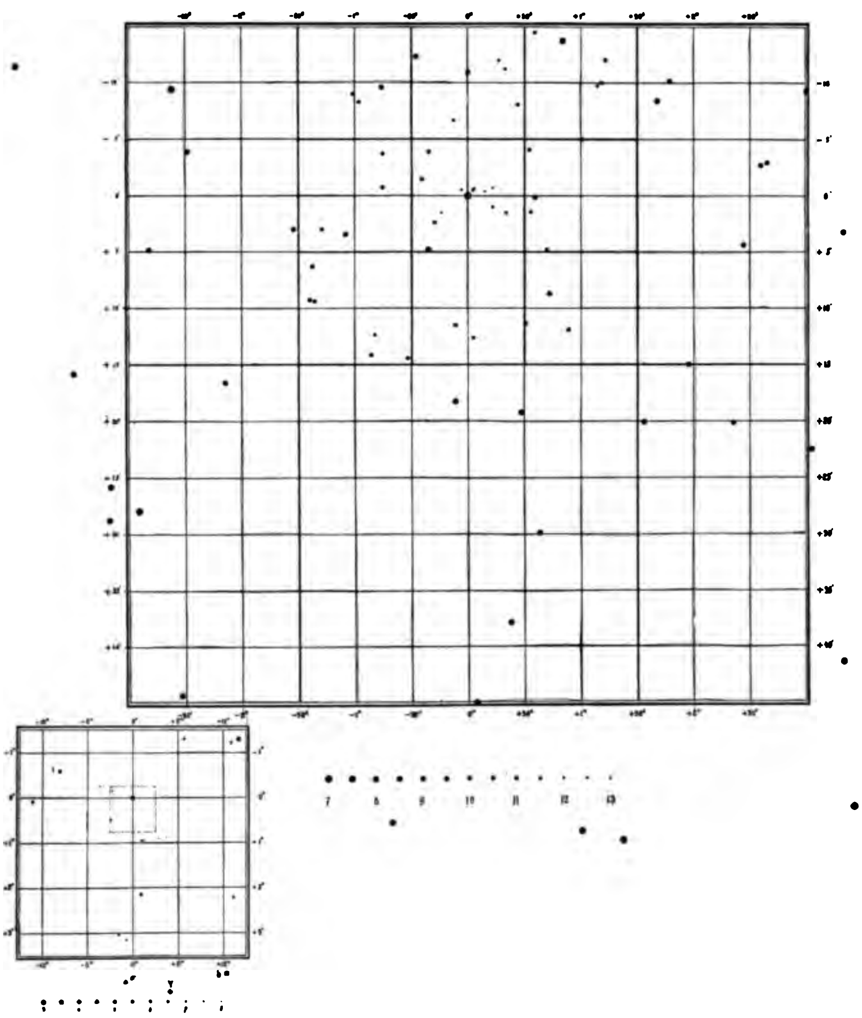
[1] Professor T. J. J. See kindly examined this object with the 26-inch refractor and found it to consist of two stars which he estimated as 14th magnitude each.

GEORGETOWN COLLEGE OBSERVATORY

## Chart II

# Nova Persei

$3^h\ 24^m\ 28^s$; $+43°\ 33'.9$



## CHANGES IN THE SPECTRUM OF *NOVA PERSEI*.[1]

Since the publication of *Bulletin* No. 16 the weather has been unusually cloudy and comparatively few observations of *Nova Persei* have been secured. The following observations have been obtained by Mr. Parkhurst. With the exception of those marked "vis." they were all made with the wedge photometer referred to in the last *Bulletin*.

| | | Gr. M. T. | Mag. |
|---|---|---|---|
| 1901 | Feb. | 25.52 | 1.0 vis. |
| | | 26.52 | 1.1 vis. |
| | | 27.56 | 2.0 vis. |
| | | 28.58 | 1.90 |
| | Mar. | 3.56 | 2.71 |
| | | 4.58 | 2.8 vis. |
| | | 5.54 | 2.71 |
| | | 6.56 | 3.08 |
| | | 7.58 | 3.2 vis. |
| | | 11.56 | 3.62 |
| | | 12.56 | 3.30 |

| | | Gr. M. T. | Mag. |
|---|---|---|---|
| 1901 | Mar. | 14.56 | 3.5 vis. |
| | | 15.56 | 3.72 |
| | | 16.6 | 3.69 |
| | | 17.56 | 3.50 |
| | | 22.59 | 4.08 |
| | | 31.58 | 4.20 |
| | Apr. | 3.60 | 5.39 |
| | | 8.58 | 4.14 |
| | | 9.56 | 4.26 |
| | | 10.58 | 5.48 |

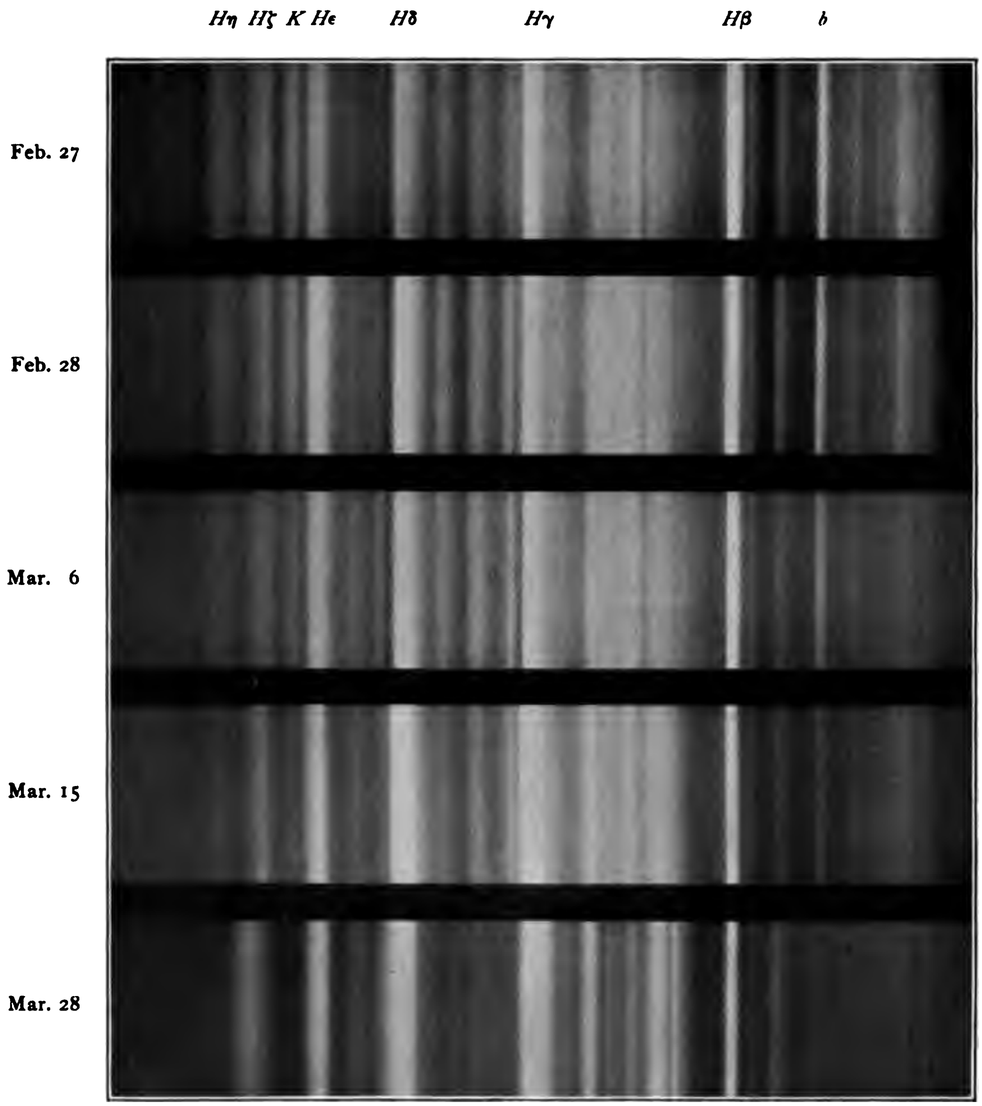
The magnitudes are based on the system of the Harvard *Photometric Durchmusterung* (*H. C. O. Annals*, XLV). There seems to be no doubt of the reality of the considerable fluctuations shown, and from the internal agreement of the measures it is probable that the amount of these fluctuations is represented within one or two tenths of a magnitude.

Since the publication of the previous list Mr. Ellerman has obtained photographs of the spectrum as follows:

| Date | No. of Plates | Dispersion | Region |
|---|---|---|---|
| 1901 March 15 | 3 | 3 prisms | D to 4400 |
| 15 | 1 | 3 prisms | 5000 to 4200 |
| 15 | 2 | 1 prism | 5700 to 3700 |
| 22 | 2 | 3 prisms | D to 4400 |
| 22 | 1 | 1 prism | 5700 to 3700 |
| 28 | 1 | 3 prisms | D to 4400 |
| 28 | 1 | 1 prism | 5700 to 3700 |
| 31 | 1 | 3 prism | D to 4400 |
| April 3 | 2 | 1 prisms | 5700 to 3700 |
| 8 | 1 | 3 prisms | D to 4400 |
| 8 | 2 | 1 prism | 5700 to 3700 |
| 9 | 1 | 1 " | 5700 to 3700 |
| 10 | 1 | 1 " | $H\alpha$ to 4000 |

[1] *Yerkes Observatory Bulletin* No. 17.

PLATE V




SPECTRUM OF *NOVA PERSEI*

PHOTOGRAPHED WITH THE 40-INCH YERKES TELESCOPE BY FERDINAND ELLERMAN

The changes which have taken place are well illustrated in the plate, which is a reproduction of direct photographic enlargements from the original negatives. On account of the cloudy weather the intervals between some of the photographs are so great that the progressive change is not always fully indicated. In general it will be noticed that the brightness of the spectrum has decreased in both the yellow and the ultra-violet. The decreased intensity of the less refrangible region is particularly striking. It should be remarked that most of the photographs reproduced in the plate were shaded in the process of enlargement so as to bring out the details as well as possible in both the brighter and the fainter regions. For this and other obvious reasons the plate must not be taken to represent the relative intensities of the various portions of the spectrum. If the yellow region in the later photographs had not been shaded in copying it would have been entirely lost in the reproduction on account of its great relative faintness.

In addition to this falling off in intensity in the less refrangible region, the continuous spectrum as a whole has become much fainter relatively to the bright lines. Exception must apparently be made of the spectrum as photographed April 8, unfortunately too late for reproduction on the plate. On this evening, as the photometric measures show, the *Nova* was brighter than on April 3. This increase in magnitude was accompanied by marked brightening of the continuous spectrum relatively to the bright lines. The $H\zeta$ line, which on the photograph of March 28 seems to have shifted toward the violet, is shown on the photograph of April 8 in its original position, but greatly decreased in intensity.

There have been many changes of importance in the relative intensities of the bright lines. The *b* line, which was so conspicuous in the earlier photographs, has greatly decreased in intensity; the K line of calcium has undergone a similar change in brightness and now seems to have disappeared entirely; $H\beta$ has become narrower and sharper and the relative intensities of its several components have undergone marked variations. The plate will serve to show in a general way the changes which the other lines have experienced. The spectrum photographed on March 28 is in some respects the most remarkable of the series on account of the apparent shifting of several of the hydrogen lines and the rise into prominence of lines which were previously inconspicuous.

The present *Bulletin* is intended merely to call attention to the

more striking changes which have taken place in the spectrum of the *Nova*. Illustrations of other changes will be published later. It should be added, however, that these changes include a duplication of the dark lines on the more refrangible edge of the bright hydrogen lines; these were at first rather broad and poorly defined, but subsequently, March 15, became sharp double lines; they have recently become much fainter and are no longer double.

Special attention has been given to the two dark D lines and the bright band upon which they were projected. This bright band has gradually moved toward the violet so that the two narrow dark lines which were at first nearly central on the band (see Fig. 1, Plate III, *Bulletin* No. 16) are now at its less refrangible edge. The dark line on the more refrangible edge of this band, which in *Bulletin* No. 16, was provisionally designated $D_8$ has given place to a much broader, but fainter, band extending toward the violet.

Various laboratory investigations on the spectrum of the spark and arc in air and in certain liquids will be described in a later paper. With the arc taken in air between carbon poles moistened from time to time with a solution of sodium hyposulphite, the appearance of the sodium lines is almost precisely like that presented by the *Nova* on February 28. The narrow dark lines due to the absorption of the cooler sodium vapor in the outer part of the arc are superposed upon a very broad bright band like that in the spectrum of the *Nova*. The experiments with the arc and spark will be continued with more powerful apparatus.

April 11, 1901.

GEORGE E. HALE.

## ULTRA-VIOLET CORONAL LINES.

IN the number of this JOURNAL for November 1900 which came to hand only last week I find M. Deslandres announcing the discovery of "two complete rings due to two new coronal radiations" in the ultra-violet as a result of his observations of the total solar eclipse of May last (page 288).

I hasten therefore to state that these two ultra-violet rings were obtained by me during the Indian eclipse of 1898 with a prismatic camera composed of two spar prisms of 60° angle and about 1 inch face and a single quartz lens of about 24 inches focus. As a *first approximation* only, the wave-lengths of the radiations came out 3456 and 3391, but I hope very soon to obtain a more correct determination.

K. D. NAEGAMVALA.

POONA, March 1, 1901.

PLATE VI

HENRY AUGUSTUS ROWLAND

THE

# ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XIII MAY, 1901 NUMBER 4

## HENRY AUGUSTUS ROWLAND.

By JOSEPH S. AMES.

IN the death of Professor Rowland, of Johns Hopkins University, the science of Astrophysics has lost its foremost investigator and its greatest authority. It may not be too much to assert that the modern study of spectroscopy as an exact science dates from the beginning of Rowland's work; and the fact that so much has been accomplished during the past twenty years by students both at home and abroad bears witness to the impulse given to research and investigation by the force of his example and by the assistance of instruments furnished by his genius. Before noting in detail, however, what were the main contributions of Professor Rowland to the science to which this JOURNAL is devoted, it may be well to record the leading features of his life.

Henry Augustus Rowland was born at Honesdale, Penn., November 27, 1848, the son of a Presbyterian minister. He received a good secondary education and entered the Rensselaer Polytechnic Institute, Troy, N. Y., from which he graduated in 1870, receiving the degree of "Civil Engineer." All of his inclinations as a boy were toward scientific work. He read with the greatest interest all the writings of Faraday and Tyndall,

and performed many chemical and physical experiments. He formed the habit at an early day of keeping accurate record not only of his observations, but also of the ideas which occurred to him from time to time. These notebooks are intensely interesting, and when Rowland's life is written, as it surely will be, they will throw the clearest light on his whole mental development. In these books are found numerous suggestions of investigations undertaken later by himself or others.

After a short experience, first as a member of a corps of railway engineers and then as teacher of general science in Wooster College, Ohio, Rowland returned to Troy as an instructor in the Polytechnic Institute, and was soon promoted to be assistant professor of physics. It was while at Troy that he performed his classical experiments on the study of the magnetization of iron. These established results and methods of investigation which are of fundamental importance. When Rowland had prepared for publication an account of his observations and theories, he sent it to an American journal; but it was returned as not suited for publication, the suggestion being made that the research was a rather presumptuous one for an unknown man to undertake, and that the paper would be improved if the theoretical part was removed. Rowland thereupon sent the manuscript to Professor Clerk Maxwell, of Cambridge, who acknowledged its receipt by saying he regarded it as of such importance that he had forwarded it at once to the *Philosophical Magazine*. One further incident in connection with this research, and one most characteristic of the man, should be recorded. He had little if any apparatus at his disposal and no laboratory rooms; so he constructed his own instruments and used the window seats of his bedroom for his laboratory piers.

In the spring of the year 1875 Mr. Gilman, who had just been elected president of Johns Hopkins University, was an official visitor at the United States Military Academy at West Point; and in talking one day to Professor Michie of that institution President Gilman mentioned the fact that he was looking for a man to take charge of the work in physics in the university

at Baltimore. Professor Michie was a connection of Rowland by marriage and was well acquainted with his work at Troy. At the former's suggestion Rowland was requested by President Gilman to come at once to West Point; and it was there that the two men met whose names are so indissolubly connected with the history of Johns Hopkins University and of education in America. Rowland's record at Troy as a teacher was not of the best, but President Gilman saw in him those qualities which were most needed as the director of a great physical laboratory. On his recommendation, therefore, the trustees of Johns Hopkins University invited Rowland to join their faculty, and gave him leave of absence for a year in which to go to Europe, with the idea that he should spend this time partly in buying apparatus and partly in becoming better acquainted with the laboratories of physics abroad and with their methods of work. Rowland spent the winter in Berlin, and took advantage of the facilities of Helmholtz's laboratory to perform his electrical convection experiment which established the fact that an electrical charge, if carried at a high speed, has the same magnetic action as an electric current. He then traveled from city to city on the continent purchasing apparatus—not for purposes of demonstration, but for use in researches and investigations: measuring apparatus of all kinds, standards of length and electrical resistance, etc. He returned to America in 1876, and began his work at Baltimore in the autumn as professor of physics.

Rowland's attention was first drawn to practical problems connected with electricity and to some theoretical questions which arose in connection with his lectures to his classes. He made a redetermination of the ohm; he measured the ratio of the electrical units; he investigated electric absorption of crystals; the Hall-effect was discovered. He then began his classical investigation on the subject of the "mechanical equivalent of heat," which resulted in his obtaining a prize offered by the Venetian Institute and also the Rumford medal given by the American Academy of Arts and Sciences. In this great research he made the first thorough study of the scientific

principles involved in the measurement of temperature and quantities of heat.

About 1881 he became interested in the subject of spectrum analysis, and realizing that the most important part of the necessary apparatus was the diffraction grating, he set himself the task of constructing a dividing-engine to be used for ruling gratings. At this time the best gratings available were those made by Mr. Rutherfurd, of New York; but they were far from being satisfactory. In the course of an investigation on the absolute wave-length of light, in which he used Rutherfurd gratings, Dr. C. S. Peirce had made a special study of the "ghosts," proving that they were due to periodic errors in the ruling of the grating. There were other errors, too, owing to the lack of uniformity in the pitch of the screw. In the dividing engine, as designed by Professor Rowland, both these sources of error were avoided as far as possible. He invented a process for cutting a screw which secured one of nearly perfectly constant pitch, and which is fully described in his article on the "Screw" in the *Encyclopædia Britannica*. He designed a method also for so moving the nut of the screw independently of the motion of the screw as would allow any periodic error—due to irregularities at the two ends of the screw—to be "corrected." This first dividing engine was arranged so as to rule 14438 lines to an inch; but machines constructed later ruled 20000 and 15000 to the inch. Having a ruling engine now at his disposal, the idea occurred to him to consider the effect of ruling the lines on a curved surface instead of a plane one, such as had always been used in the past. He attacked the problem mathematically, and discovered that a grating ruled on a spherical concave surface would have certain most distinctive properties: the spectrum could be maintained "normal" by a simple form of mounting, and always in focus along a certain line, without the use of lenses; the spectrum would be astigmatic; dust in the slit would cause no difficulty. Concave gratings were immediately made and were found to be perfectly satisfactory. When his own laboratory was supplied with the necessary gratings, others were ruled and were

distributed throughout the world. The immediate result was a most wonderful development of the science of spectroscopy and its applications.

Rowland himself had two great ideas or projects in regard to the use of the grating; to map the solar spectrum and to make a careful study of the metallic spectra. He saw the importance of having accurate measurements of both solar and metallic lines so as to serve as standards for reference and as means for verifying various solar or stellar theories; but the main reason for his interest in this prolonged investigation which lasted over so many years lay in the belief that by means of its conclusions some definite ideas might be deduced as to the nature of the molecules of matter. The measurement of spectrum lines as such was without the slightest interest to him; it was only when these measurements could be used in connection with some fundamental theory that they possessed value. "Where is the Kepler for molecules?" he would often say. It is hardly necessary to recall to the readers of this JOURNAL what Professor Rowland did for the science of spectroscopy in addition to the invention of the concave grating: the preparation of the wonderful map of the solar spectrum; the measurement of tables of "standard" lines, both solar and metallic, the measurement of the arc-spectra of the elements; the first theory of the diffraction grating which took into account the effect of the shape of the groove and the complete action of periodic errors.

It is well to recall, too, the difficulties under which Professor Rowland labored when he began his spectrum work, and the way he overcame them. He made his concave grating and its mounting, but was at first obliged to confine himself to eye-measurements. The art of photography was in its infancy. He had to flow his own plates and to learn how to sensitize them for different portions of the spectrum and how to develop them properly. This he did with great success. He further had to make a complete study of absorbing liquids so as to prevent the objectionable effects of overlapping spectra.

During the past few years Professor Rowland was interested

largely in the theory of alternating electric currents and in their application to motors, measuring instruments, and in particular to a multiplex printing telegraph system which achieved a most striking success at the Paris Exposition of 1900.

Only a word need be said in regard to the last days of his life. He was ill from a nervous disorder during the months of January and February, but then recovered sufficiently to return to his duties and work at the laboratory. After a few weeks, however, he was again confined to his home by a trivial illness, when suddenly he was taken critically ill and died within twenty-four hours, on the morning of Tuesday, April 16. He had known for more than ten years that his end would come as it did; and the realization of this fact was always in his mind. The manner in which he hid it from others and went on his life's way was but one of many illustrations of his self-control and bravery.

Professor Rowland's services to science were recognized both at home and abroad, as is shown by the list of honors which came to him. Among the societies to which he was elected are these:

FOREIGN

The British Association for the Advancement of Science.
The Physical Society of London.
The Philosophical Society of Cambridge, England.
The Royal Society of London.
The Royal Society of Göttingen.
The Gioenian Academy of Natural Sciences, Catania, Sicily.
The French Physical Society.
The French Academy of Sciences.
The Literary and Philosophical Society of Manchester.
The Royal Lyncean Academy, Rome.
The Academy of Sciences, Stockholm.
The Italian Society of Spectroscopists.
The Royal Society of Edinburgh.
The Society of Arts, London.
The Royal Astronomical Society of England.
The Royal Society of Lombardy.
The Royal Physiographic Society of Lund.
The Royal Academy of Sciences, Berlin.

AMERICAN

The American Philosophical Society, Philadelphia.
The American Academy of Arts and Sciences, Boston.
The National Academy of Sciences, Washington.
The American Physical Society — its first president.

His academic degrees were these:

Civil Engineer (C. E.), Rensselaer Polytechnic Institute, 1870.
Doctor of Philosophy (hon.) (Ph.D.), Johns Hopkins University, 1880.
Doctor of Laws (LL.D.), Yale University, 1895.
Doctor of Laws (LL. D.), Princeton University, 1896.

Among other distinctions may be named:

Officer of the Legion of Honor of France.
Rumford Medallist of the American Academy of Arts and Sciences.
Draper Medallist of the National Academy of Sciences.
Matteucci Medallist (Italian).
Recipient of the prize of the Venetian Institute, for his work on the Mechanical Equivalent of Heat.

Delegate from the United States government to the

International Congress of Electricians, Paris, 1881.
International Congress for the Determination of Electrical Units, Paris, 1882.
Electrical Congress, Philadelphia, 1884 — President.
International Chamber of Delegates for the Determination of Electrical Units, Chicago, 1893 — President.

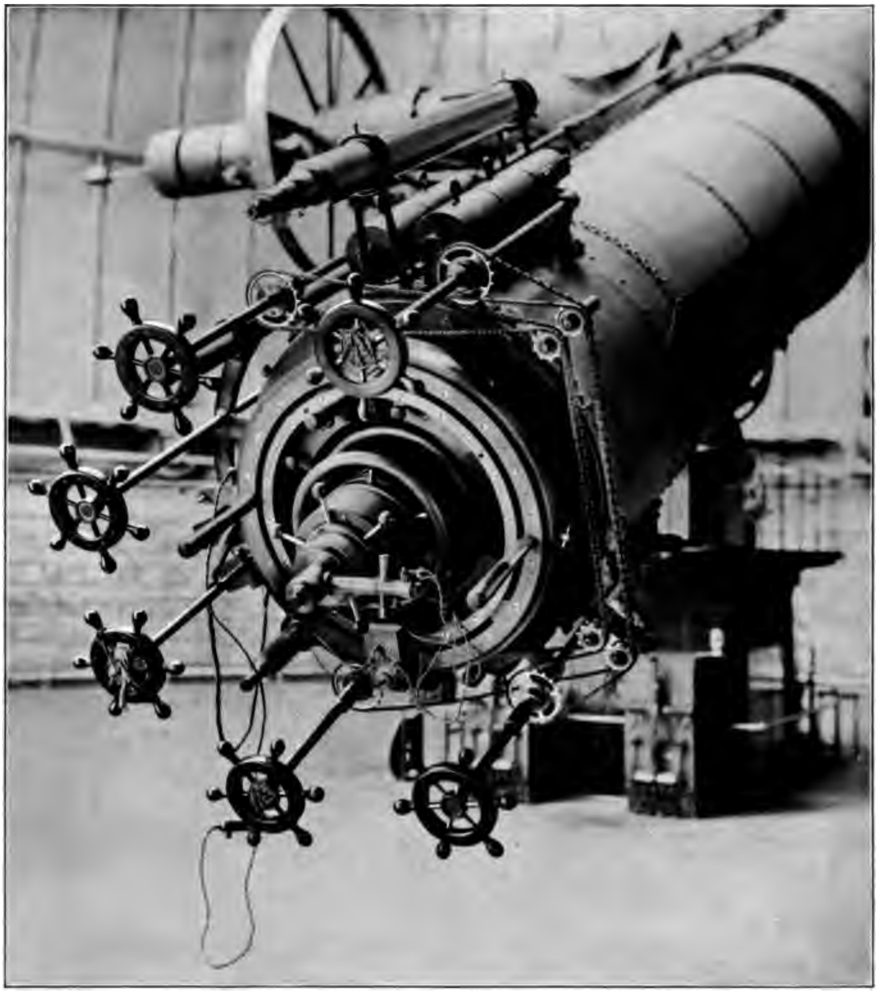
Even if one takes into account the inventions and discoveries of Professor Rowland and his many scientific researches, it is not upon these alone, or even in the main, that his reputation and renown rest; they are not his greatest gift to the world. Much more important than any of his individual pieces of work was his influence on his generation by his spirit, his aims, and through the many students and associates who came to know and appreciate him. It is quite impossible to estimate the effect he has had on all branches of science, both theoretical and practical. His most striking qualities of mind were clear vision, absolute self confidence, simplicity, generosity, moral courage. His intuitive knowledge of physical laws was simply marvelous; and his assurance in his own judgment was complete. It was largely

owing to these qualities that he accomplished what he did. The suggestions as to theoretical or experimental work which he offered in the course of his daily lectures were always of the greatest help to his students; and his criticisms of their work or of that of others were even more so. He was continually inspiring his students to aim at the highest ideals, not to be satisfied with ordinary things. The great simplicity and truthfulness of his character made him beloved by every one who came near him. He was generous to a fault in his life as a citizen and always took most seriously his civic duties.

He was interested intensely in everything pertaining to the improvements of his city.

The loss to science in America, occasioned by his death, cannot be estimated, it can be only dimly felt. It is a personal one to everyone who worked with him or who knew him and even to those who have merely seen him. There is no one to take his place.

PLATE VII



STELLAR PHOTOMETER ATTACHED TO 40-INCH YERKES TELESCOPE

# DETERMINATION OF THE WEDGE CONSTANT OF A STELLAR PHOTOMETER.

By J. A. PARKHURST.

IN pursuance of a plan for coöperation in determining standards for faint stellar magnitude, Professor E. C. Pickering sent to the Yerkes Observatory in April 1900, one of the five wedge photometers which he had devised for the work. This was to be used with the 40-inch refractor in the measurement of the faintest stars included in the plan. The construction of the instrument is shown in Fig. 1 and Plate VII, and will need but few words in description. The tube *T*, carrying the ocular *O*, slides into the tailpiece of the telescope. At right angles to this is the tube *C*, carrying the essential parts of the photometer. The light from a 1½ candle-power incandescent lamp *L* shines through a minute hole in the diaphragm *D* upon a piece of ground glass *G*, forming an artificial star. In contact with *G* is a piece of blue glass to render the light of the star less yellow. An image of this star is thrown by the projecting lens *P* upon a plate of plane-parallel glass *B* and reflected from both surfaces into the focus of the ocular *O*, forming at *E* and *F* two images of the artificial star. Interposed in the path of these rays is the photographic wedge *W*, movable at right angles to *C* by the rack and pinion *R*. The short tube carrying the ground glass *G* is movable away from the diaphragm *D* by means of the head of the screw *S*, projecting through an inclined slot in the farther side of the tube *C*. By this means the artificial star can be made larger and less sharply defined, thus resembling more closely a real star under poor atmospheric conditions. Finally, a pair of shade glasses at *A* can be moved, either both or singly, into the path of the rays.

In photometers made on this principle the all-important condition to be fulfilled is that the images of the real and artificial stars should closely resemble each other. The range of

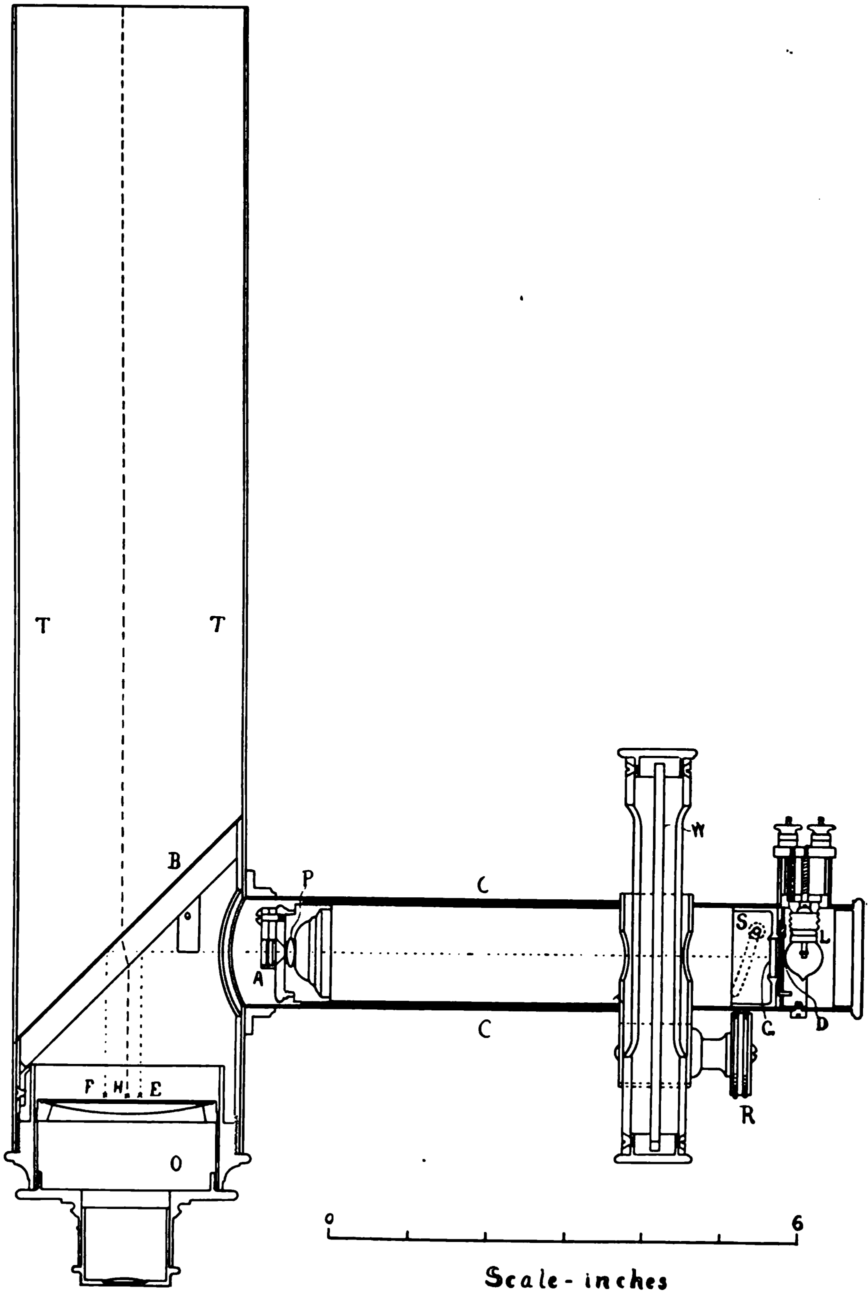



FIG. 1.

adjustment of the ground glass *G* was found to be insufficient to meet this condition with the different telescopes on which the photometer was to be used; therefore the diaphragm *D*, originally provided, which had a single aperture 0.17 mm in diameter, was replaced with a movable diaphragm carrying three apertures, 0.11, 0.20, and 0.30 mm in diameter. By choosing the most suitable aperture and combining with it a slight movement of the ground glass, it was possible to give the disk of the artificial star any required size and sharpness, to suit the various telescopes used, and the different atmospheric conditions.

In order to use the photometer with a 6½-inch reflector and a 2-inch refractor, a smaller tube was provided carrying an ocular and a diagonal reflecting plate to replace *B*. After several trials good images of the real and artificial stars were given by a diagonal plate with surfaces correct to $\frac{1}{10}$ of a wave-length, furnished by Mr. O. L. Petitdidier of Chicago.

The use of this instrument is very simple and convenient. The image of the star to be measured (shown at *H* in the drawing) is brought between the two images of the artificial star and the wedge is moved by the pinion *R* till the light of the real star is matched by *E*, the image formed by reflection from the first surface of the plate *B*. The position of the wedge is then read on a scale divided to twenty-fifths of an inch, tenths of a division being estimated. If now the light of a star of known magnitude be measured, the only unknown quantity is the "constant of the wedge," the value of one division of the scale expressed in magnitudes.

The methods available for determining the wedge constant can be classed under two heads: (1) Measurements of standard stars whose magnitudes have been well fixed; (2) measurements of an artificial star whose light can be reduced by a known amount either by (*a*) polarization, (*b*) a revolving wheel, (*c*) reduced apertures by stationary diaphragms. The last method can be used either with real or artificial stars. The method by standard stars seems to give the best results, as it has the great advantage that the measurements are made under precisely the

same conditions as in actual practice; therefore the main dependence was placed upon it. Under (2) the reduction of apertures by stationary diaphragms seems to give the least reliable results, so was not used. As no proper polarizing photometer was available the choice was restricted to (*b*), the "wheel photometer," consisting of a revolving disk out of which sector-shaped openings were cut.

MEASURES WITH THE "WHEEL PHOTOMETER."

This arrangement consisted of an artificial star formed in a manner similar to the photometer star, by a small incandescent lamp, diaphragm, oiled paper (found to be a good substitute for ground glass), and a piece of the same blue glass used with the photometer lamp, thus insuring stars of the same color. Half an inch from the blue glass was placed the "wheel," a brass disk out of which were cut two opposite sectors, either of which could be covered by black photographic paper. The wheel was turned by clockwork at a speed of about forty revolutions per second. The wheel could be moved out of the path of the rays, giving the light of the wheel lamp unreduced. This position is called "sector off" in Tables I and II, "sector on" indicating that the light was cut down by the revolving wheel. The photometer was placed with the tube *T* pointing at the wheel, and the diagonal plate *B* 30 inches from the wheel. The light of the wheel star was focused by an achromatic lens of 1 inch aperture, inserted in the tube *T*.

Table I gives a specimen set of measures made in the following order (both sectors *E* and *F* being uncovered). With the wheel out of the path of the rays ten settings in the first column were made, then the wheel was turned into the path, the clockwork started and the ten settings in the second column were made, and so on, the quantities recorded being the readings of the wedge scale.

For the accurate values of the sector angles used in the reductions I am indebted to the kindness of Dr. E. S. Johonnott, of the Rose Polytechnic Institute, Terre Haute, Ind., who measured the angles on an excellent circular dividing engine made by the Société Genévoise.

It is evident that the ratio of the unreduced to the reduced light is that of 360° to the combined sector angles, in this case 20° 42′.68, and that this ratio can be expressed in difference of magnitudes by dividing its logarithm by 0.4. The reductions are given in detail at the foot of Table I; the difference in scale readings, 24.03 divisions, corresponding to a difference of 3.100 magnitudes, from which the value of one scale division, which is the wedge constant $C$, is found to be 0.1290 magnitudes.

In Table II are collected the results of thirteen sets of measures made with the "wheel photometer," each quantity in the second and third columns being the mean of ten settings of the wedge. Columns 4 and 5 give the difference between the unreduced and reduced light, expressed in scale divisions and magnitudes, respectively; column 6 gives the resulting value of $C$, which, with equal weights, give the mean value 0.1283 mag, with a probable error of 0.0008 mag.

TABLE I.

SPECIMEN SET OF MEASURES WITH "WHEEL PHOTOMETER."

1900 November 22, sectors E and F.

| | Off | On | Off | On | Off |
|---|---|---|---|---|---|
| | d | d | d | d | d |
| | 16.0 | 36.5 | 13.9 | 41.6 | 15.5 |
| | 16.2 | 39.2 | 15.2 | 40.4 | 17.0 |
| | 14.9 | 40.7 | 17.0 | 40.0 | 16.9 |
| | 16.3 | 40.4 | 16.8 | 41.0 | 16.8 |
| | 17.1 | 39.0 | 15.9 | 41.4 | 17.9 |
| | 16.2 | 41.2 | 18.0 | 41.5 | 16.2 |
| | 17.2 | 41.4 | 15.7 | 39.0 | 15.5 |
| | 16.9 | 40.8 | 15.9 | 41.0 | 15.0 |
| | 16.5 | 40.4 | 17.0 | 40.9 | 16.8 |
| | 16.9 | 39.5 | 16.4 | 41.3 | 16.4 |
| Means..... | 16.42 | 39.91 | 16.18 | 40.81 | 16.40 |

| | Off | On | Sectors E and F |
|---|---|---|---|
| | d | d | |
| Means - | 16.33 | 40.36 | Angle = 20° 42′.68 |

$$\text{Log ratio } \frac{360^\circ}{20^\circ\ 42'.68} = 1.2402$$

$$\frac{\text{Log ratio}}{0.4} = \Delta\ \mathrm{M} = 3.100$$

Result, $\Delta$ d = 24.03, $\Delta$ M = 3.100, $C = 0.1290$ mag.

TABLE II.

RÉSUMÉ OF MEASURES WITH "WHEEL PHOTOMETER."

| Sectors | Means Off | Means On | Δd | ΔM | C | No. of settings |
|---|---|---|---|---|---|---|
| | d | d | | | M | |
| E & F........ | 10.48 | 35.08 | 24.60 | 3.100 | 0.1260 | 30 |
| E & F........ | 16.33 | 40.36 | 24.03 | 3.100 | .1290 | 50 |
| F............ | 16.66 | 45.52 | 28.86 | 3.629 | .1257 | 15 |
| F............ | 14.72 | 43.09 | 28.37 | 3.629 | .1280 | 30 |
| E & F........ | 13.82 | 37.83 | 24.01 | 3.100 | .1291 | 50 |
| E & F........ | 13.67 | 36.38 | 22.71 | 3.100 | .1365 | 50 |
| E............ | 13.70 | 47.20 | 33.50 | 4.136 | .1235 | 50 |
| E & F........ | 13.31 | 36.68 | 23.37 | 3.100 | .1326 | 50 |
| E & F........ | 17.56 | 40.87 | 23.31 | 3.100 | .1330 | 50 |
| E & F........ | 17.89 | 42.29 | 24.40 | 3.100 | .1271 | 50 |
| E & F........ | 15.03 | 38.81 | 23.78 | 3.100 | .1304 | 50 |
| F............ | 14.67 | 43.82 | 29.15 | 3.629 | .1245 | 50 |
| E............ | 15.12 | 48.98 | 33.86 | 4.136 | 0.1222 | 50 |
| Mean ...... | | | | | 0.1283 | |
| P. e........ | | | | | ± 0.0008 | |

CONSTANTS OF SECTORS.

| Sector | Angle | Log ratio $\frac{360°}{\text{Angle}}$ | $\frac{\text{Log ratio}}{0.4}$ |
|---|---|---|---|
| E ............. | 478°.62 | 1.6545 | 4.136 |
| F ............. | 764.06 | 1.1514 | 3.629 |
| E & F ......... | 1242.68 | 1.2402 | 3.100 |

MEASUREMENTS OF STANDARD STARS.

Advantage was taken of Müller and Kempf's excellent "Determination of the Brightness of Ninety-six *Pleiades* Stars,[1] which furnished a conveniently placed list of well-determined standards. It was found by experiment that the best results were obtained by comparing bright stars between magnitudes 6 and 8, with faint ones between magnitudes 9.5 and 10.5. The stars used are collected in Table III, which gives the list number, approximate place for 1900, and magnitude; all taken from Müller and Kempf's list. Table IV is a specimen set of these measures, made with the 6-inch reflector, and the Petitdidier diagonal plate. Four settings were made on a bright star and faint star alternately. In this case the measures began with star

[1] *A. N.*, 150, 193.

No. 25 in M. and K.'s list, and the four settings are given in the first line, followed by "Mean d," the mean of the scale readings; and the star's magnitude according to M. and K. The ninth column gives the difference between the "Mean d" for this star and the faint star in the following line; the tenth column the corresponding difference in magnitude; the eleventh the resulting value of $C$. Column 12 gives the difference in magnitude computed with $C = 0.1334$ mag. (the mean value from the *Pleiades* measures), and the last gives the difference between this and M. and K.'s $\Delta$ M.

The measures of the *Pleiades* stars are collected in Table V, for which the headings of the columns are self-explanatory, except that the last column gives the number of nights on which the pairs were measured. Weighting the values of $C$ according to the number of nights, gives the mean value 0.1334 mag. The probable error 0.0004 mag. is obtained, not from the values of $C$ in this table, but from the eighty-four separate results of a single night's measure of the different pairs. No correction has been made for change in zenith distance, since the series begin and end with a bright star.

TABLE III.

MÜLLER AND KEMPF'S *PLEIADES* STARS USED IN DETERMINING WEDGE CONSTANT.

| | 1900 | | | | 1900 | | |
|---|---|---|---|---|---|---|---|
| No. | R. A. | Dec. | Mag. | No. | R. A. | Dec. | Mag. |
| | h m s | | | | h m s | | |
| IV | 3 39 57 | +24° 15′ | 6.17 | 55 | 3 41 15 | +23° 28′ | 9.57 |
| 14 | 43 1 | 23 33 | 6.72 | 56 | 42 10 | 23 50 | 9.58 |
| 15 | 40 5 | 24 13 | 6.75 | 57 | 44 16 | 24 22 | 9.63 |
| 17 | 44 2 | 23 33 | 7.10 | 58 | 40 43 | 23 48 | 9.65 |
| V | 41 2 | 24 13 | 7.15 | 60 | 39 45 | 23 59 | 9.70 |
| 19 | 41 32 | 23 59 | 7.18 | 61 | 41 13 | 23 58 | 9.76 |
| 20 | 42 33 | 24 3 | 7.23 | 64 | 39 42 | 24 19 | 10.00 |
| 21 | 41 22 | 23 25 | 7.24 | 66 | 41 56 | 23 38 | 10.06 |
| 22 | 44 56 | 23 40 | 7.28 | 67 | 44 1 | 23 39 | 10.08 |
| 23 | 41 25 | 23 30 | 7.31 | VIII | 43 34 | 24 20 | 10.14 |
| 25 | 41 28 | 23 36 | 7.53 | 71 | 42 3 | 23 48 | 10.20 |
| 26 | 40 30 | 22 57 | 7.63 | 72 | 42 29 | 23 45 | 10.21 |
| 27 | 44 30 | 24 12 | 7.78 | 73 | 42 22 | 23 20 | 10.29 |
| 29 | 43 59 | 24 3 | 7.84 | 74 | 42 9 | 23 4 | 10.31 |
| VI | 3 41 26 | +24 17 | 7.99 | 75 | 40 42 | 23 29 | 10.35 |
| | | | | 76 | 39 39 | 23 46 | 10.36 |
| | | | | 77 | 41 5 | 24 20 | 10.42 |
| | | | | 79 | 3 42 22 | +23 35 | 10.52 |

## TABLE IV.

### SPECIMEN SET OF MEASURES OF *PLEIADES* STARS.

1900 Nov. 15, 23 h. Sid. T.

**With 6-inch reflector, aperture reduced to 4½ inches**

| Z | Star | Scale readings | Mean d | M. & K. Mag. | Δd | ΔM | C | ΔM P | M. & K.—P. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | M | M | M | M |
| 53° | 25 | 18.0 19.4 19.2 19.4 | 19.00 | 7.53 | | | | | |
| | | | | | 19.23 | 2.67 | 0.1388 | 2.57 | +0.10 |
| | 71 | 38.6 38.0 37.5 38.8 | 38.23 | 10.20 | | | | | |
| | | | | | 21.90 | 3.05 | 1393 | 2.92 | + 13 |
| | V | 16.7 15.7 16.1 16.8 | 16.33 | 7.15 | | | | | |
| | | | | | 26.62 | 3.27 | 1228 | 3.55 | — 28 |
| | 77 | 43.0 43.2 42.8 42.8 | 42.95 | 10.42 | | | | | |
| | | | | | 21.80 | 2.43 | 1115 | 2.91 | — 48 |
| | VI | 20.5 21.1 21.3 21.7 | 21.15 | 7.99 | | | | | |
| | | | | | 18.88 | 2.37 | 1255 | 2.52 | — 15 |
| | 76 | 40.2 40.0 40.1 39.8 | 40.03 | 10.36 | | | | | |
| | | | | | 24.20 | 3.18 | 1314 | 3.23 | — 5 |
| | 19 | 15.3 16.2 15.0 16.8 | 15.83 | 7.18 | | | | | |
| | | | | | 25.05 | 3.34 | 1333 | 3.34 | 00 |
| | 79 | 41.8 41.2 39.4 41.1 | 40.88 | 10.53 | | | | | |
| | | | | | 25.35 | 3.28 | 1294 | 3.38 | — 10 |
| | 21 | 15.0 15.2 15.9 16.0 | 15.53 | 7.24 | | | | | |
| | | | | | 25.05 | 3.05 | 1218 | 3.34 | — 29 |
| | 73 | 42.0 40.4 40.6 39.3 | 40.58 | 10.29 | | | | | |
| | | | | | 22.33 | 2.98 | 1334 | 2.98 | 00 |
| | 23 | 17.8 18.0 18.2 19.0 | 18.25 | 7.31 | | | | | |
| | | | | | 20.23 | 2.90 | 1434 | 3.00 | — 10 |
| | 72 | 37.9 39.4 38.0 38.6 | 38.48 | 10.21 | | | | | |
| | | | | | 19.73 | 2.68 | 1358 | 2.53 | + 15 |
| | 25 | 19.6 17.7 19.2 18.5 | 18.75 | 7.53 | | | | | |
| | | | | | 21.08 | 2.78 | 1318 | 2.81 | — 3 |
| | 74 | 39.8 39.7 39.9 39.9 | 39.83 | 10.31 | | | | | |
| | | | | | 23.65 | 3.08 | 1303 | 3.14 | 5 |
| | 20 | 15.9 15.9 17.0 15.9 | 16.18 | 7.23 | | | | | |
| | | | | | 23.30 | 3.12 | 1339 | 3.11 | + 1 |
| | 75 | 38.0 40.9 40.0 39.0 | 39.48 | 10.35 | | | | | |
| | | | | | 22.50 | 3.11 | 1382 | 3.00 | + 11 |
| | 21 | 16.5 17.2 17.2 17.0 | 16.98 | 7.24 | | | | | |
| | | | | | 22.42 | 3.12 | 1392 | 2.99 | + 13 |
| | 76 | 38.2 41.0 39.1 39.3 | 39.40 | 10.36 | | | | | |
| | | | | | 23.87 | 3.21 | 1345 | 3.18 | + 3 |
| | V | 15.6 15.3 16.0 15.2 | 15.53 | 7.15 | | | | | |
| | | | | | 23.10 | 3.06 | 1325 | 3.08 | — 2 |
| | 72 | 37.0 40.2 38.0 39.3 | 38.63 | 10.21 | | | | | |
| | | | | | 17.05 | 2.22 | 0.1302 | 2.27 | —0.05 |
| 43 | VI | 20.0 23.0 21.6 21.7 | 21.48 | 7.99 | | | | | |

## TABLE V.

### MEASURES OF *PLEIADES* STARS.

| Pairs | Δd | ΔM M. & K. | C | ΔM P | M. & K.—P. | No. |
|---|---|---|---|---|---|---|
| | | | M | M | M | |
| V-71 | 22.64 | 3.05 | 0.1348 | 3.02 | +0.03 | 3 |
| V-77 | 25.46 | 3.27 | .1285 | 3.40 | — .13 | 4 |
| V-VIII | 23.04 | 2.99 | .1299 | 3.07 | — .08 | 4 |
| 19-VIII | 23.49 | 2.96 | .1261 | 3.13 | — .17 | 2 |
| 19-76 | 23.40 | 3.18 | .1360 | 3.12 | + .06 | 4 |
| 19-74 | 23.81 | 3.13 | .1316 | 3.18 | — .05 | 4 |
| 19-72 | 22.63 | 3.03 | .1337 | 3.02 | + .01 | 4 |
| 19-77 | 24.19 | 3.24 | .1339 | 3.23 | + .01 | 2 |
| 20-VIII | 21.59 | 2.91 | .1349 | 2.88 | + .03 | 2 |
| 20-71 | 21.17 | 2.97 | .1404 | 2.82 | + .15 | 2 |
| 20-72 | 21.46 | 2.98 | .1389 | 2.86 | + .12 | 4 |
| 21-73 | 23.20 | 3.05 | .1318 | 3.09 | — .04 | 5 |
| 21-79 | 24.55 | 3.28 | .1337 | 3.27 | + .01 | 5 |
| 23-75 | 22.98 | 3.04 | .1324 | 3.06 | — .02 | 4 |
| 23-79 | 24.55 | 3.21 | .1309 | 3.27 | — .06 | 4 |
| 25-73 | 20.55 | 2.76 | .1345 | 2.74 | + .02 | 4 |

TABLE V.—*Continued.*

MEASURES OF *PLEIADES* STARS.

| Pairs | Δd | ΔM M. & K. | *C* | ΔM P | M. & K.—P. | No. |
|---|---|---|---|---|---|---|
| | | | M | M | M | |
| 25–74 | 20.97 | 2.78 | .1327 | 2.80 | − .02 | 5 |
| 25–75 | 20.94 | 2.82 | .1349 | 2.79 | + .03 | 4 |
| 25–76 | 20.41 | 2.83 | .1388 | 2.72 | + .11 | 4 |
| V –76 | 23.87 | 3.21 | .1345 | 3.18 | + .03 | 1 |
| V –72 | 23.10 | 3.06 | .1325 | 3.08 | − .02 | 1 |
| 19–79 | 25.05 | 3.34 | .1333 | 3.34 | .00 | 1 |
| 20–74 | 23.65 | 3.08 | .1303 | 3.15 | − .07 | 1 |
| 20–75 | 23.30 | 3.12 | .1339 | 3.11 | + .01 | 1 |
| 21–75 | 22.50 | 3.11 | .1382 | 3.00 | + .11 | 1 |
| 21–76 | 22.42 | 3.12 | .1392 | 2.99 | + .13 | 1 |
| 23–72 | 20.23 | 2.90 | .1434 | 2.70 | + .20 | 1 |
| 23–73 | 22.33 | 2.98 | .1334 | 2.98 | .00 | 1 |
| 25–71 | 19.23 | 2.67 | .1388 | 2.56 | + .11 | 1 |
| 25–72 | 19.73 | 2.68 | .1358 | 2.63 | + .05 | 1 |
| VI–72 | 17.05 | 2.22 | .1302 | 2.27 | − .05 | 1 |
| VI–76 | 18.88 | 2.37 | .1255 | 2.52 | − .15 | 1 |
| VI–77 | 21.80 | 2.43 | .1115 | 2.91 | − .48 | 1 |
| Means | | | 0.1334 ±0.0004 | | 0.10 | |

TABLE VI.

PAIRS OF STARS FROM THE *POTSDAM PHOTOMETRIC DURCHMUSTERUNG*, PART I.

| Pairs | Settings | | Mags. | | Δd | ΔM | *C* | R |
|---|---|---|---|---|---|---|---|---|
| | d | d | | | | M | M | d |
| 2306–00 | 14.13 | 40.38 | 4.42 | 7.73 | 26.25 | 3.31 | 0.1261 | 0.65 |
| 2368–87 | 12.02 | 40.48 | 4.10 | 7.64 | 28.46 | 3.54 | .1244 | .70 |
| 2430–35 | 12.37 | 34.47 | 4.17 | 6.62 | 22.10 | 2.45 | .1109 | .47 |
| 2439–17 | 11.74 | 36.13 | 4.00 | 7.02 | 24.39 | 3.02 | .1238 | .65 |
| 2415–03 | 15.39 | 36.28 | 4.23 | 7.04 | 20.89 | 2.81 | .1345 | .39 |
| 2581–78 | 16.43 | 35.93 | 4.44 | 7.28 | 19.50 | 2.84 | .1456 | .64 |
| 2879-77 | 11.12 | 32.98 | 3.90 | 6.37 | 21.86 | 2.47 | .1130 | .90 |
| 2875–78 | 18.49 | 37.78 | 4.85 | 7.48 | 19.29 | 2.63 | .1302 | .60 |
| 2875–66 | 18.49 | 37 83 | 4.85 | 7.24 | 19.34 | 2.39 | .1236 | .62 |
| Means | | | | | | | 0.1158 | 0.62 d |
| P. e | | | | | | | 0.0024 | |

Table VI gives the results of measures of nine pairs of stars taken from the *Potsdam Photometric Durchmusterung*, Part I.

For these measures a Brashear objective of 2.1 inches aperture was used with the photometer in its original form. The order of settings was: (1) four on the bright star; (2) eight on the faint star; (3) four on the bright star. The table gives in the successive columns the *P. DM.* numbers of the stars of the pair, the mean of the settings on the bright and faint stars, their magnitudes, the difference in scale readings and magnitudes, the resulting value of *C*, and the mean residual of the separate settings from the mean of eight.

Table VII gives the details of a single night's measures of six of the comparison stars for the variable *7792 SS Cygni*,

TABLE VII.

MEASURES OF COMPARISON STARS FOR *7792 SS CYGNI.*

1900 June 14, with 12-inch refractor.

| | Settings | | | | Means | | |
|---|---|---|---|---|---|---|---|
| | | | | | 1st | 2d | R. |
| | d | d | d | d | d | d | d |
| *b* | 16.8 | 14.2 | 15.2 | 16.6 | 15.70 | 15.17 | 1.00 |
| *c* | 24.0 | 23.8 | 23.5 | 23.3 | 23.65 | 23.22 | 25 |
| *a* | 26.8 | 26.2 | 24.7 | 26.8 | 26.13 | 26.02 | 69 |
| *d* | 32.2 | 33.7 | 31.9 | 32.8 | 32.65 | 32.78 | 60 |
| *p* | 33.3 | 34.2 | 35.0 | 34.3 | 34.20 | 32.90 | 45 |
| *m* | 37.8 | 38.3 | 38.9 | 37.0 | 38.00 | 38.59 | 60 |
| *m* | 39.1 | 38.1 | 39.5 | 40.0 | 39.18 | | 58 |
| *p* | 30.5 | 31.8 | 31.1 | 33.0 | 31.60 | | 80 |
| *d* | 33.0 | 32.8 | 32.6 | 33.2 | 32.90 | | 30 |
| *a* | 24.9 | 26.5 | 26.4 | 25.8 | 25.90 | | 55 |
| *c* | 20.8 | 23.9 | 22.5 | 23.9 | 22.78 | | 1.13 |
| *b* | 13.4 | 15.1 | 15.6 | 14.4 | 14.63 | | 0.73 |
| Mean | | | | | | | 0.64 |

whose magnitudes were kindly communicated to me by Professor E. C. Pickering.[1] Four settings were made on each of the six stars, then these were repeated in reverse order. Column 6 gives the mean of each set of four, column 7 the mean of the eight settings on each star. The method of deducing the value of *C* is given at the foot of the table.

[1] For the positions and notation of the comparison stars see the ASTROPHYSICAL JOURNAL, 12, 260.

Table VIII collects the final results of these measures. The mean value

$$C = 0.1300$$

is obtained by weighting the separate determinations according to the number of settings. It would seem to be reliable within two or three units in the third decimal place. The last two columns of the table give the average residual of the separate settings from the mean of four, eight, or ten as the case may be.

RESULTS.

By comparing the mean settings and magnitudes of the three brighter stars, *a*, *b*, and *c*, with those of the three fainter stars, *d*, *m*, and *p*, we have

| | d | M | | d | M |
|---|---|---|---|---|---|
| *b* | 15.17 | 8.50 | *d* | 32.78 | 10.92 |
| *c* | 23.22 | 9.39 | *p* | 32.90 | 10.90 |
| *a* | 26.02 | 9.62 | *m* | 38.59 | 11.17 |
| Means | 21.47 | 9.17 | | 34.76 | 11.00 |

$\Delta d = 13.29$, $\Delta M = 1.83$, $C = 0.1377$.

TABLE VIII.

FINAL RESULTS FOR VALUE OF THE WEDGE CONSTANT.

| | C | No. of settings | P. e. of mean | Mean R. | |
|---|---|---|---|---|---|
| | M | | M | d | M |
| Wheel photometer........... | 0.1283 | 575 | 0.0008 | 0.71 | 0.09 |
| *Pleiades* stars .............. | 0.1334 | 360 | 0.0004 | 42 | 05 |
| *P. DM.* stars................ | 0.1258 | 136 | 0.0024 | 62 | 08 |
| Comparison stars for *SS Cygni*. | 0.1377 | 48 | | 0.64 | 0.08 |
| Weighted mean....... | 0.1300 | | | | |

In conclusion I may say that this form of photometer is certainly very convenient in use, and seems to give good results. It has the advantage over other forms of wedge photometers that the light of the real star does not pass through the wedge, thus avoiding the danger of systematic errors arising from star colors.

YERKES OBSERVATORY,
March 1901.

# ON THE TYPES OF SUN-SPOT DISTURBANCES.

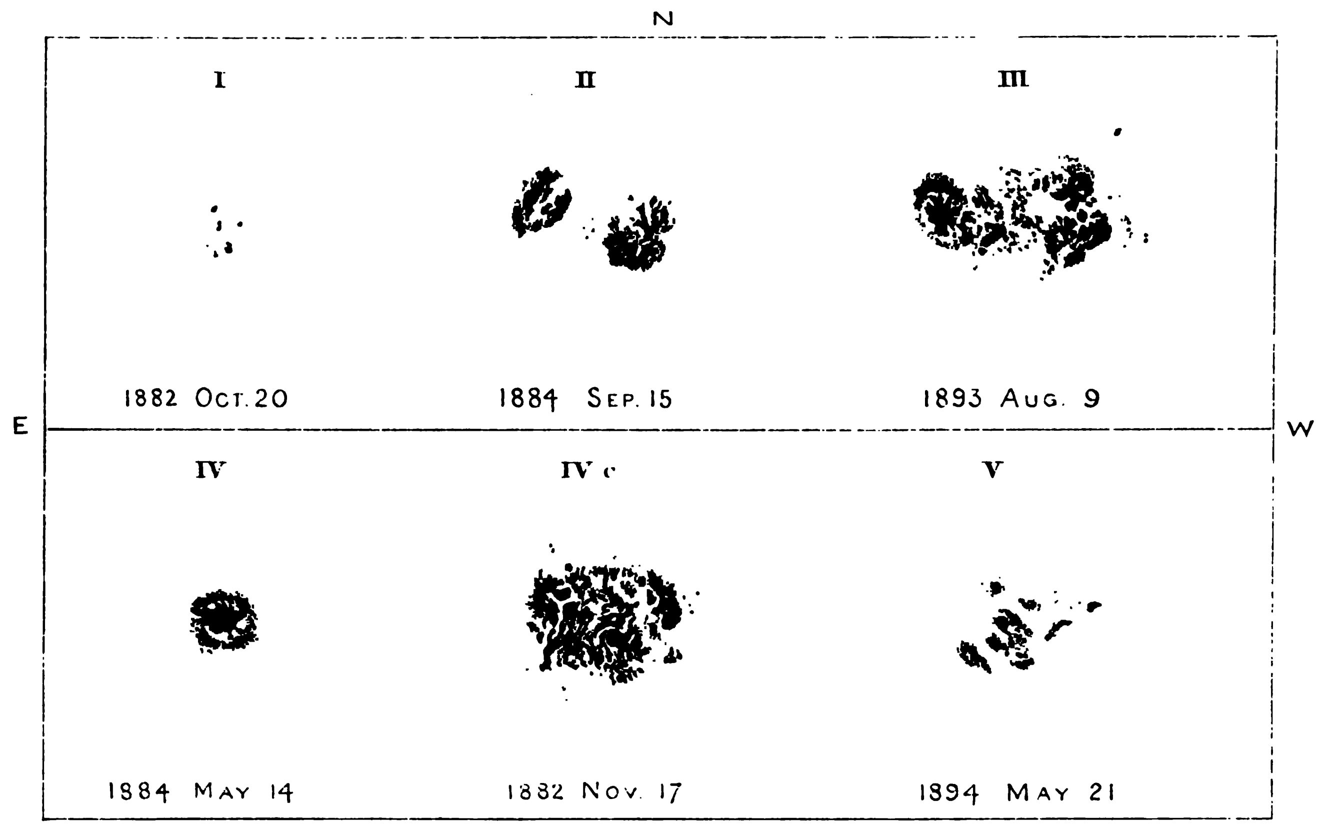
By A. L. Cortie, S.J.

As an aid to researches connected with Sun-spots an attempt is made in the present paper to classify them according to some prevailing typical forms. Descriptions of the varying phases presented by a group of Sun-spots are generally and necessarily somewhat verbose, and one aim of the present attempt at classification is to be able, by a succession of type numbers, to succinctly describe the various phases in the life-history of a spot-group. These classes cannot pretend to describe in minutest detail all the varying aspects presented in the life-history of a group of Sun-spots, but their most salient features can be succinctly represented by them. The classification adopted in this paper is merely tentative, and is submitted as such to the criticism and discussion of solar observers. It has been derived from a study of some 3500 drawings of Sun-spots secured at the Stonyhurst College Observatory during the last twenty years.

Among the groups represented upon these drawings, 296 were selected for discussion, belonging to 117 either greater Sun-spot disturbances, or disturbances in some way connected with these greater outbursts. By a greater disturbance is meant one which during any part of its life-history covered an area of $\frac{1}{1000}$ of the Sun's visible hemisphere. A full list of such disturbances is given in a paper recently read before the Royal Astronomical Society (*Monthly Notices*, Vol. LX, No. 8, May 1900).

The Stonyhurst drawings show that spots appear as scattered groups of small spots, as trains of spots, as composite groups consisting of three or more larger spots, as single spots of round and regular outline, which may or may not be accompanied by smaller companions, and as single spots of irregular outline, either accompanied by a train of smaller companions, or with outliers not arranged in the form of a train.

PLATE VIII




TYPES OF SUN-SPOTS

The chief type, however, of which the above mentioned are in most, possibly in all, cases but phases, is the double spot formation, with a train of smaller spots between the two principal spots of the group, the whole group generally drifting into more or less parallelism with the solar equator. In this form the principal spot, which eventually becomes a normal spot of regular outline, is generally the leading spot, but in many cases it is the following spot, while sometimes the preponderance in area alternates between the two, as the group traverses the disk. In yet rarer instances both the chief spots develop as regular spots. The following are the types which will probably be found to cover most cases that may arise:

Type I. A group of one or more small scattered spots.
Type II. The two-spot formation:
- II*a*. In which the leader is the principal spot.
- II*b*. In which the following spot is the principal spot.
- II*c*. In which both spots are more or less equal.

Type III. A train of spots:
- III*a*. With well-defined principal spots.
- III*b*. Without well-defined principal spots, but consisting mostly of penumbral patches with shattered irregular umbra.

Type IV. Single spots:
- IV*a*. A single spot of round and regular outline.
- IV*b*. A single spot of round and regular outline with small companions.
- IV*c*. A single spot of irregular outline.
- IV*d*. A single spot of irregular outline with a train of smaller companions.
- IV*e*. A single spot of irregular outline with smaller companions not in a train.

Type V. An irregular group of larger spots.

In most cases it is comparatively easy to assign Sun-spots to the various types selected, but in some the line of demarcation is not very marked, and consequently it becomes more difficult to do so. For instance, Type IV*d* approximates very often to II*a*. The order of types has been chosen as indicating the succession of phases through which a normal Sun-spot disturbance generally passes.

The first type covers the period of the birth of a Sun-spot group which almost invariably appears as a few small scattered dots, surrounded by flecks of brilliant faculæ. Whether these flecks of faculæ precede or follow the first appearance of the dots, is yet an unsettled point. The weight of evidence, however, seems to favor the first supposition. These scattered dots next coalesce into two principal spots, as indicated by Type II, the preceding spot of the couple being generally the more compact, while the following spot presents a broken appearance, though in many cases it may cover a larger area than its fellow. The space between these two spots begins to fill up with a train of smaller spots, the process being completed in 5 to 7 days after the birth of the group. This phase is represented by Type III. In a few days the train of spots between the two principal spots disappears, the process being followed in most cases by the disintegration of the following of the pair of principal spots. This leads to the stage represented by Type IV, where the group consists mainly of a single spot, generally of a round and regular outline, with the penumbra arranged symmetrically about a densely black central umbra. This single spot may, however, be of quite irregular outline—witness, the great spot of February 1892—and each of these subclasses, again, may or may not have accompanying smaller spots. In the great majority of cases it is the leading spot which becomes round and regular, and which also, during the earlier stages in the life of the group, frequently has a very rapid forward proper motion in longitude. The cases in which the following spot of the group, which often in the earlier stages of the life-history of the group is of far greater area than the leader, develops into a round spot are comparatively rare (*e. g.*, 1892, July 4–October 5). However, instances occur in which both spots become round spots (*e. g.*, 1881, October 14, December 17). The stage of a single round spot lasts frequently for two solar rotations, and has in one case (1897, April 28–August 27) been recorded during five successive rotations. In some cases the single spot gradually decreases in

size, until it becomes a mere dot, when other small spots will spring up around it, and the cycle of phases will be again repeated (*e. g.*, 1886, March 29–September 15).

During the earlier stages of the life-history of a Sun-spot group the faculæ are intensely bright and cling closely to the component members of the group. As the group grows in age, the faculæ gradually extend, until during the single spot stage of the group and its evanescence, they cover a very considerable area. With their greater extension they lose their brilliancy. Even when the spot has finally disappeared, the faculæ may remain extended in the same region of the Sun for two or more rotations. In the scheme of classification submitted the gradual dying away of a spot is represented as a recurrence to Type I, especially in view of the repetition of the cycle which then sometimes supervenes. Irregular groups of larger spots have been put into a class apart, Type V. Allowing for the necessary imperfection of our record, on account of days when it was impossible to secure drawings, the prevailing types are IV, II and I, arranged in provisional order. It would seem, too, from a study of the material at hand, that the ordinary process of spot-formation and life-history could be represented by the following sequence of types: I, II*b*, II*a*, III*a*, II*a*, IV*d*, IV*a*, I.

One advantage of describing the life-history of a spot-group by means of these type-numbers is that it indicates fairly accurately the age of a spot. Thus a group marked II is in the earlier stages of its life, while one marked III*a* is, at the most, ten days old, while one marked IV*a* is at least thirteen or fourteen days old. Two examples will suffice to illustrate the application of these type numbers.

The first was a spot-group seen during seven rotations, from 1886, March 29–September 15, its mean heliographic positions being longitude 71° and latitude —10°.

The separate rotations are denoted by vertical lines: 1, 2*b*, 1, 3*b* | 1, 2*b* (April 29) 2*a* | 4*a* | 4*a*, 4*c* | 1, 2*a* | 4*c* | 1 | . The

second was a composite group which was born and died on the visible disk, and was seen from 1887, May 14–September 4, in mean position longitude 92, latitude —8°. Its history reads thus: 1, *2b*, *3a* | *4b*, *4a* | *4d*, *4a* | *4a*, companion 1, *2a* | 1 | . In this case the companion group did not appear until the fourth rotation.

STONYHURST COLLEGE OBSERVATORY,
September 4, 1900.

PLATE IX

FIG. 1.—INTERFERENCE FRINGES OF GREEN MERCURY LINE

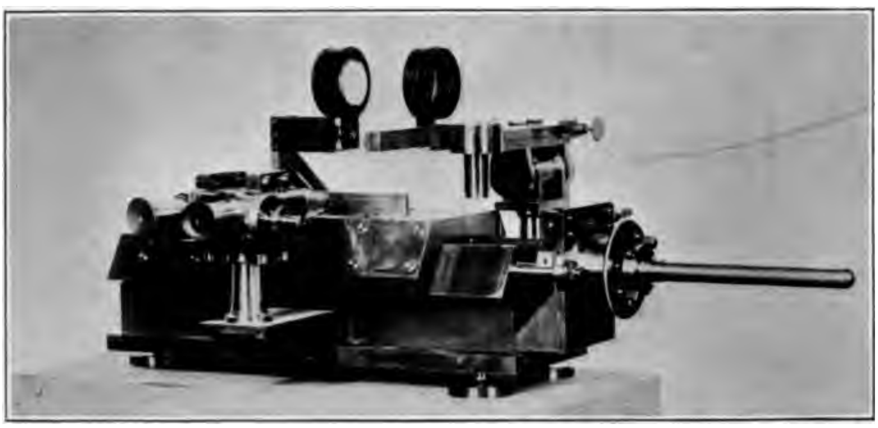


FIG. 2.—NEW FORM OF INTERFEROMETER

# ON A NEW FORM OF INTERFEROMETER.[1]

By CH. FABRY and A. PEROT.

IN a series of memoirs[2] published by us during the last three years we have described various applications of fringes produced by silvered plates. The special properties of these fringes will, in our opinion, render possible a notable extension of the already wide field comprising the applications of interference fringes. The applications which we have already described relate to the optical measurement of lengths, to the comparison of wave-lengths, and to spectroscopy.

All of these experiments have been made with apparatus which we have constructed with no aid from the instrument-maker other than that required in preparing plane glass surfaces. These are the instruments, ordinarily of very simple construction, which we have described in the memoirs already referred to. These instruments, improvised for the occasion in order to avoid serious loss of time, necessarily contain various imperfections. We have always taken care to conduct our experiments in such a way as to avoid the influence of these imperfections on the accuracy of the results. A characteristic feature of our methods is that the observer has constantly before his eyes, in the very appearance of the phenomenon under observation, a proof that the adjustments are rigorously exact. But imperfections of the apparatus nevertheless make the preliminary steps in

[1] Translated from an advance proof from *Annales de Chimie et de Physique*, communicated, with additions and illustrations, by the authors.

[2] "Sur les franges des lames minces argentées et leur application à la mesure des petites épaisseurs d'air." (*Ann. de Chim. et de Phys.*, 7e série, t. XII, p. 459; 1897.) "Théorie et applications d'une nouvelle méthode de spectroscopie interférentielle." (*Ibid.*, 7e série, t. XVI, p. 115; 1899.) "Méthodes interférentielles pour la mesure des grandes épaisseurs et la comparaison des longueurs d'ondes." (*Ibid.*, 7e série, t. XVI, p. 289; 1899.) "Sur les sources de lumière monochromatique." (*Journ. de Phys.*, 3e série, t. IX, p. 369; 1900.) "Électromètre absolu pour petites différences de potentiel." (*Ann. de Chim. et de Phys.*, 7e série t. XIII, p. 404; 1898.) "Mesure du coefficient de viscosité de l'air." (*Ibid.*, 7e série, t. XIII, p. 275; 1898.)

A brief résumé of these investigations was published in the ASTROPHYSICAL JOURNAL, Vol. IX, p. 87, February 1899.

every investigation much more troublesome, and may even render impracticable an application which would be easily effected with a more perfect instrument.

These considerations have led us to order from M. Jobin an interferometer suitable for the convenient observation of the phenomena of silvered plates, and consequently for the realization of the various applications described in our papers. This apparatus, which has been constructed in a most perfect manner, is described in the present article.

The greater part of our applications of the interference phenomena of silvered films depend upon interference at great difference of path, produced by transmission through two plane surfaces, rigorously parallel, with transparent silver surface; the interference rings are observed by means of a telescope focused for parallel rays. These conditions determine the essential parts of the interferometer with silvered plates; they consist simply of two plane surfaces, provided with all necessary means of adjustment for orientation and displacement. It must be possible to adjust their relative orientation and particularly to render them rigorously parallel. Their distance must be susceptible of varying from contact up to 10 cm; it is very convenient to have this displacement effected by an exactly parallel motion in such a way as to preserve the parallelism of the surfaces. It must be possible, during this parallel displacement, to stop at any desired distance within a few thousandths of a micron, but displacements of several centimeters must not require too much time. This leads to the use of three different rates of adjustment: (1) rapid motion; (2) motion slow enough to permit the fringes to be counted; (3) displacement by flexure through a range of a few microns, as slow and as delicate as may be desired.

Similarly, there are two distinct adjustments for orientation: a quick motion of great amplitude for approximate adjustment, and a very slow motion of orientation of small range, produced by flexure.

The adjustments by flexure are all obtained by the pressure

on pieces of steel of small rubber bags filled with water and connected by means of a long rubber tube to a funnel containing water whose height can be varied; by changing the height a variable force is applied by means of the bag upon the piece of steel against which it presses. This arrangement has the following advantages: the bag being wider than the metallic piece against which it presses, the tension of the rubber does not enter

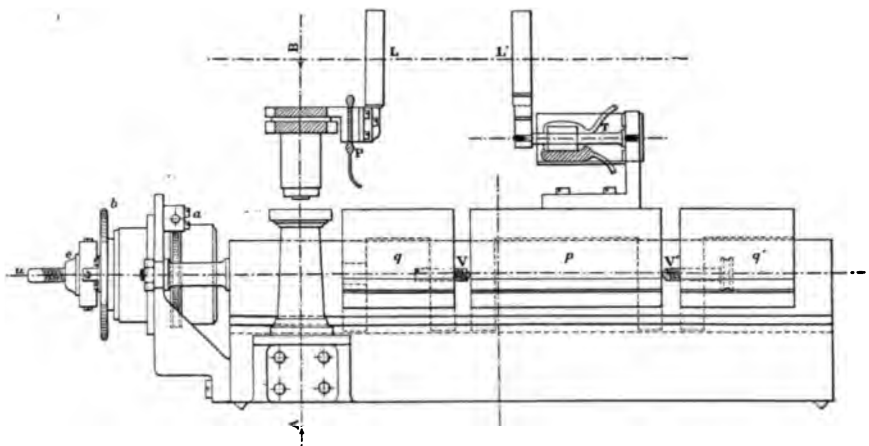



FIG. 1.

and the force depends only upon the pressure exercised by water; as this is defined by the height of the funnel, there need be no fear of a change in the adjustment from this cause during the progress of the work. Moreover, the pressure may be varied as slowly as is desired, and with this arrangement absolutely perfect adjustments are obtained; it may be added that the pressures are produced without giving any shock to the system, which is indispensable in order to avoid any disarrangement.

These are the essential elements of the instrument. Let us now proceed to the detailed description.

*L*, *L'* (Fig. 1) are the two plate-carriers in which the silvered plates are supported; each of these is a disk 40 mm in diameter with projecting shoulder, by means of which it can be fastened in the plate-carrier without danger of distortion. The silvered

face is rigorously plane, the reverse only approximately so. The two faces are not parallel, but make with each other an angle of about 1′, to prevent interference in a single plate, which would interfere with the phenomenon under observation.

The plate-carriers have the following adjustments: $L$, toward the observer (Fig. 2), may be given quick motions for orientation and very small parallel displacements. The first of these motions is obtained by rotation about two axes, one of them vertical, $O$, the other horizontal, $O'$, as in a theodolite. The small parallel displacement is produced by bending a strong steel spring $P$ (Figs. 1 and 2), attached at its middle point to the theodolite axis $O$ on one side, and to the plate-carrier on the other; this spring consists of two steel bars 16 cm long by 2 cm wide and 4 mm thick, connected at their two extremities $E$ and $E'$ by means of metal plates. The rubber bag used to produce the displacement is shown in position in the cuts. A change of 1 cm in the height of the funnel produces a parallel displacement of $0.15\,\mu$. The total range employed has never exceeded $20\,\mu$.

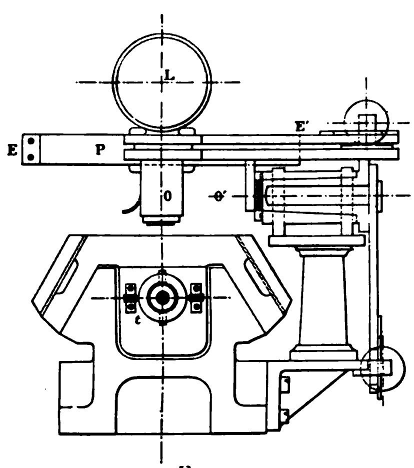


FIG. 2.

The plate-carrier $L'$ (Fig. 1) may be given fine adjustments for parallelism and large parallel displacements, either rapid or slow. $L'$ is carried at the extremity of a steel shaft $T$ rigidly held at the other end, on which is fitted a bronze block, against which two rubber bags press in two directions at right angles to each other. A displacement of one centimeter of each of the corresponding funnels produces an angular displacement of 0.″25.

To obtain perfect parallelism it is frequently necessary to adjust the height of the funnels within a millimeter.

Finally, large parallel displacements are obtained by means

of a bronze carriage on guides, $p$ (Fig. 1), the bearing surfaces of which are worked with great accuracy. The displacement of the carriage is not produced by operating directly upon it; for this purpose it is placed between two shorter carriages, $q$, $q'$, rigidly fixed with respect to each other; it may be moved by these in either direction by means of butting screws $V$, $V'$, bearing on suitable points, which allow a little play. $p$ is thus always free on the guides, where it rests by its own weight, and is acted upon when in motion only by forces parallel to the displacement, which produce no tendency to rock; it is doubtless on account of this device that it is possible to follow the fringes even when the carriage is moving. The two carriages $q$, $q'$, are connected to a screw $u$ whose head $t$ (Fig. 2) is attached by a Cardan suspension, of which the nut $e$ (Fig. 1), susceptible of rotation only, is also carried by a Cardan suspension. Lateral strains due to imperfect centering are thus avoided. There can thus be transmitted to the principal carriage only such impulses as are exactly longitudinal. The nut $e$ can be moved rapidly by a milled head $b$ or slowly by a tangent screw $a$. In the latter case one turn of the screw corresponds to about 15 fringes; it is possible to count the fringes.

It is very convenient to be able to quickly determine at any time the distance between the two silvered surfaces within a few microns; a scale divided to fifths of a millmeter is provided for this purpose, attached to the carriage $p$, and read by a microscope with micrometer eyepiece fastened to the bed of the apparatus. One division of the head equals $1 \mu$. The zero is determined by setting the two surfaces at a known and easily calibrated distance; for example, that which corresponds to the first resolution of the two yellow lines of mercury ($40 \mu$).

Finally, to prevent vibrations the apparatus is carried on a small table hanging by four rubber cords, whose points of support are adjustable so as to permit the apparatus to be leveled.

A solid body[1] whose dimensions are to be determined is suspended between the interferometer plates in such a way that its

[1] *Ann. de Chim. et de Phys.*, 7 ser., **16**, 289.

faces can be made parallel to those of the silvered glass planes. The form of the support depends upon the dimensions of the solid body; it is always such as to permit displacements in distance or in inclination. In certain cases, such as the measurement of the quartz cube employed in the determination of the kilogram, perfect parallelism could be obtained through flexure produced by the rubber bags. The apparatus itself does not include this support and it is not represented in the figures.

Such, in general, is the new interferometer constructed by M. Jobin; it will be seen that it is especially adapted for the observation of interference fringes at great difference of path and for the applications of these phenomena. It is evident that it may also be employed to produce the phenomena of thin plates in parallel light; it is only necessary to put the plates a short distance apart and to give them the desired angle by means of the corresponding adjustments; the distance can then be varied without changing this angle.

It is also very easy to observe *superposition fringes* in white light, the numerous applications of which we have already indicated. If the fringes of thin plates[1] are desired the silvered surfaces of the interferometer are placed a short distance apart and upon the thin layer of air thus obtained there is projected the image of a *standard film*, which may be easily constructed by placing two surfaces of silvered glass in contact. By varying the path at a uniform rate by means of the tangent screw of the apparatus the various systems of fringes which correspond to the various simple ratios of the two thicknesses of air will be seen to appear successively. In this way at least ten systems of fringes in white light, easily distinguished from one another by their appearance, can be observed successively.

If thick layers are to be employed[2] both must have parallel surfaces which must be capable of orientation with reference to each other. One of these layers will be in the interferometer itself; the other may be a layer of air with parallel faces at a

[1] *Ann. de Chim. et de Phys.*, 7 ser., 12, 459.

[2] *Ann. de Chim. et de Phys.*, 7 ser., 16, 289.

fixed distance apart. We have constructed such layers and named them *standards of thickness.*[1] For experiment we have constructed in the laboratory standards of thickness varying from 0.25 cm to 12 cm. Fig. 3 represents a 1 cm standard made by M. Jobin. It consists of a steel plate *A* pierced by a circular opening in which are fastened three small steel screws *P*, the ends of which are carefully rounded and polished. Against these three curved surfaces plates of plane silvered glass *L*, *L'*, are held by Brunner spring clamps, and are thus maintained at a fixed distance. By carefully scraping the steel pins the silvered plates are brought to perfect parallelism. Experience has shown that after dismounting and replacing the glass plates their parallelism is preserved and the thickness of the standard does not change.

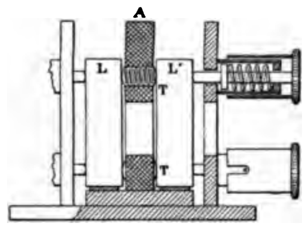


FIG. 3.

If then a convergent beam of monochromatic light is passed through the standard the phenomena of thick silvered plates will be visible. The adjustment of the standard, begun by the observation of multiple images, is completed by observation of the rings themselves. When the thickness exceeds about 20 cm the fringes in monochromatic light are no longer visible, and it becomes necessary to employ other methods which will be described later.

For the observation of superposition phenomena this plate is carried by an adjustable support standing beside the interferometer.

The use of these superposition fringes permits the interferometer surfaces to be set always at the same distance (that of the standard or one of its multiples or submultiples); this distance will be exactly known if the standard has been measured. It is thus possible to graduate the scale of the interferometer by investigating the various intervals successively and moving along the scale. Conversely, the fringes permit the measurement in wave-lengths of a constant standard, either by a direct

[1] *Comptes Rendus*, 130, 492, 1900.

determination on the interferometer or by two sets of measurements if its length is too great for the first method.

We believe that the applications of these fringes are far from being exhausted. They adapt themselves to the most varied combinations. As an example we may cite the possibility of measuring in a single operation the sum or difference of two different lengths which may or may not be capable of direct comparison, or any quantity of the form $pe+p'e'$, $p$ and $p'$ being small positive or negative integers. We shall have occasion to return to these various applications at some future time. In our present investigations the apparatus which M. Jobin has constructed in such perfection is proving itself to be of the greatest service.

Fig. 1, Plate IX, is a photograph of interference rings obtained with this apparatus, corresponding to the green mercury line ($\lambda = 5460.7424$) given by the mercury arc in a vacuum. The distance between the silvered surfaces was only 5 mm. Although this distance was so small the complex constitution of this radiation is clearly shown: each bright ring is accompanied by a fainter first interior ring, by a second interior ring, which is much fainter still, and finally by an exterior ring, which is double.[1] Each of these rings indicates the presence of a faint radiation which accompanies the principal line.

The existence of these complex radiations has been announced by Michelson, but we believe that hitherto no spectroscopic apparatus has permitted them to be *seen* and their position in the spectrum to be fixed without any hypothesis. Our interferometer permits this to be done, and thus probably constitutes the most powerful spectroscope hitherto constructed. The resolving power of this spectroscope increases with the distance between the silvered plates, and can thus be increased as long as the fringes remain visible; the finer the lines examined the further can the resolution be carried; in other words, the power of the apparatus is in every case sufficient to show the finest details that it is possible to distinguish.

UNIVERSITY OF MARSEILLES,
January 1901.

[1] Unfortunately the delicate details of the original photograph are lost in the reproduction.—Eds.

# *Minor Contributions and Notes*

## VARIABILITY IN LIGHT OF *EROS*.[1]

THE discovery by Dr. Oppolzer that the light of *Eros* is variable suggests some photometric problems of great interest. If, as seems probable, we assume that the variation is due to the rotation of the planet, we can, from measures of its light, determine the time of rotation, and the direction in space of the axis of rotation. Owing to the varying position of the observer with regard to the planet, much information can be obtained which is impossible in the case of a variable star.

Four corrections must be applied to the observations. First, for the velocity of light; second, for the distance of the Sun and Earth; third, for phase; and fourth, for the direction of the axis of rotation. If this axis were pointed toward the observer, no variation would be perceptible, while the range in brightness would attain its maximum value when the axis was at right angles to the line of sight. Neither of these conditions can be fulfilled exactly, since the position of the axis is probably nearly fixed, and the inclination of the orbits of *Eros* and the Earth would make great changes in this angle. Let $p$ represent the complement of the altitude of the Earth above the equator of *Eros*, which will be equal to the angle between the axis of *Eros* and the direction of *Eros* as seen from the Earth. Let $v$ denote the angle between the plane passing through the Earth and the axis of *Eros*, and any other plane passing through the axis of *Eros*, assumed as an origin. A most important correction depends upon $v$. The time of all the observations must be corrected by an amount equal to $v$ divided by $360^\circ$ and multiplied by the period of variation. As a first approximation, we may assume that the axis of *Eros* is parallel to that of the Earth, and that the plane passing through the vernal equinox is taken as an origin. In that case, $v$ will equal the right ascension of *Eros*. As stated above, if $p = 0^\circ$ there will be no variation in light, and the range will be zero. If $p = 90^\circ$, the range will attain its maximum

[1] *Harvard College Observatory Circular* No. 58.

value. For intermediate values of $p$, we may assume that the range will be proportional to $\cos p$. The changes in the range may be used to determine the value of $p$, and from it the position of the axis of *Eros*. Equations may be formed in which $p$ and $v$, or $p$ alone, are the unknown quantities from which we may derive the approximate position of the axis. Besides observations at the present time, it will be necessary to determine the light curve when *Eros* is in several other portions of the sky, determining the range and also the times of maximum and minimum as accurately as possible. The rapid motion of *Eros* renders it difficult to compare the observations on different nights, without using different, and in some cases, distant comparison stars. Fortunately, the change in light is so rapid that consecutive observations of a large part of the light curve can generally be made. The opposition of 1894 would have been particularly favorable for these studies, since the declination changed from $+57°$ to $-14°$ in a few months, and would thus have furnished large coefficients for determining the value of $p$, although, as shown below, the range seems to have been small at that time.

Assuming that the variation in light of *Eros* is due to its rotation, two explanations may be offered, as in the case of variable stars of short period (*Proc. Amer. Acad.*, 1881, XVI, 257). First, that *Eros* is darker on one side than on the other, as is probably the case with *Iapetus*, the outer satellite of *Saturn*, and secondly, that it is elongated, or double, as has been assumed by M. André and others (*Astron. Nach.*, 155, 30). In the first case, the successive maxima would always have the same intensity, and would succeed each other at equal intervals which would be equal to the period of revolution. The same would be true for the minima. In the second case, if the two bodies differed in diameter, the successive maxima and minima might have unequal intensities, and if the orbit were elliptical the intervals between them would be alternately long and short. This seems to be the case with *Eros*, and the first hypothesis seems therefore improbable.

On the other hand, if the variation in light is caused by two similar bodies alternately eclipsing each other, it is difficult to see how more than half the light can be cut off in each case, and the minima more than three quarters of a magnitude fainter than the maxima. It then becomes necessary to assume that the two bodies are of unequal brightness, that they are elongated, or that we have a single body of the shape of a dumb-bell. Some observers have found the minima two

magnitudes fainter than the maxima. To account for this, we should be obliged to assume that one axis of the body was six times as long as that at right angles to it. Observations show that the light of *Eros* is continually varying, while if the case were that of a simple eclipse, as in the stars of the *Algol* type, we should expect that it would retain its full brightness for a large portion of the time.

If the bodies were of the same size, and the orbit circular, it might be impossible, from the light curve, to distinguish between the two hypotheses. The fourth of the corrections mentioned above, however, furnishes a means of distinguishing between them in any case. If the body is dark on one side, the time of revolution will equal the interval between the successive maxima, and the correction for the position of the observer will be proportional to this quantity. If then the position changes 180°, the correction will be one half the interval between the successive maxima. In the second case, the time of revolution will be double this, that is, equal to the interval between a given maximum and the next but one, so that the correction for position will now be twice as great as before, and approximately equal to the interval between the successive maxima.

Much material already exists for determining the constants mentioned above. Several of the photographs of *Eros* taken in 1893, 1894, and 1896, had an exposure of an hour or more. Owing to the motion of *Eros*, it formed a trail on each of these plates, which in some cases show distinct variations in brightness. This was noticed when the plates were first examined, but was supposed to be due to changes in the haziness of the air. As this is an easy method of discovering the variability of an asteroid, it is hoped that astronomers engaged in a photographic search for such objects will examine carefully all trails, to detect any changes in intensity. An examination of forty-one asteroid trails photographed with the Bruce telescope, seven of them on a single plate, failed to show, except in one or two instances, any change beyond that apparently due to varying atmospheric absorption. Generally, more than one asteroid appeared on each plate, and in such cases all showed the same changes in intensity.

The photographs of *Eros* taken in 1893 and 1894 failed to show any marked variations in light, and it is probable that the range was, at that time, small. The first three photographs were taken on October 28, 30, and 31, 1893, and included the same region, so that *Eros* could be compared with the same stars on all. On the first photograph it

was estimated to be 0.20 mag. fainter, and on the second 0.17 mag. fainter than on the third. The corresponding times, expressed in Julian Days and fractions following Greenwich Mean Noon, are 2,412, 765.913, 2,412,767.846, and 2,412,768.890, respectively. The corrections mentioned above for velocity of light, and for the position of the Earth, have not been applied. No conclusions can be drawn from the plates taken on January 1 and 8, 1894. The plate taken on January 30, 1894, shows that the light was nearly constant during the first 30 minutes of the exposure. The Bruce plate taken on February 5, 1894, shows that the light was nearly constant during the first 12 minutes of the exposure, diminishing by about 0.4 mag. during the remainder of the exposure. A maximum is therefore indicated at about 2,412,865.622.

The plates taken during 1896 give more conclusive evidence of changes. The plate taken on April 6 showed an increase of light during the first part of the exposure, and indicated a probable but somewhat uncertain maximum at 2,413,656.890. One plate was taken on June 4, and two on June 5. The first of these images was estimated to be 0.20 mag. fainter, and the third 0.83 mag. fainter than the second. The first also indicated a maximum at about 2,413,715.702. The times of the three plates were 2,413,715.694, 2,413,716.829, and 2,413,716.-919. A maximum is indicated by the plate taken on June 29, at about 2,413,740.803. The Bruce plate taken on June 30, shows a probable increase, followed by a very marked decrease, and indicating a maximum at 2,413,741.561.

The photometric measures made in 1898, and described in *H. C. O. Circular* No. 34 (*Astron. Nach.* **147**, 363), furnish an accurate determination of the times of maximum, and of the range for that epoch.

A very large number of photometric measures of *Eros* have been made since July 1900. Observations have been obtained with the 15-inch equatorial on 51 nights, the number of photometric settings each night being, in general, 32, but sometimes more. It has often been observed on 56 nights with the 12-inch horizontal telescope, 32 or more settings being made each night. Some months will be required to reduce these observations completely, owing to delay in adopting magnitudes of the comparison stars. It is hoped, however, to issue shortly another *Circular* giving the results of a preliminary discussion of these observations, and of those described above.

EDWARD C. PICKERING.

April 24, 1901.

## VARIATION IN LIGHT OF *EROS.*

THE range of variation in the light of *Eros*, which has been diminishing during the spring, has now become zero. In February 1901 it was found by European astronomers to amount to 2.0 mag. Observations by Professor O. C. Wendell, with the Harvard Equatorial, showed that the range on March 12, 1901 was 1.13 mag.; on April 12 it was 0.40 mag. and on May 6 and 7 it was imperceptible and apparently less than 0.1 magnitude.

EDWARD C. PICKERING.

May 8, 1901.

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## *NOVA PERSEI.*

IT was pointed out in *Astronomy and Astro-Physics*, 13, 201, that all the phenomena connected with the spectrum of a *Nova* could be readily explained if we supposed the appearance to be caused by an outburst of hot gases, which cooled as they receded from the star. The approaching gases being comparatively cool on the side turned toward us would present a spectrum of dark lines. The receding gases being hot on the side toward us would give a bright line spectrum. Since the direction of the velocity of the gases on the further side of the star in the line of sight was the reverse of that of the nearer gases, the latter could not mask the bright lines, and we should accordingly have a superposed spectrum of bright and dark lines as shown.

Perhaps the more generally accepted explanation of the phenomenon presented by a *Nova* is that it is due to the collision of two bodies, solid, meteoric, or gaseous, moving in opposite directions nearly in the line of sight.

There is, it appears, a crucial test of the validity of these two hypotheses, which we are now for the first time able to apply. On the latter theory, the relative velocity of the sources of the dark and bright lines must be less after the collision than while it is taking place, and while the *Nova* is at its brightest, and under no circumstances can it be much greater. On the former theory, on the other hand, while the gases are working their way through the resisting surface of the star, and heating it, their relative velocity will be less than after the surface has given way, and they are free to expand unresisted. In other words, the question is, was the relative velocity of the two sources at the time of maximum brilliancy greater or less than it was afterward?

*Nova Persei* is the first of these objects whose spectrum it has been possible to study photographically at the time of maximum brilliancy. The answer to our question is given very definitely by the *Harvard Circular* No. 56. It there appears that upon the night of February 23, the maximum, the spectrum consisted of a bright band crossed by dark lines. Of these, the hydrogen lines were seen, on careful examination, to be bright on the side of greater wave-length. That is to say, the relative velocity of the sources of the bright and dark lines was small. On the next night, however, they were well separated, and the spectrum presented the usual appearance of a *Nova*. At the same time the brilliancy of the object was appreciably reduced.

From these facts I conclude that, as far as these observations go, the collision theory has been rendered untenable, and the explosion theory has been corroborated.

William H. Pickering.

Harvard Observatory,
Cambridge, Mass., March 21, 1901.

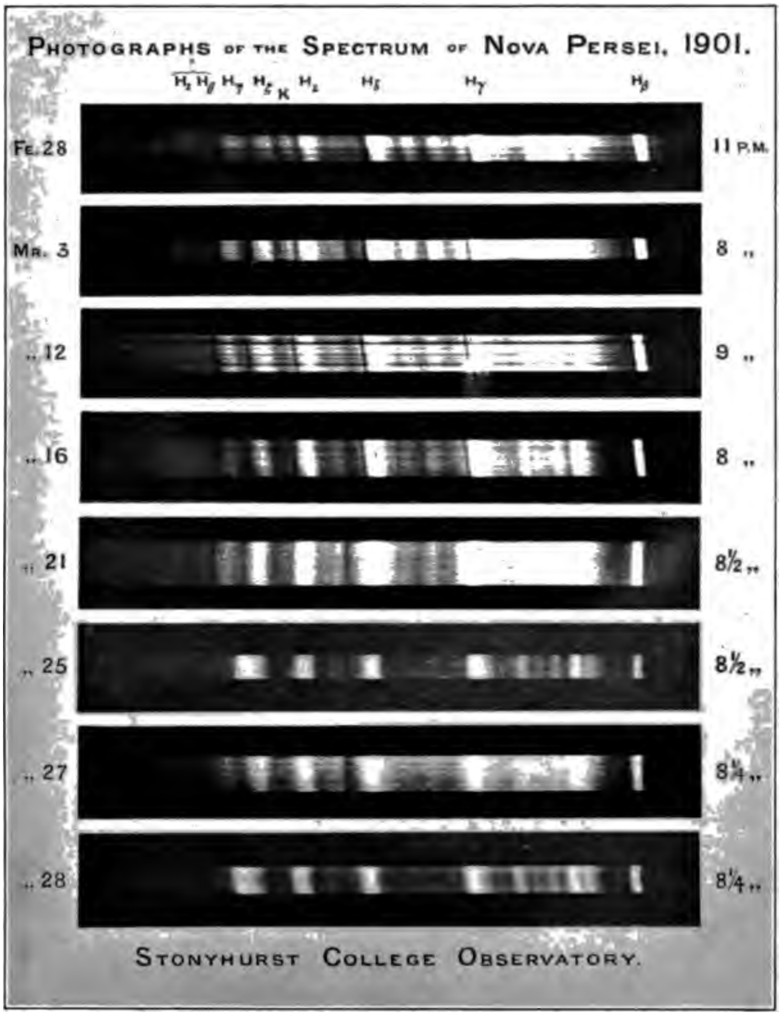
## *NOVA PERSEI.* FEBRUARY 28 TO APRIL 4: STONYHURST COLLEGE OBSERVATORY.

The finer details of the negatives are mostly lost on the enlargements (Plate X). But a feeble central shading of bright $H\beta$ can be detected on all the dates except March 25.

More details appear on the photograph of March 12, which is the best plate for definition, but is disfigured at $H\gamma$ by a flaw on the film. The double character of the dark $H$ lines is well seen on this enlargement in $\gamma$, $\delta$, $\epsilon$, $\zeta$, and fairly well in $\eta$.

The variable spectrum corresponding with the three-day light-period is shown in the last four spectra. The photograph of the 27th was not very successful; but on examination it will be found to be closely the same as the better photograph of the 21st. These dates are the days preceding a minimum of the light curve. The spectrum on the minimum of the 22d has been omitted; but it is in every detail the same as that of the 25th, so that the three minima are represented by the two photographs of the 25th and 28th, which show a remarkable extension of bright $H\zeta$ nearly covering the entire space up to $H\eta$, and the bright blue bands asserting their gaseous origin.

PLATE X


PHOTOGRAPHS OF THE SPECTRUM OF NOVA PERSEI, 1901.
FE. 28
11 P.M.
MR. 3
8 „
„ 12
9 „
„ 16
8 „
„ 21
8½ „
„ 25
8½ „
„ 27
8¼ „
„ 28
8¼ „
STONYHURST COLLEGE OBSERVATORY.

On March 31 and April 3, supposed minima, the sky was overcast. On April 4 a good photograph showed the same spectrum as on March 27 except that $H\zeta$ was marked by a strong line of maximum brightness considerably to the violet side of its wave-length center. The star was brighter; its continuous spectrum was stronger; and the $H\zeta$ extension absent.

WALTER SIDGREAVES, S. J.

# *Reviews*

*Annals of the Astrophysical Observatory of the Smithsonian Institution.*[1] Volume I. By S. P. LANGLEY, Director, aided by C. G. ABBOT.

IN the ASTROPHYSICAL JOURNAL for February 1895 a brief account was given of the bolometric investigations then in progress at the Smithsonian Institution. Since the publication of this paper important improvements have been made in the apparatus and in the conditions under which it is used. The effect of these changes upon the bolometric results has been marked. In fact, the experimental period of the research is so far past that it has become possible to make a long series of bolometric records of the lines in the infra-red solar spectrum upon a uniform system. The present volume contains a detailed description of the apparatus and methods employed in this research, together with an extensive map of the infra-red spectrum and a table of wave-lengths of 740 lines determined from the bolographs.

It is interesting to remember in the present connection that the development of an astrophysical observatory as a part of the Smithsonian Institution is in harmony with the ideas regarding the purpose of the institution entertained by certain prominent members of Congress at the time of its foundation. In 1838 ex-President Adams, when asked by the secretary of state as to the proper disposition to be made of the Smithson legacy, strongly urged the establishment of a great astronomical observatory. For many years Mr. Adams continued to advocate in Congress that the income of the Smithson fund for seven successive years should be used to found an observatory. Subsequently, in his opinion, the income should be employed "to promote establishments for increasing and diffusing knowledge among men." The present volume of *Annals* is satisfactory evidence that, although Mr. Adam's wishes were not complied with in the early days of the Smithsonian Institution, an observatory has nevertheless been established, which with very modest equipment has produced results of the first importance.

[1] Reviewed from advance sheets.

The volume opens with a brief historical statement, from which it appears that the work of the Observatory was begun in 1890 in a temporary shed erected immediately south of the main Smithsonian building. From time to time various additions have been made to the "temporary shed," but it still continues to be the principal Observatory building. The needs of the work were reported to Congress in 1891 by the secretary and since that time an annual appropriation has been made for the maintenance of the Astrophysical Observatory. The researches of the Observatory, which have been carried on under the general supervision of Secretary Langley, have been in direct continuation of his well-known investigations at Allegheny. Credit is given to the assistants who have been in immediate charge of the work, and particularly to Mr. C. G. Abbot, whose labors during the last few years have done much to establish the high degree of efficiency of which the results here recorded offer abundant evidence.

As might have been expected in an investigation of this kind, the early years of the Observatory were devoted to the development of apparatus and methods capable of giving the desired results in spite of the unfavorable character of the surroundings. Anyone who has used the bolometer is aware of the difficulty of obtaining good results when a high degree of sensitiveness is required. In visual observations the fluctuations due to "drift" may not be serious unless the drift is very rapid. But in cases like the present, where the galvanometer deflections are photographically recorded, it becomes necessary to eliminate the drift as far as this can possibly be done. For this purpose automatic control of the heating supply was introduced in 1896, and a cooling system, also controlled automatically, was provided in 1897. Successive improvements in the bolometer case and in the temperature control of the various resistances in the bridge circuit have finally resulted in reducing the drift to a minimum. As more and more sensitive galvanometers were constructed, the difficulty of protecting these delicate instruments from disturbances of various kinds became greater and greater, particularly in view of the fact that wagons and railway cars are constantly passing near the Observatory. The detailed description of the apparatus finally adopted, which will prove of the greatest interest to spectroscopists, is given in a later section of the volume, and will be referred to more particularly below.

Chapter 1 contains a short account of previous investigations of the infra-red spectrum. After referring briefly to the work of Herschel,

Lamansky, Abney, and others, Dr. Langley's early researches at Allegheny are described somewhat more fully. The solar spectrum, as mapped with the bolometer at Allegheny, ended near the band Ω, at wave-length about $2\mu$. In 1883 Dr. Langley conducted an expedition to Mt. Whitney in southern California, where at an altitude of 12,000 feet he detected for the first time an extensive region of greater wave-length, the detailed investigation of which forms the principal subject of the present volume. Chapter II outlines the progress of the research at the Smithsonian Astrophysical Observatory up to July 1, 1897, by which time the position of 222 lines in the infra-red spectrum between wave-lengths $0.76\mu$ and $5.3\mu$ had been accurately determined. Chapter III brings us to a description of the Observatory and the apparatus at present employed.

There can be no doubt that delicate bolometric work of this character, which requires great uniformity of temperature, could be done to better advantage in a building of brick or stone. To obviate as far as possible the fluctuations in temperature, which would be especially troublesome in a light wooden building, the sensitive parts of the apparatus are placed within an interior chamber, which is maintained at as nearly as possible a constant temperature of 20° C. by means of heating and cooling coils provided with a system of automatic regulation. This has proved so successful that the extreme variation within the inner chamber during twenty-four hours is frequently as small as 0°.1 C.

The immediate object of the investigation is to obtain a photographic record of the galvanometer deflections caused by the passage of the infra-red spectrum of sunlight over a fixed bolometer. A large siderostat of the Foucault type, with plane mirror 46 cm in diameter, stands to the north of the building and sends a beam of sunlight to a slit mounted upon a pier within the Observatory. After passing through the slit the rays encounter a convex cylindric mirror from which they are reflected to a concave cylindric mirror, the two being so adjusted as to collimate the beam. The parallel rays now fall upon a rock-salt prism of about 60° refracting angle, and are then reflected from a plane mirror whose silvered face forms an extension of the base of the prism. This prism mirror system, which was introduced at the Smithsonian Observatory by Professor F. L. O. Wadsworth, has the advantage that after being once adjusted for minimum deviation it will remain in adjustment when rotated, at the same time causing

the spectrum to move over a fixed bolometer strip. The image is formed upon the bolometer by a concave mirror, and a double convex cylindric lens of rock salt is interposed in front of the bolometer case in order to reduce the height of the spectrum to that of the bolometer strip. It will be understood that all parts of the spectroscopic apparatus remain fixed, except the spectrometer table upon which the prism-mirror combination stands. This table is rotated at a uniform rate by an accurately constructed driving-clock, which also moves a photographic plate in a vertical direction behind a horizontal slit. In an adjoining room a spot of sunlight is reflected from the galvanometer mirror through the horizontal slit to the sensitive plate. When, through the rotation of the prism, a dark band in the spectrum comes into coincidence with the bolometer strip, the spot of light from the galvanometer mirror moves horizontally across the photographic plate, which thus records the deflection.

The curves thus photographed accurately represent the distribution of energy in the infra-red spectrum, except in so far as they are affected by changes in the battery current, passing clouds, magnetic or mechanical disturbances of the galvanometer needle, lack of perfect synchronism in the motion of the prism and the plate, etc. The close agreement of bolographs of the same region of the spectrum is sufficient evidence that the greater part of the disturbing causes have been eliminated. The principal difficulty is that arising from the "drift." The use of a constant temperature laboratory has not proved sufficient to prevent drift. The regulation, as remarked above, is accurate to about $0^\circ.1$ C. Fluctuations of temperature of this magnitude in various parts of the bridge circuit are quite great enough, however, to cause large deflections. In order to keep all parts of the bridge circuit at practically the same temperature, Mr. Abbot has designed an ingenious form of bolometer case which contains not only the bolometer itself and the coils that form the other two arms of the bridge circuit, but also the balancing device with which the perfect equilibrium of the circuit is secured. The bolometer is protected from the air by a rock-salt cylindric lens which closes the front of the case. The resistance coils, which are of platinoid wire wound in a double spiral, surround a tube which contains the eyepiece for viewing the bolometer from behind. The entire case is enclosed in a waterjacket. Balancing is easily effected without opening the case.

The elimination of drift depends not only upon the constancy of

the battery and the uniformity of the temperature in the bolometer circuit but also upon the construction of the bolometer itself. Difficulty sometimes arises from the use of very thin platinum for the bolometer strip. The reviewer has found that electrolytically deposited platinum of thicknesses ranging from $0.3\mu$ to $0.6\mu$ frequently contains minute holes and is liable to give trouble when employed for bolometers. The experience of the Smithsonian observers has led them to employ rolled platinum not less than $1\mu$ thick. The only flux which can safely be used for soldering is resin.

No part of the apparatus is more important than the galvanometer, and here marked advances have been made at the Smithsonian Observatory. Through successive improvements the galvanometer constant (current required to give a deflection of 1 mm on a scale at 1 m with a time of single swing of the needle of 10 s) has been reduced from $2.4\times 10^{-7}$ to $2.0\times 10^{-11}$. In order to protect it against disturbances the needle system is suspended in an air-tight case and the entire galvanometer is floated upon mercury in a lead pan hung from a modified form of Julius suspension system. A battery of sixty storage cells protected from sudden temperature changes supplies the current. This has proved to be satisfactory, but its performance has recently been surpassed by that of a ten-cell Cupron battery.

All of these parts of the apparatus are fully described, in many cases with working drawings, in chapter III, which also explains the various adjustments. Chapter IV gives the exact procedure in preparing and comparing bolographs. An important part of the process, which furnishes a check on the reality of the deflections, should be mentioned here. After the apparatus has been set in motion and before the bolometer has been exposed, a run of several minutes is always made to determine the magnitude of accidental disturbances. If there were no disturbances of any kind and if the battery current were perfectly constant throughout this run, it is evident that the trace recorded on the photographic plate would be a straight line. In practice it is found that the line is broken by numerous small deflections, averaging about 0.4 mm. In comparing bolographs, a record of these accidental deflections is indispensable in deciding as to the reality of the less conspicuous solar lines.

Chapter V contains an interesting discussion of limitations of the method and existing sources of error in the apparatus. Here may be found statements regarding the resolving power of the rock-salt prism,

the slit-width employed in practice, a discussion of the use of an uncollimated beam, and an investigation of the loss of energy by diffraction with narrow slit. It is shown that with a slit-width somewhat less than that used (0.15 mm) the effects of diffraction would be very considerable. This result led to the adoption of a pair of cylindric mirrors in place of the spherical mirror system formerly used. In a discussion of resolving power it is concluded that bolographic resolution may in certain cases exceed the visual because of the bolometer's power of detecting differences of brightness which are inappreciable to the eye. Much space is then given to an investigation of errors arising from a variety of causes, such as shifting of the slit image due to temperature changes in the mirror supports, effect of temperature changes on the index and angles of the rock-salt prism, unsteadiness of galvanometer, drift, unsteadiness of the battery, irregularities in the motion of the plate and circle, errors of comparator observations in measuring bolographs, lag of galvanometer record, errors due to the width of the slit and the bolometer strip, etc. From the entire discussion the conclusion is drawn that the probable error in a determination of the relative angular deviation of any well-marked absorption line in the infra-red is less than one second of arc. In view of the difficulties of the investigation, this result, which is often not greatly surpassed in visual measurements with a spectrometer, may be considered extremely satisfactory.

The positions of the lines measured in the infra-red solar spectrum, together with a table of wave-lengths expressed to the nearest tenth-meter, are given in chapter VI. The plates which accompany this part of the volume reproduce bolographs of the infra-red solar spectrum made with a 60° rock-salt prism on various dates, together with "cylindrics" and line drawings of the spectrum made from the galvanometer curves. As in several cases bolographs of the same region taken on different dates are reproduced on the same plate, it is possible to make a comparison of the corresponding maxima in the energy curves. In general it may be said that the agreement of the curves is surprisingly close and that there is much less drift than is ordinarily encountered with less highly developed apparatus.

The bolographs were taken with a large rock-salt prism at the speed ratio 1 cm of plate = 1′ of spectrum = 1 minute of time. This applies to the linear spectrum shown in Plate XX, which represents graphically the results given in the table of wave-lengths. The map is

in two parts, the first extending from the limit of the visible spectrum to the Ω line at $1.8\mu$. The less refrangible portion begins at Ω and extends to $5.3\mu$, thus including the region discovered by Dr. Langley on Mt. Whitney. The solar radiation below this point is not sufficiently marked to be included in this map. Some of the bolographs, covering limited regions of the spectrum, were taken with the greater dispersion of a 60° glass prism. These curves show that certain bands which were formerly supposed to be single lines can be resolved into many fine lines. This is true of the bands $\omega_1$ and $\omega_2$, which are now found to be similar to the A line, both in constitution and in variability with the altitude of the Sun. The reader's confidence in the identification of the smaller deflections will be increased by the full discussion of the effect of various disturbances and the evident pains which have been taken to eliminate all possible sources of error. Such records must almost inevitably contain a great number of small irregularities, which a too sanguine observer might be inclined to consider as real lines. Some of the earlier publications regarding the Smithsonian bolographic results, as was pointed out in the article in the ASTROPHYSICAL JOURNAL already referred to, overestimated the number of solar lines actually recorded.

The present table of wave-lengths contains 740 lines measured by two observers on twenty-one bolographs. Some of these bolographs were taken with a rock-salt prism and others, of the region A to Ω, with the glass prism. Some twenty auxiliary bolographs were used to check the results in special cases. Deflections smaller than 0.4 mm, the average accidental deflection in curves made with the bolometer covered, were rejected. All of the examinations and measurements of the bolographs were made by two observers independently, and the tables include only such deflections (with a few exceptions) as were agreed upon by both observers.

In notes which accompany the tables the data for the various bolographs are given. Tables 18 and 19 contain all the measures of each line made by both observers. Table 20 contains the ordinates of the minima of the rock-salt bolographs, which serve to give an approximate measure of the relative intensity of various lines. The drift enters here as a disturbing factor and in some cases an error as great as 5 mm may occur. It may be noted, however, that many of the results contained in this table should have a far greater claim to accuracy than the very rough estimates of intensity which usually accompany

wave-length tables. In fact, the galvanometer record represents the spectrum in a much more satisfactory way than a photograph showing the lines themselves. Shadings, reversals, and other peculiarities of lines, which in an ordinary spectrum photograph are seen as it were in plan, are here shown in elevation, to the great convenience of the observer.

A full statement of the method of reducing the bolographs is next given. The most important point here is the determination of the wave-length. For this purpose a new investigation of the dispersion curve of rock salt was made with the large prism; a full account of this work is given in Part II. The angular distances from the A line of lines of known wave-length were first determined directly. The wave-lengths corresponding to the measured positions of the absorption lines were then interpolated from a curve plotted with wave-lengths and corresponding distances from A as coördinates. The table of wave-lengths of 740 lines follows.

Some space is given to a comparison of the bolographic results with the solar spectrum maps of Higgs and Abney and the infra-red metallic lines of Snow, Lewis, and others. The detailed comparison with Higgs' map, in view of the much greater dispersion of the latter, the employment of a narrower slit, and the consequent possibility of photographing faint lines which are not recorded bolographically, is satisfactory. Wave-length differences of more than two Ångström units are uncommon, and in general the agreement is closer than this. For some reason one of the double lines in the bolograph of the A series is absent, and the existence of several lines, all except one of which were marked doubtful in the bolographs, is not confirmed. It is unfortunate, probably because of the small amount of work which has hitherto been done on the infra-red spectra of the metals, that no conclusions could be drawn regarding the origin of the solar lines.

But while it has not yet been possible to determine the presence of metallic lines in the infra-red spectrum of the Sun, there is evidence that at least seven of the nine great bands are largely due to absorption in the Earth's atmosphere. In this connection an interesting discovery has been made. In the first place there seems to be evidence of a seasonal change in the infra-red absortion, typical bolographs taken in the spring and autumn resembling each other more closely than a pair corresponding to spring and summer, for example. These comparisons are very difficult to make, and some doubt may be left in

the reader's mind as to the reality of the change referred to. For this reason it is not claimed that the results given are absolute, though it is considered that they contain facts of interest. Irregular variations of absorption are more clearly indicated. A low Sun, as Dr. Langley showed many years ago, produces little general diminution of energy in the region less refrangible than $1\mu$, in spite of its marked effect in the visible spectrum; but the present results seem to show that great changes in local absorption can take place during a brief time interval. A table giving the relative ordinates of nearly 600 lines on plates taken in 1896 accompanies this discussion. It seems probable that if this portion of the investigation is followed up, results of value to meteorology will be established.

Part II contains an account of various subsidiary researches, the most important of which relates to the dispersion of rock salt and fluorite. This investigation deserves more extended mention than can be given here. The dispersion curve of rock salt on a scale adequate to retain the accuracy of the data accompanies the text and an interpolation table for determining the wave-length for any desired deviation is also given. A comparison of different rock-salt prisms seems to indicate that all such prisms at the same temperature and air pressure have equal refractive indices. The other investigations of Part II include a study of the accuracy of the bolometer, measures of radiation from terrestrial sources, particularly the Welsbach burner, and valuable details regarding the construction by Mr. Abbot of a sensitive galvanometer.

The Appendix discusses various dispersion formulae and their application to the infra-red spectrum, gives a determination of the constants for rock salt in Ketteler's formula, and concludes with a study of the minute structure of the absorption band, $\omega_1$.

Secretary Langley and his associates are certainly to be congratulated on the character of the first volume of the Astrophysical Observatory's *Publications;* with this evidence of work accomplished before them, members of Congress can hardly fail to see the importance of establishing the Smithsonian Observatory on a permanent foundation. This is especially to be desired because of the fact that the U. S. Naval Observatory properly devotes little or no attention to astrophysical research. The United States should not fail to follow the lead of Germany and France by establishing a national Astrophysical Observatory under the auspices of the Smithsonian Institution.

G. E. H.

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# NOTES ON THE ZEEMAN EFFECT.

By N. A. KENT.

## INTRODUCTION.

IT has been shown by H. M. Reese[1] that the separation of the external components of the regular Zeeman triplet or quadruplet, as seen perpendicular to the lines of force, does not vary proportionally with the strength of the magnetic field in which the luminous source is placed. This fact was established for the zinc lines 4680.38, 4722.26, and 4810.71, and for the three homologous cadmium lines up to a field of about 26,000 c. g. s. units.

Reese also states, in referring to certain lines in the spectrum of iron, that "in comparing the separation of the lines between 3900 and 4450 it was at once observed that the lines could be broken up into two classes, in each of which the separation of the various lines was of the same magnitude. These two classes are identical with those for which Humphreys found that the shift due to pressure was the same. On these plates the separation is very small in all cases, owing to a weak field, and no accurate measurements were taken of the separation."

[1] ASTROPHYSICAL JOURNAL, 12, 120–135, 1900.

It appeared, then, to the author of the following paper to be a matter of no little interest to extend Reese's investigations on zinc, using higher field strengths, and also to make a more exhaustive investigation of the spectrum of iron, measuring with care the separations, and comparing the values so obtained with the shift for the various lines as given by Humphreys.[1]

The following investigation was made primarily with these two ends in view. From time to time, however, as subjects worthy of attention presented themselves, the scope of the work was broadened. In short, the results obtained deal with the following subjects:

1. The variation of the separation with the strength of the magnetic field for zinc and iron to a field of about 33,000 c.g.s. units.

2. The spectrum of iron, including: (*a*) The relation of the separation to the pressure shift as given by Humphreys; (*b*) a study of the iron lines which are affected by the magnetic field in an anomalous manner; (*c*) the laws governing the separation of the iron lines.

3. The spectra of nickel and cobalt—a search for peculiarities and for a law governing the separation.

4. The Zeeman effect along the lines of force.

5. The extension of Preston's law for the homologous lines of the spectroscopic series, namely, that for these lines the expression $\frac{\Delta\lambda}{\lambda^2 H}$ is a constant for lines whose wave-lengths are given by equal values of $n$ in the formula

$$\lambda^{-1} = A + Bn^{-2} + Cn^{-4} ,$$

where $\Delta\lambda$ stands for the separation of the outer components of the Zeeman triplet, quadruplet, etc.; $\lambda$ signifies the wave-length of the line in question; $H$, the strength of the magnetic field; $A$, $B$, and $C$ are constants; and $n$ is an integer, 3, 4, 5. . . . .

These subjects will subsequently be discussed in the order given above.

[1] ASTROPHYSICAL JOURNAL, 6, 169–232, 1897.

APPARATUS.

The grating employed was that used by Reese: Rowland mounting, concave, radius of curvature thirteen feet three inches, of 15,000 lines to the inch, fitted with the ordinary adjustable slit and camera-box with shutter. The first spectrum was the most brilliant; the third could not be used beyond $\lambda$ 4600 owing to mechanical interference of the beam and truck at that point. The plates used were Seed's "Gilt Edge" and Cramer's "Isochromatic Fast," and the "Erythro" plates of the International Color Photo Company, all cut to 1¼ by 11½ inches. The light from the luminous source passed through a Nicol prism (or doubly refracting rhomb) when such was necessary, a total reflecting prism of quartz (cut with its optical axis parallel to its edge), a condensing lens of quartz (cut with its optical axis parallel to its principal axis), 2½ inches in diameter and of 21 inches focal length; and thence to the spectrometer slit. The total reflecting prism was necessary owing to the size of the magnet and the shape of the room.

The electromagnet was that employed by Reese in his investigations conducted during the scholastic year 1899-1900. The cores, cylinders of soft iron 78.3 cm. long by 15.1 cm. in diameter, are each fitted with two coils of about 1600 turns each, formed of No. 9 B. & S. cloth-covered copper wire. The pole heads are bored to admit of viewing the luminous source along the lines of force if so desired; and, being movable normally to the axis of the cores, are held in position by bolts sunk in the cores themselves. The pole tips proper, of conical form, have a semi-angle of 15°. As used by Reese in his work perpendicular to the lines of force, these pole tips had screwed into them solid pieces of the same semi-angle. The flat surfaces of these latter had a diameter of 3.2 cm. In the following investigations these small terminal pieces were replaced by a pair whose semi-angle was 45° and whose diameter was but 1.5 cm. The result was about 15 per cent. increase in field strength. The field was very uniform at all sizes of gap used — 3 to 7 mm.; in fact, it varied by an amount about equal to the error of reading of the ballistic

galvanometer used to measure its strength. The two coils on each core were connected in series, the two pairs in parallel. A current of thirty amperes in the main circuit and twelve amperes in the coils gave a field of over 33,000 c.g.s. units for a gap of 3 mm.

The luminous source was a spark between metal terminals, which were ground flat and firmly held in the arms of an adjustable device fitted with two racks and pinions. Thus was furnished an exceedingly efficient means of adjusting the spark gap even during exposure; and, as a result, small terminals could be used and the metal fed in at any desired speed.

Nickel wire of about 1 mm diameter, and small pieces of cobalt soldered with silver solder to brass terminals and then ground flat, were used for the spectra of those metals and furnished no trouble even in strong fields. Iron wire of about 1½ mm diameter, ground flat to less than 1 mm thickness, was used for the iron spectrum. To keep these highly magnetic terminals away from the pole tips, adjustable bracings of asbestos, wood fiber and brass were used; and no difficulty was experienced in feeding in the metals during exposure. Zinc, cadmium and magnesium are handled easily. Mercury was fed from a reservoir of adjustable height through a rubber tube to the lower brass terminal, which was hollow. The upper electrode was of brass. The spectra of calcium and of strontium were obtained as follows: The chlorides of these metals were quickly ground fine in a mortar and put at once in test tubes in a desiccator. Copper wire of about 2½ mm diameter was drilled to the depth of about 2 cm so that merely a shell remained. The salt was then packed tightly in the tubular cavity, the end pinched and the wire pounded flat. The hygroscopic character of these salts is the objectionable feature.

The spark used as luminous source was produced as follows: An alternating current of 133 cycles per second was received at about 110 volts difference of potential. This was passed through an adjustable impedance of closed magnetic circuit and through a transformer whose ratio of transformation was 110 to 8000.

From the secondary of the transformer connections were made to a condenser placed as near as possible to the spark gap. From the condenser short thick wires lead directly to the spark gap. The impedance could be so adjusted that the spark was either brilliant, excessively disruptive and "cool" (if, indeed, it be proper to speak of the temperature of a spark), or less brilliant, more continuous and "hot," approaching—though but to a very slight degree—an arc. The difference of potential at the terminals of the transformer varied from 4 to 28 volts according as the spark gap used was small (about 1 mm) or large (about 7 mm). This gave a potential difference at the terminals of the secondary of from 300 to 2100 volts. The condenser was built of forty glass plates, of ⅛ inch thickness, separated by 36 square inches of brass foil. Thus was obtained a capacity of about 0.014 microfarads.

The adjustable impedance transformer and condenser were kindly loaned by Dr. E. F. Northrup and proved of great service.

Several turns of an air-coil of about 15 cm diameter were often inserted in the discharge circuit for various reasons which will be touched upon later.

The dividing engine used to measure the separation of the lines was that designed by Professor Rowland and used for the construction of his table of the solar spectrum. The error of the screw is far less than the error of setting on even the sharpest lines.

ACCOUNT OF EXPERIMENTS.

1. *The variation of the separation with the strength of the magnetic field.*—For Zinc: Table 1 gives the distance in Ångström units between the outer components of the sharp triplet zinc $\lambda$ 4680.38, between the means of the outer components of the sextuplet zinc 4722.26 (see Fig. 1), and between the outer components of the diffuse triplet zinc 4810.71. The values of the separation given represent the measurements on plates taken when the requisite conditions were obtained in two different ways: set I with a small magnetic

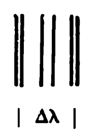
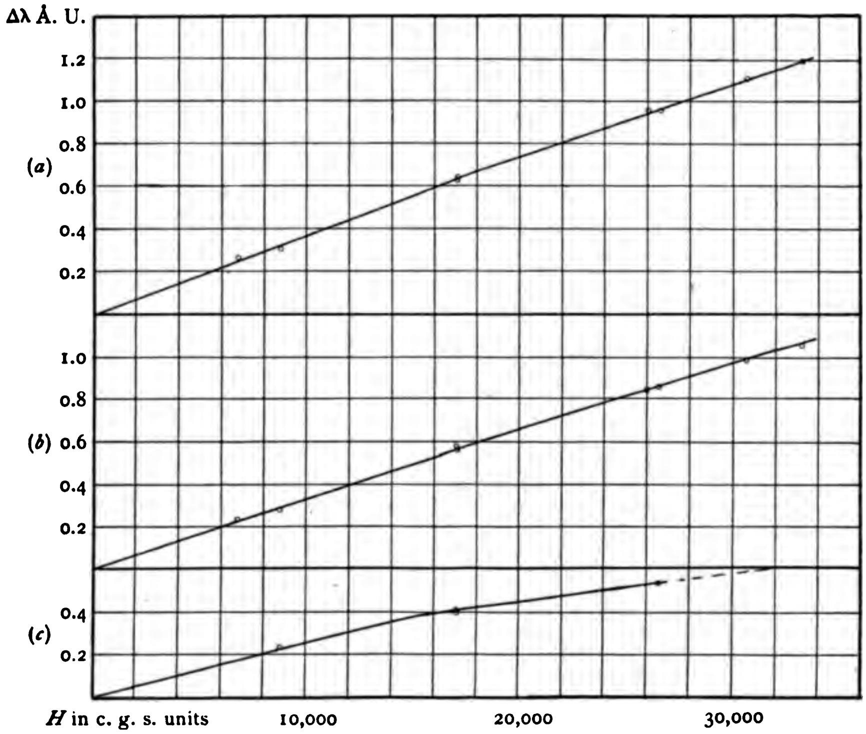



FIG. 1

gap and the magnetizing current changed to obtain different strengths of field, set II with the current constant and the width of gap changed. This shows the uniformity of the field and the ease of control of the conditions. The line 4722.26 appears as a




Zinc lines: (*a*) λ 4680.38; (*b*) λ 4722.26; (*c*) λ 4810.71.

FIG. 2. SEPARATION Δλ AS A FUNCTION OF FIELD STRENGTH *H* (TABLE I).

sextuplet only with the strongest fields, otherwise as a quadruplet.

The accompanying curves (Fig. 2) explain themselves. Reese's maximum field was 26,600 c. g. s. units. His results are in general confirmed. The drop in the curve is very marked in the case of 4810.71. The sharp triplet and the quadruplet give a curve which approximates a straight line.

TABLE I.

Zinc. $\Delta\lambda$ as a function of $H$.[1]

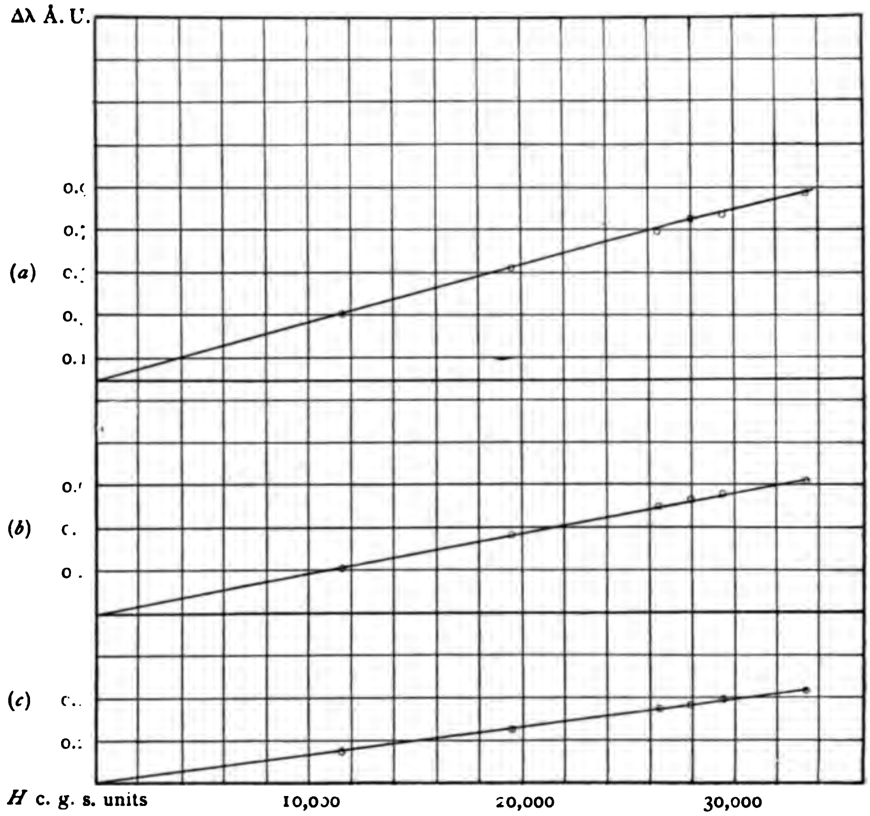
| $\lambda$ | Intensity[2] and character | $\Delta\lambda$ for given values of $H$ | | | |
|---|---|---|---|---|---|
| | | SET I | | | |
| | | 8.820 | 17.110 | 26.060 | 30.660 |
| 4680.38 | 10 r | 0.313 | 0.646 | 0.961 | 1.106 |
| 4722.26 | 10 r | .278 | .578 | .852 | 0.988 |
| 4810.71 | 10 r | .240 | .412 | .... | .... |
| | | SET II | | | |
| | | 6.800 | 17.140 | 26.630 | 33.320 |
| 4680.38 | 10 r | 0.266 | 0.630 | 0.959 | 1.194 |
| 4722.26 | 10 r | .238 | .568 | .864 | 1.060 |
| 4810.71 | 10 r | .... | .400 | .543 | ..... |

For Iron: Tables II to IV and Figs. 3 to 5 explain themselves. It will be noted that with this metal the drop appears to be less. The three lines, 3737.27, 3920.40, 4033.24, which as a result of the magnetic field appear in the spectrum as sharp triplets, one of small, one of medium, and one of large separation, were chosen at random; as were also the two lines, 3887.18 and 3903.09, which appear as quadruplets, and the two 3834.38 and 4030.89, which appear as somewhat diffuse triplets—in each of these last sets one of larger and one of smaller separation. 3834.38 is not very diffuse, while 4030.89 is quite so, and indeed, is classed as "nebulous" in Exner and Haschek's table.

The curves show that the separation is not proportional to the field strength for strong fields—the curves are not straight lines, but droop; and it is apparent that the division into classes of small and large droop is dependent upon the character of the line and not on the degree of initial separation, whether large or small.

[1] $\lambda$ and $\Delta\lambda$ in Ångström units. Wave-lengths as given in Kayser and Runge's tables. $H$ in c. g. s. units.

[2] Maximum intensity = 10; r stands for "easily reversed."




Sharp Iron Triplets: (*a*) λ 4033.24; (*b*) λ 3920.40; (*c*) 3737.27.

FIG. 3.—Δλ AS A FUNCTION OF $H$ (TABLE II).

The plates which gave these results both in the case of zinc and iron, were taken in the second spectrum; the time of exposure varied from 20 to 120 minutes; and the deviation of the mean in measurement was such that, in general, the values may be considered correct to 0.01 of an Ångström unit in the case of zinc and to 0.007 Å. U. in that of iron.

TABLE II.

Iron. $\Delta\lambda$ as a function of $H$. Sharp triplets.[1]

| Plate | Field | $\lambda$ | Intensity (10-max.) | $\Delta\lambda_s$ [2] | Mean $\Delta\lambda_s$ | $\Delta\lambda$ (A.U.) |
|---|---|---|---|---|---|---|
| 387 | 11.600 | 3737.27 | 8 | 0.0740 | | 0.160 |
| 387 | 11.600 | 3920.40 | 5 | .1000 | | .216 |
| 387 | 11.600 | 4033.24 | 1 | .1433 | | .310 |
| 386 | 19.600 | 3737.27 | 8 | .1203 | | .260 |
| 386 | 19.600 | 3920.40 | 5 | .1705 | | .369 |
| 386 | 19.600 | 4033.24 | 1 | .2425 | | .524 |
| 359 | 26.460 | 3737.27 | 8 | .1665 | | .360 |
| 359 | 26.460 | 3920.40 | 5 | .2328 | | .503 |
| 359 | 26.460 | 4033.24 | 1 | .3195 | | .691 |
| 345 | 28.000 | 3737.27 | 8 | .1715 | 0.1725 | .373 |
| 350 | 28.000 | 3737.27 | 5 | .1735 | | |
| 345 | 28.000 | 3920.40 | 1 | .2488 | .2488 | .538 |
| 350 | 28.000 | 3920.40 | 8 | .2488 | | |
| 345 | 28.000 | 4033.24 | 5 | .3460 | .3445 | .745 |
| 350 | 28.000 | 4033.24 | 1 | .3430 | | |
| 355 | 29.500 | 3737.27 | 8 | .1838 | | .397 |
| 355 | 29.500 | 3920.40 | 5 | .2595 | | .561 |
| 355 | 29.500 | 4033.24 | 1 | .3585 | | .775 |
| 400 | 33.400 | 3737.27 | 8 | .2025 | | .438 |
| 400 | 33.400 | 3920.40 | 5 | .2890 | | .625 |
| 400 | 33.400 | 4033.24 | 1 | .4018 | | .869 |

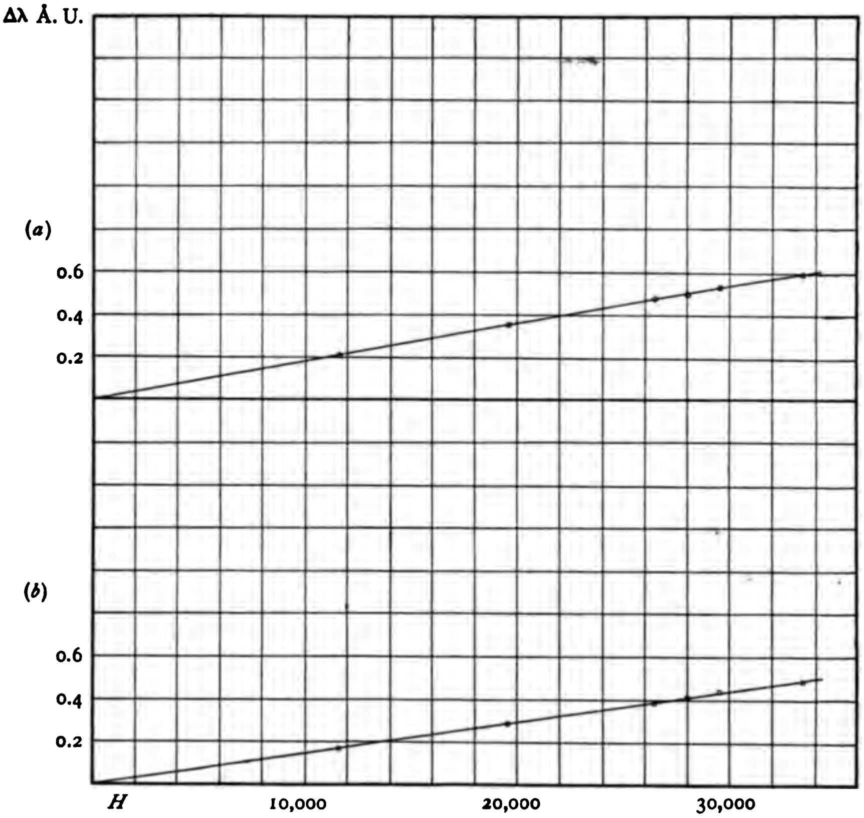
TABLE III.

Iron. $\Delta\lambda$ as a function of $H$. Quadruplets.

| Plate | Field | $\lambda$ | Intensity | $\Delta\lambda_s$ | Mean $\Delta\lambda_s$ | $\Delta\lambda$ (Å.U.) |
|---|---|---|---|---|---|---|
| 387 | 11.600 | 3887.18 | 5 | 0.0958 | | 0.207 |
| 387 | 11.600 | 3903.09 | 7 | .0803 | | .174 |
| 386 | 19.600 | 3887.18 | 5 | .1648 | | .356 |
| 386 | 19.600 | 3903.09 | 7 | .1333 | | .288 |
| 359 | 26.460 | 3887.18 | 5 | .2195 | | .475 |
| 359 | 26.460 | 3903.09 | 7 | .1793 | | .388 |
| 345 | 28.000 | 3887.18 | 5 | .2310 | | .499 |
| 345 | 28.000 | 3903.09 | 7 | .1905 | 0.1902 | .411 |
| 350 | 28.000 | 3903.09 | 7 | .1900 | | |
| 355 | 29.500 | 3887.18 | 5 | .2468 | | .533 |
| 355 | 29.500 | 3903.09 | 7 | .2043 | | .442 |
| 400 | 33.400 | 3887.18 | 5 | .2741 | | .593 |
| 400 | 33.400 | 3903.09 | 7 | .2258 | | .488 |

[1] Nearly all of the iron lines are sharp. With difficulty could diffuse lines be found. 4030.89 is the most diffuse of any and is marked "nebulous" in Exner and Haschek's table. From this table are taken also the wave-lengths and the intensity of the iron lines.

[2] $\Delta\lambda$ as measured in screw-turns of dividing engine.




Iron quadruplets: (*a*) λ 3887.18; (*b*) λ 3903.09.

FIG. 4.—Δλ AS A FUNCTION OF *H* (TABLE III).

TABLE IV.

Iron. Δλ as a function of *H*. Somewhat diffuse triplets.

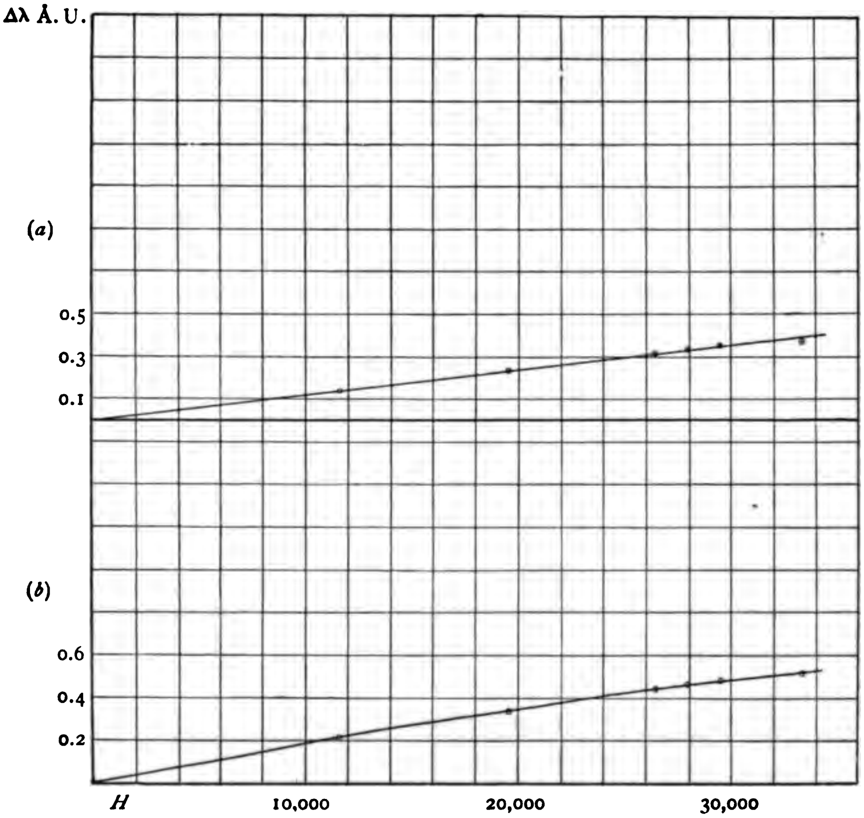
| Plate | Field | λ | Intensity and Character | $\Delta\lambda_s$ | Mean $\Delta\lambda_s$ | Δλ (Å. U.) |
|---|---|---|---|---|---|---|
| 387 | 11.600 | 3834.38 | 8 | 0.0638 | | 0.138 |
| 387 | 11.600 | 4030.89 | 2n[1] | .0981 | | .212 |
| 386 | 19.600 | 3834.38 | 8 | .1115 | | .241 |
| 386 | 19.600 | 4030.89 | 2n[1] | .1562 | | .338 |
| 359 | 26.460 | 3834.38 | 8 | .1495 | | .323 |
| 359 | 26.460 | 4030.89 | 2n[1] | .2035 | | .440 |
| 345 | 28.000 | 3834.38 | 8 | .1602 | | .346 |
| 345 | 28.000 | 4030.89 | 2n[1] | .2147 | } 0.2145 | .464 |
| 350 | 28.000 | 4030.89 | 2n[1] | .2144 | | |
| 355 | 29.500 | 3834.38 | 8 | .1658 | | .358 |
| 355 | 29.500 | 4030.89 | 2n[1] | .2223 | | .487 |
| 400 | 33.400 | 3834.38 | 8 | .1733 | | .375 |
| 400 | 33.400 | 4030.89 | 2n[1] | .2420 | | .523 |

[1] Nebulous.

TABLE Va.

Small Δλ. (0.28 to 0.55 Å. U.) and small ΔP (0. 01 to 0.02 Å. U.)

| Spark[1] | | Δλ[2] | ΔP[3] | Arc[4] | | C[6] |
|---|---|---|---|---|---|---|
| λ | $I_s$ | | | $I_a$ | λ | |
| 3969.40 | 8 | 0.55 | 0.01 | 8 | 3969.34 | P s r |
| 97.52[5] | 4 | .41 | .01 | 6 | 97.49 | B s u |
| 4030.89 | 2n | .46 | .01 | 6 | 4030.84 | B s r |
| 45.98[5] | 10 | .47 | .02 | 10 | 45.90 | B s r |
| 63.75[5] | 10 | .41 | .01 | 10 | 63.63 | B s r |
| 71.92 | 10 | .28 | .01 | 10 | 71.79 | B s r |
| 4118.72[5] | 5 | .41 | .02 | 10 | 4118.62 | B s u |
| 81.94 | 7 | .52 | .02 | 8 | 81.85 | B s u |
| 99.27[5] | 8 | .43 | .01 | 10 | 99.19 | B s u |
| 4219.51[5] | 6 | .46 | .01 | 8 | 4219.47 | B s u |
| 71.93 | 10 | .52 | .01 | 10 | 71.93 | B s r |
| 82.60 | 6 | .54 | .01 | 10 | 82.58 | B s u |
| 4308.06 | 10 | .48 | .01 | 10 | 4307.96 | B s r |
| 25.94[5] | 10 | .39 | .01 | 10 | 25.92 | B s r |
| 83.71[5] | 10 | .51 | .01 | 10 | 83.70 | B s r |
| 4404.94[5] | 10 | .51 | .01 | 10 | 4404.88 | B s r |




Somewhat diffuse iron triplets: (a) λ 3834.38; (b) λ 4030.89.

FIG. 5—.Δλ AS A FUNCTION OF $H$ (TABLE IV).

TABLE Vb.

Large Δλ (0.59 to 0.03 Å. U.) and large Δ P (0.05 to 0.12 Å. U.)

| Spark[1] λ | Spark[1] $I_s$ | Δλ[2] | ΔP[3] | Arc[4] $I_a$ | Arc[4] λ | C[6] |
|---|---|---|---|---|---|---|
| 4187.22 | 7 | 0.61 | 0.07 | 10 | 4187.17 | P n u |
| 88.00 | 7 | .60 | .10 | 10 | 87.92 | P n u |
| 98.50 | 6 | .59 | .08 | 10 | 98.42 | P n u |
| 4208.73 | 2 | .93 | .05 | 4 | 4208.71 | P n u |
| 22.35[5] | 4 | .77 | .09 | 8 | 22.32 | P n u |
| 36.09[5] | 8 | .67 | .06 | 10 | 36.09 | P n u |
| 60.64[5] | 10 | .68 | .10 | 10 | 60.64 | P n r |
| 99.43 | 7 r | .63 | .10 | 10 | 99.42 | P n u |
| ..... | .. | .71 | .12 | 10 | 5573.05 | P n u |
| ..... | .. | .80 | .10 | 10 | 86.92 | B n r |

TABLE Vc.

Large Δλ (0.60 to 0.81 Å. U.) and small Δ P (0.01 to 0.02 Å. U.)

| Spark[1] λ | Spark[1] $I_s$ | Δλ[2] | ΔP[3] | Arc[4] $I_a$ | Arc[4] λ | C[6] |
|---|---|---|---|---|---|---|
| 4005.40[5] | 8 | 0.60 | 0.01 | 8 | 4005.33 | P s r |
| 33.24 | 1 | .75 | .01 | 6 | 33.16 | B r & P n r |
| 4132.24 | 8 | .67 | .01 | 10 | 4132.15 | P s r |
| 44.06 | 7 | .60 | .01 | 10 | 43.96 | P n r |
| 4315.26 | 5 | .81 | .02 | 10 | 4315.21 | B s u |
| 76.10 | 4 | .65 | .02 | 8 | 76.04 | B s u |

TABLE Vd.

Small Δλ (0.41 to 0.48 Å. U.) and large Δ P (0.08 to 0.09 Å. U.)

| Spark[1] λ | Spark[1] $I_s$ | Δλ[2] | ΔP[3] | Arc[4] $I_a$ | Arc[4] λ | C[6] |
|---|---|---|---|---|---|---|
| 4233.74[5] | 6 | 0.41 | 0.08 | 10 | 4233.76 | P n u |
| ........ | .. | .48 | .09 | 10 | 5569.67 | B s u |

NOTES ON TABLES V a, b, c, d:

1. Exner and Haschek — Rowland's scale. Intensity and character that of original line.

2. Δλ, in Ångström units, taken, with three exceptions, from Table VII.

3. Δ P = approximate pressure shift in Å. U. as measured by me on plates taken by Dr. Huff at pressure of about 9 atmospheres.

4. Kayser and Runge; wave-lengths Rowland's scale. Intensity and character that of original line.

5. Lines which appear in Humphrey's table, page 200. ASTROPHYSICAL JOURNAL, **6**, October 1897, as corrected by Ames and others (see ASTROPHYSICAL JOURNAL, **8**, 50).

6. Character of original line and that under pressure as apparent on Dr. Huff's plates.

| | | |
|---|---|---|
| B n r | signifies: | Both original line and that under pressure non-symmetrical and reversed. |
| B s r | " | Both original line and that under pressure symmetrical and reversed. |
| B s u | " | Both original line and that under pressure symmetrical and unreversed. |
| B r | " | Both original line and that under pressure reversed. |
| P s r | " | Line under pressure symmetrical and reversed. |
| P n u | " | Line under pressure non-symmetrical and unreversed but very much broadened. Original line symmetrical and unreversed. |
| P n r | " | Line under pressure non-symmetrical and reversed. |

2. *The spectrum of iron.*

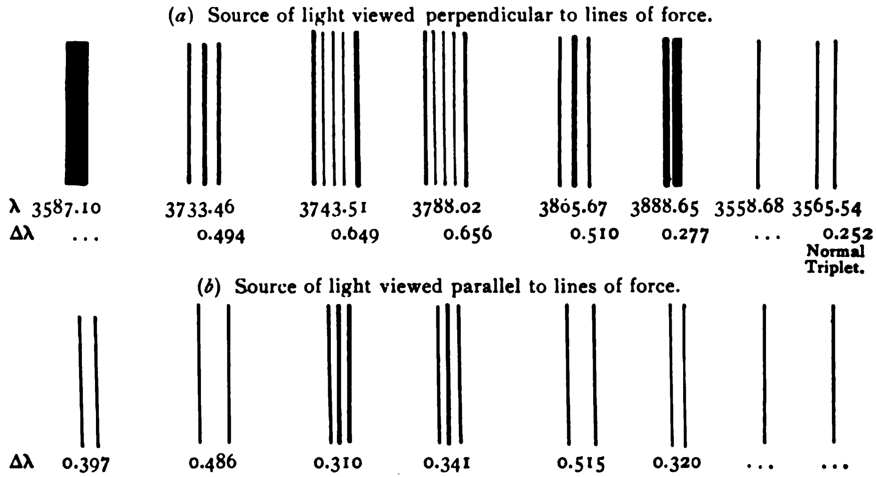
As an introduction it may be said that nearly all the results given under this section were obtained from plates taken in the second spectrum. The time of exposure varied from 30 to 135 minutes; and the values of the separation given are correct to 0.008 Å. U., generally speaking.

(*a*) The relation of the Zeeman effect to the pressure shift.

As, upon a preliminary survey, Reese's statement before mentioned was not confirmed by measurements of the separation made on my plates, and, as Humphrey's table had been shown incorrect in several particulars (see Ames, Earhart, and Reese, ASTROPHYSICAL JOURNAL, **8**, 50, 1898), it seemed best to go over Humphrey's work with care. This was rendered possible by the kindness of Dr. Huff, who happened to be working upon the pressure shift. He furnished me with three excellent plates taken (for a pressure of about 9 atmospheres) in the second spectrum of the 20-foot Rowland grating used by Humphreys. Tables V, a, b, c, d contain the results. It is apparent that Reese's statement, that the lines could be broken up into two classes in each of which the separation, $\Delta\lambda$, was of the same magnitude, these two classes being identical with those for which Humphreys found that the shift due to pressure, $\Delta P$, was the

same, is not verified. The statement was true of the lines on his plates—a mere coincidence. As appears from the tables given, the majority of lines obey Reese's law, 26 against 8; or 24 per cent. are exceptions, 76 per cent. follow the rule.

The conclusions which may be drawn from this study are that in general the lines for which $\Delta\lambda$ is large (1) show a large pressure shift and (2) as a result of pressure are broadened




Field $(H)$=28,000 c. g. s. units.

FIG. 6.—PECULIAR IRON LINES.

excessively and rendered unsymmetrical—whether reversed or unreversed—being sharper on the violet side and shaded toward the red. Again, in general, the lines which show a small $\Delta\lambda$ are, under pressure, symmetrical whether reversed or not. The exceptions are clear and unmistakable.

(*b*) A study of the anomalous iron lines.

Becquerel and Deslandres[1] have made quite an exhaustive study of those iron lines which do not appear as triplets or quadruplets of the ordinary type. I have studied these lines with care and made some measurements upon them. The conclusions reached agree with those of Becquerel and Deslandres in all cases save one—that of line 3888.63.

[1] *Comptes Rendus*, 127, 18–24, 1898.

The types appearing on my plates are most easily explained graphically, as given in Fig. 6. Tables VIa and VIb contain the measurements taken.

Vertical vibrations or those perpendicular to the lines of force:

3587.10 appears on all my plates as an indefinable band.

3733.46 is a sharp triplet of inner component twice as strong as the two outer.

3743.51 is a quintuplet. Of the two outermost components the red is the more intense. Between the two lies a band completely filling the gap, and this band is, on several plates, clearly marked by three very fine faint lines which appear to be separated by distances about equal to those components which are polarized in the other plane (see below). These three fine lines appear of equal intensity. The red component of the strong doublet appears the more intense irrespective of the field — a fact which proves that the asymmetry is not due to the interference of another line. Note the accompanying component's asymmetry.

3788.02 resembles 3743.51, but the two strong components are of equal intensity.

3865.67 resembles 3733.46 exactly.

3888.65 appears distinctly on several plates as two diffuse lines, the red being a trifle more intense. Becquerel and Deslandres describe this line as a characterless band instead of two diffuse lines.

TABLE VIa.[1]

Anomalous lines in the iron spectrum.

$\Delta\lambda$, or the separation of the components whose vibrations are perpendicular to the lines of force. $H = 28{,}000$.

| Plate | $\lambda$ | Intensity | $\Delta\lambda_s$ | Mean $\Delta\lambda_s$ | $\Delta\lambda$ (Å. U.) |
|---|---|---|---|---|---|
| 345 | 3733.46 | 6 | 0.2290 | 0.2284 | 0.494 |
| 350 | 33.46 | 6 | .2278 | | |
| 345 | 43.51 | 7 | .2965 | .3001 | .649 |
| 350 | 43.51 | 7 | .3038 | | |
| 345 | 88.02 | 5 | .3032 | ...... | .656 |
| 345 | 3865.67 | 6 | .2325 | .2360 | .510 |
| 350 | 65.67 | 6 | .2395 | | |
| 350 | 88.65 | 6 | .1279 | ...... | .277 |

[1] Notation used similar to that of Table II.

TABLE VIb[1].

Anomalous lines in the iron spectrum.

$\Delta\lambda'$, or the separation of the components whose vibrations are parallel to the lines of force. $H = 26{,}000$.

| Plate | $\lambda$ | Intensity | $\Delta\lambda'$s | Mean $\Delta\lambda'$s | $\Delta\lambda'$ (Å. U.) |
|---|---|---|---|---|---|
| 351 | 3587.10 | 5 | 0.1838 | ...... | 0.397 |
| 345 | 3733.46 | 6 | .2245 | ...... | .486 |
| 345 | 43.51 | 7 | .1403 | ...... | .310 |
| 351 | 88.02 | 5 | .1598 | 0.1578 | .341 |
| 345 | 88.02 | 5 | .1558 | | |
| 351 | 3865.67 | 6 | .2373 | .2382 | .515 |
| 345 | 65.67 | 6 | .2390 | | |
| 351 | 88.65 | 6 | .1490 | .1487 | .320 |
| 345 | 88.65 | 6 | .1483 | | |

Horizontal vibrations or those parallel to the lines of force:

3587.10 is a doublet.

3733.46 is a doublet whose components are separated (to within the error of observation) by a distance equal to the separation, $\Delta\lambda$, of the vibrations perpendicular to lines of force.

3743.51 is asymmetrical as noted above, the two red components being of equal intensity and both stronger than the violet component. Note that $\Delta\lambda =$ approximately $2\ \Delta\lambda'$.

3788.02 is symmetrical but otherwise similar to 3743.51. Here again note that $\Delta\lambda =$ approximately $2\ \Delta\lambda'$.

3863.67 is exactly similar to 3733.46.

3888.65 is a sharp doublet of $\Delta\lambda' > \Delta\lambda$. Therefore 3888.65 as a whole is of the form of an inverse quadruplet, $\Delta\lambda$ being unusually diffuse.

With regard to asymmetry, Zeeman[2] has, in weak fields verified to some extent Voigt's[3] theory. The only one of these lines which is given in Zeeman's list is 3733.46. This in weak fields is said to show reversed asymmetry—the violet component is nearer the central component than is the red. Line 3743.51 is not noted by Zeeman as unsymmetrical in the intensity of its components.

[1] Limit of error in $\Delta\lambda$ and $\Delta\lambda'$ about 0.01 Å. U.

[2] *Proceedings Royal Acad. Sci. Amsterdam*, **2**, 298–301, 1900.

[3] *Ann. der Physik*, **1**, 376–388, February 1900.

All these variations are interesting, but at present mean little. They merely show that the mathematical and mechanical theories which have been advanced in explanation are too simple —the complexity of separation and polarization has not yet been accounted for.[1]

(*c*) The laws governing the separation of the iron lines. Becquerel and Deslandres have made the following statements:

That the complexity of phenomena present in the iron spectrum renders it difficult to form a law governing the separation of the different lines, but that the following general chracteristics are apparent:

1. The separation in the ultra-violet is notably less than that in the blue, and the phenomenon appears a function of the wave-length, which increases with that variable.

2. If a restricted region—that very rich in lines—be examined it appears that many anomalous separations and separations of very different magnitudes lie in the immediate neighborhood of the lines which are insensible to magnetic influence. If an effort is made to classify the separations as a function of the wave-length of the corresponding lines it is evident that "for the most part" they can be put in different classes such that for lines of neighboring wave-length the separation is proportional to the numbers 1, 2, 3, 4. . . . . and that in one and the same class the separation varies as the square of the wave length.

3. Again, for the lines which are divided into fine components or inversely polarized, $\Delta\lambda$ and $\Delta\lambda'$ are for the same line, "exactly proportional to 1, 2, 3, 4 . . . ."

4. The distribution of the separation in the spectrum as a function of the wave-length shows in general a sort of periodicity.

The first statement is approximately true. The meaning of the second is not at all clear. No numerical data are given in their paper. The third statement is true for 3733.46, 3865.67, 3743.51 and 3788.02. The fourth is not apparent from the measurements made on my plates. To settle this last point

[1] See article by Lorentz, *Proceedings Royal Acad. Sci.*, 1, 340–359, 1899.

TABLE VII.

Plates 345 and 346. $H = 28{,}000$.

Iron spectrum. Lines appearing as triplets.

| $\lambda$[1] | Intensity | $\Delta\lambda$ | $\frac{\Delta\lambda}{\lambda^2} \times 10^{10}$ | $\lambda$[1] | Intensity | $\Delta\lambda$ | $\frac{\Delta\lambda}{\lambda^2} \times 10^{10}$ |
|---|---|---|---|---|---|---|---|
| 3565.54 | 8 r | 0.252 | 198 | 28.09 | 7 | 0.533 | 345 |
| 70.18 | 8 | .326 | 256 | 30.43 | 6 | .532 | 345 |
| 81.36 | 10 r | .359 | 279 | 69.40 | 8 | .545 | 345 |
| 3618.92 | 10 | .245 | 187 | 97.52 | 4 | .408 | 255 |
| 31.64 | 10 | .284 | 216 | 4005.40 | 8 | .594 | 370 |
| 48.00 | 9 | .291 | 219 | 30.89 | 2 n | .464 | 286 |
| 87.55 | 6 | .471 | 348 | 33.24 | 1 | .748 | 460 |
| 3709.40 | 6 | .473 | 344 | 45.98 | 10 | .473 | 289 |
| 20.10 | 8 | .393 | 284 | 63.75 | 10 | .411 | 249 |
| 27.78 | 6 | .478 | 344 | 71.92 | 10 | .277 | 167 |
| 35.01 | 10 | .453 | 325 | 4118.72 | 5 | .406 | 240 |
| 37.27 | 8 | .371 | 266 | 32.24 | 8 | .674 | 394 |
| 48.41 | 7 | .256 | 182 | 4144.06 | 7 | .603 | 351 |
| 49.64 | 10 | .432 | 307 | 81.94 | 7 | .521 | 298 |
| 58.39 | 8 | .403 | 286 | 87.22 | 7 | .611 | 348 |
| 63.91 | 7 | .328 | 231 | 88.00 | 7 | .599 | 342 |
| 65.70 | 5 | .339 | 239 | 98.50 | 6 | .589 | 334 |
| 95.15 | 6 | .508 | 353 | 99.27 | 8 | .426 | 242 |
| 98.68 | 6 | .491 | 340 | 4208.73 | 2 | .925 | 522 |
| 99.70 | 7 | .487 | 338 | 19.51 | 6 | .458 | 257 |
| 3813.12 | 5 | .312 | 214 | 22.35 | 4 | .768 | 431 |
| 15.99 | 9 | .382 | 262 | 27.60 | 7 | .483 | 270 |
| 3820.57 | 9 | .425 | 291 | 33.74 | 6 | .412 | 230 |
| 24.58 | 7 | .504 | 345 | 36.09 | 8 | .673 | 375 |
| 26.04 | 9 | .386 | 264 | 60.64 | 10 | .676 | 372 |
| 27.98 | 9 | .346 | 236 | 71.93 | 10 | .524 | 287 |
| 34.38 | 8 | .347 | 236 | 82.60 | 6 | .544 | 297 |
| 40.61 | 8 | .261 | 177 | 99.43 | 7 r | .632 | 342 |
| 41.21 | 8 | .273 | 185 | 4308.06 | 10 | .480 | 259 |
| 56.51 | 8 | .501 | 337 | 15.26 | 5 | .813 | 437 |
| 60.07 | 9 | .527 | 354 | 25.94 | 10 | .390 | 209 |
| 86.41 | 8 | .523 | 347 | 76.10 | 4 | .652 | 333 |
| 95.78 | 5 | .516 | 340 | 83.71 | 10 | .514 | 267 |
| 99.84 | 6 | .520 | 342 | 4404.94 | 10 | .512 | 258 |
| 3920.40 | 5 | .532 | 347 | 15.29 | 8 | .540 | 277 |
| 23.05 | 6 | .531 | 345 | 4528.80 | 6 | .569 | 277 |

three plots were made by me from values of $\Delta\lambda$ for lines 3565.54 to 4528.80 as given in Table VII. As abscissæ $\lambda$, and as ordinates, $\Delta\lambda$, $\frac{\Delta\lambda}{\lambda}$, $\frac{\Delta\lambda}{\lambda^2}$, respectively, were plotted. No periodic variation of any of these three quantities with $\lambda$ was shown.

[1] $\lambda$ and $\Delta\lambda$ expressed in Ångström units. Wave-lengths and intensities as given by Exner and Haschek. The character of the lines is sharp. r means "easily reversed."

The subject does not merit discussion. The only method which at the present time seems justifiable and productive of results is that which deals with homologous lines in the spectra of different elements or lines of the same series in the spectra of any one element.

The following Tables VII and VIII explain themselves. One thing is certain. Becquerel and Deslandres are not justified in attempting to make a classification or formulate a law from data obtained with such a high field as used by them — namely, that of 35,000 c. g. s. units. From the curves of Figs. 3 to 5 it is apparent that $\Delta\lambda$ does not increase proportionally with the field for all lines. The difference at 28,000 is considerable; at 35,000 it would be much greater.

TABLE VIII

Plates 345 and 346. $H = 28,000$.
Iron Spectrum. Lines appearing as quadruplets.[1]

| $\lambda$ | Intensity | $\Delta\lambda$ | $\Delta\lambda'$ | $\frac{\Delta\lambda}{\Delta\lambda'}$ | $\frac{\Delta\lambda}{\lambda^2}\times 10^{10}$ | $\frac{\Delta\lambda'}{\lambda^2}\times 10^{10}$ |
|---|---|---|---|---|---|---|
| 3680.06 | 5 | 0.440 | ..... | ... | 325 | ... |
| 3705.73 | 6 | .435 | 0.181 | 2.4 | 317 | 132 |
| 22.73 | 6 | .424 | .279 | 1.5 | 307 | 200 |
| 3872.65 | 6 | .424 | .315 | 1.3 | 283 | 210 |
| 87.18 | 5 | .499 | .150 | 3.3 | 331 | 99 |
| 3903.09 | 7 | .412 | .214 | 1.9 | 270 | 141 |
| 4202.20 | 9 | .494 | .218 | 2.3 | 280 | 124 |
| 94.32 | 6 | .511 | .238 | 2.1 | 277 | 129 |

Lines which are apparently unaffected:

| $\lambda$ | 3558.68 | 3609.02 | 3767.32 | 3850.15 |
|---|---|---|---|---|
| Intensity | 6 | 9 | 7 | 6 |

3. *The spectra of nickel and cobalt.*—A search for peculiarities and for a possible law connecting the separation.

Most of the work done with nickel and cobalt was confined to the first spectrum, owing to lack of brilliancy in the spectra

[1] Separation of inner components given as $\Delta\lambda'$.

of these metals. This reduced very much the chances of the discovery of any peculiar lines, if such indeed exist. No law governing the separation is evident in either of these metals, nor is any periodic variation of $\Delta\lambda$, $\frac{\Delta\lambda}{\lambda}$, or $\frac{\Delta\lambda}{\lambda^2}$ with $\lambda$ apparent.

TABLE IXa.[1]

Plates 314, 319, 328, and 332.

Nickel. Lines appearing as triplets.

| Plate | Spectrum | Field | $\lambda$ | Intensity | $\Delta\lambda_s$ | $\Delta\lambda_m$ $H=32,800$ | $\Delta\lambda$ (Å. U.) $H=32,800$ | $\frac{\Delta\lambda}{\lambda^2}\times 10^{10}$ |
|---|---|---|---|---|---|---|---|---|
| 314 | 1 | 32,740 | 3391.21 | 6 | 0.0842 | 0.0844 | 0.365 | 318 |
| 314 | 1 | 32,740 | 93.10 | 7 | .0920 | .0921 | .399 | 346 |
| 328 | 1 | 32,800 | 93.10 | | .0920 | | | |
| 328 | 1 | 32,800 | 3414.90 | 8 | .0938 | .0938 | .406 | 348 |
| 314 | 1 | 32,740 | 33.71 | 7 | .0920 | .0922 | .399 | 339 |
| 314 | 1 | 32,740 | 46.34 | 8 | .0807 | .0809 | .350 | 295 |
| 314 | 1 | 32,740 | 52.98 | 7 | .0920 | .0922 | .399 | 335 |
| 319 | 1 | 32,500 | 61.78 | 8 n r | .0875 | | | |
| 314 | 1 | 32,740 | 61.78 | | .0945 | .0917 | .397 | 330 |
| 328 | 1 | 32,800 | 61.78 | | .0920 | | | |
| 314 | 1 | 32,740 | 72.68 | 7 n r | .1112 | .1114 | .482 | 401 |
| 328 | 1 | 32,800 | 93.10 | 9 n r | .0720 | .0720 | .311 | 255 |
| 314 | 1 | 32,740 | 3501.00 | 6 | .0850 | .0852 | .368 | 301 |
| 314 | 1 | 32,740 | 14.06 | 5 | .0930 | .0932 | .403 | 327 |
| 314 | 1 | 32,740 | 15.17 | 9 n r | .0777 | | | |
| 328 | 1 | 32,800 | 15.17 | | .0715 | .0785 | .339 | 275 |
| 332 | 2 | 33,400 | 15.17 | | .1677 | | | |
| 328 | 1 | 32,800 | 24.65 | 10 n r | .0992 | .0947 | .410 | 330 |
| 332 | 2 | 33,400 | 24.65 | | .1885 | | | |
| 328 | 1 | 32,800 | 66.50 | 9 n r | .0725 | .0725 | .313 | 246 |
| 314 | 1 | 32,740 | 71.99 | 7 n r | .0862 | .0864 | .374 | 294 |
| 314 | 1 | 32,740 | 3610.60 | 4 r | .1112 | .1114 | .482 | 371 |
| 314 | 1 | 32,740 | 19.52 | 10 n r | .0915 | .0902 | .390 | 298 |
| 332 | 2 | 33,400 | 19.52 | | .1825 | | | |
| 314 | 1 | 32,740 | 3769.58 | 2 | .1320 | .1322 | .572 | 403 |
| 314 | 1 | 32,740 | 75.71 | 9 | .0972 | .0974 | .421 | 296 |
| 314 | 1 | 32,740 | 83.67 | 8 | .1262 | .1280 | .554 | 387 |
| 319 | 1 | 32,500 | 83.67 | | .1283 | | | |
| 319 | 1 | 32,500 | 3807.30 | 8 | .1393 | .1453 | .628 | 433 |
| 314 | 1 | 32,740 | 07.30 | | .1497 | | | |
| 314 | 1 | 32,740 | 58.40 | 9 r | .1085 | | | |
| 328 | 1 | 32,800 | 58.40 | | .1085 | .1086 | .470 | 315 |
| 332 | 2 | 33,400 | 58.40 | | .2210 | | | |
| 319 | 1 | 32,500 | 4401.70 | 9 | .1573 | .1587 | .686 | 354 |
| 319 | 1 | 32,500 | 59.21 | 9 | .1560 | .1574 | .681 | 342 |

[1] $\lambda$ as given in Exner and Haschek's (spark spectrum) and Hasselberg's (arc spectrum) tables. Wave-lengths on Rowland's scale. Intensities as given above are taken from these two tables. n = nebulous; r = easily reversed.

From the following tables, IX to XI, it will be noted that for nickel the separation does not increase proportionally with the field, else $\frac{\Delta\lambda}{\lambda^2 H}$, as calculated for fields of different strength, would be approximately the same. This probably holds good in the case of cobalt also.

In Tables IXa and XI, $\Delta\lambda_s$ signifies $\Delta\lambda$ in terms of the dividing engine screw, $\Delta\lambda_m$ the mean separation in screw units for $H = 32{,}800$ as calculated from the values of $\Delta\lambda_s$ given for different fields, 32,500, 32,740, and 33,400. The calculation is based on the assumption that for the short intervals employed $\Delta\lambda$ varies proportionally with $H$; and double weight is given to measurements made on plate No. 332, which was taken in the second spectrum.

TABLE IXb.

Plate 317. $H = 29{,}100$.

Nickel. Lines appearing as triplets.

| Spectrum | $\lambda$ | $\Delta\lambda$ | $\frac{\Delta\lambda}{\lambda^2} \times 10^{10}$ |
|---|---|---|---|
| 2 | 3775.71 | 0.384 | 269 |
| 2 | 83.67 | .503 | 352 |
| 2 | 3807.30 | .584 | 403 |
| 2 | 58.40 | .422 | 290 |

TABLE X.

$\frac{\Delta\lambda}{\lambda^2 H} \times 10^5$ as calculated from fields of 32,800 and 29,100 c. g. s units.

Nickel.

| $\lambda$ | $H = 32{,}800$ | $H = 29{,}100$ |
|---|---|---|
| 3775.71 | 9.0 | 9.5 |
| 83.67 | 11.8 | 12.1 |
| 3807.30 | 13.2 | 13.9 |
| 58.40 | 9.6 | 10.0 |

TABLE XI.[1]

First spectrum. Plates 321, 322, 323, 324, and 329.
Cobalt. Lines appearing as triplets.

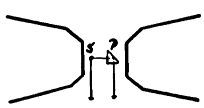
| Plate | Field | $\lambda$ | Intensity | $\Delta\lambda_s$ | $\Delta\lambda_m$ $H=32,800$ | $\Delta\lambda$ (Å. U.) $H=32,800$ | $\frac{\Delta\lambda}{\lambda^2}\times 10^{10}$ |
|---|---|---|---|---|---|---|---|
| 321 | 32,300 | 3521.70 | 6 r | 0.1363 | 0.1377 | 0.595 | 480 |
| 321 | 32,300 | 3704.17 | 6 | .1398 | .1412 | .610 | 455 |
| 329 | 32,800 | 3845.59 | 9 n | .1158 | .1158 | .501 | 339 |
| 321 | 32,500 | 73.25 | 9 n | .1208 | .1220 | .528 | 355 |
| 324 | 32,200 | 94.21 | 10 n r | .0828 | .0884 | .382 | 252 |
| 329 | 32,800 | 94.21 | | .0925 | | | |
| 324 | 32,200 | 3933.32 | 2 | .1235 | .1268 | .548 | 354 |
| 321 | 32,500 | 33.32 | | .1265 | | | |
| 321 | 32,500 | 41.034 | 5 | .1676 | .1693 | .732 | 471 |
| 323 | 32,200 | 41.87 | 6 | .1155 | .1169 | .505 | 325 |
| 322 | 32,500 | 41.87 | | .1150 | | | |
| 324 | 32,200 | 3969.25 | 5 | .1308 | | | |
| 322 | 32,500 | 69.25 | | .1325 | .1335 | .577 | 366 |
| 329 | 32,800 | 69.25 | | .1332 | | | |
| 322 | 32,500 | 4118.92 | 9 | .1008 | | | |
| 321 | 32,500 | 18.92 | | .1067 | .1055 | .456 | 269 |
| 329 | 32,800 | 18.92 | | .1069 | | | |
| 323 | 32,200 | 21.47 | 9 | .1118 | .1142 | .492 | 290 |
| 322 | 32,500 | 21.47 | | .1133 | | | |
| 323 | 32,200 | 4225.28 | 3 | .1100 | | | |
| 324 | 32,200 | 25.28 | | .1145 | .1167 | .505 | 283 |
| 322 | 32,500 | 25.28 | | .1200 | | | |
| 322 | 32,500 | 4629.515 | 9 | .2210 | .2232 | .965 | 450 |

The nickel lines which appear unseparated are: 3423.80, 3483.1,[2] 3510.47, 3518.80, 3597.84.

The values of $\Delta\lambda$ in Tables IX to XI are correct to at least 0.02 Å. U.

4. *The Zeeman effect along the lines of force.*

Previous investigators in examining the Zeeman effect along the lines of force have used a pierced magnet pole. This rendered the field non-uniform and prevented accurate measurement of its strength. To obviate this a small total reflecting prism was used, as shown in the adjoining cut.



Two images appear near the spectrometer slit, one giving the spectrum along the lines of force, the other perpendicular to them.

[1] Same notation as table IXa. $\lambda$ taken from Hasselberg's table (arc spectrum). Wave-lengths on Rowland's scale.

[2] $\lambda$ as given by Liveing and Dewar.

The prism used was 12 mm long by 2 mm across its perpendicular faces. The crystal was cut so as to have the optic axis parallel to one of the faces of the prism, and perpendicular to its edge. The prism was supported by a delicate adjustable framework bound to the pole-pieces of the magnet and a piece of fine microscopic cover glass protected it from the heat of the spark. Mica may be used when ultra-violet light is desired.

The method proved a success. In one exposure during a period of 70 minutes, the images were alternately focused upon the slit and the shutter in the camera turned accordingly. The direct image was first used for 20 minutes; then the shutter was changed and the image which had passed through the small prism was employed for 30 minutes; then again the direct image for another 20 minutes. The two 20-minute exposures were thus thrown on the edges of the photographic plate, the 30-minute one on the center. Thus would be shown any change of field strength, which, indeed, was a highly improbable occurrence.

Table XII shows that within the limits of error of observation, the separation of the external plane polarized components of the triplet is equal to that of the circularly polarized components of the doublet which is expressed by the symbol $\Delta\lambda_0$.

TABLE XII.

Zinc. $H=18,760$.

| Line | Intensity and character | $\Delta\lambda$ | $\Delta\lambda_0$ | $\Delta\lambda-\Delta\lambda_0$ |
|---|---|---|---|---|
| *Zn* 4680.38 | 10 r | 0.703 | 0.700 | +0.003 |
| *Zn* 4722.26 | 10 r | 0.614 | 0.607 | + .007 |
| *Zn* 4810.71 | 10 r | 0.422 | 0.425 | − .003 |

5. *The extension of Preston's law that* $\frac{\Delta\lambda}{\lambda^2 H}$ *is a constant for homologous lines of Kayser and Runge's spectroscopic series.*

This part of the investigation has proved to be one of great interest, and promises even better results if continued under slightly better conditions, notably with a grating which is bright in the second or third spectrum.

The most brilliant lines, especially those of zinc and cadmium, and those of the best character as to symmetry and sharpness, have been repeatedly photographed by various investigators. Preston derived the law mentioned from results of experiments upon the zinc, cadmium, and magnesium lines whose wave-lengths are given approximately by putting $n$ equal to 3 in Kayser and Runge's formula,

$$\lambda^{-1} = A + Bn^{-2} + Cn^{-4},$$

where $A$, $B$, and $C$ are constants for any one element. It appeared of interest to investigate the homologous lines of the other metals occurring in the second column of Mendelejeff's chart — that is, of the metals mercury, calcium, strontium, and barium; and also to study the separation of the lines forming the first subordinate series. For barium, Kayser and Runge have found no series. Strontium has only the first subordinate. To obtain in a sharp and measurable form some of the diffuse and unsymmetrical lines of mercury and the diffuse though symmetrical lines of the first subordinate series of some of the members of this chemical group, is a matter of no little difficulty. Total failure has been the result in many cases. It has long been known that self-induction in the spark circuit tends to sharpen diffuse lines. This method has been used and has proved indispensable. A better method would be the use of a vacuum tube. This the author intends to try if later an opportunity offers itself.

In the following investigation the first question was one of the choice of field and magnetic gap. It was advisable to use a large current in the magnetizing coils, for the higher the point chosen in the magnetization curve the less will the field vary for a small given change in the current. Secondly, the gap must be neither too large, else the field would be too weak and the lines of small separation would not be resolved; nor too small, else both the size of spark would be limited and consequently the exposures necessarily prolonged, and also the field would be so strong that the droop on the $H$–$\Delta\lambda$ curve would enter to a too great degree. A current of 15 amperes in the coils and a gap

of 7 mm were chosen. This combination gave a field of about 26,460 c. g. s. units; much too strong, but quite necessary to resolve some lines, notably *Zn* 3282.42 and *Cd* 3403.74.

By referring to Fig. 2c it will be seen that zinc 4810.71 at $H = 26{,}460$ shows a separation of 0.53 Å. U., while if $\Delta\lambda$ increased proportionately with the field, its value would be 0.71 Å. U. or 34 per cent. greater. Lines homologous with zinc 4810.71 show a similar droop.[1] This explains the discrepancies between Reese's values (Table XV) and Preston's and mine for the three lines—one each in cadmium, mercury, and magnesium—which are homologous with zinc 4810.71. Of the other lines investigated by me, it must be said that the characters of all the curves are not known. To obtain the variation of $\Delta\lambda$ with the field for all would call for more labor than time at present permits.

Tables XIVa and XIVb go together, the homologous lines occupying homologous positions. Table XIII contains the data on which are based the tables following it. Certain points should be noted relative to some of the lines given in this table.

Cadmium 3252.63 was difficult to measure because of small intensity, but the value of the separation given is probably accurate to 5 per cent.

Mercury 5460.97 and magnesium 5183.84 are unsymmetrical. The value of $\Delta\lambda$ given for each represents the distance between the center of the symmetrical violet component and the most dense portion of the red component, which latter is sharp on the violet edge and shades off toward the red. Zinc 4810.71 is somewhat similar but the asymmetry is not so marked, while cadmium 5086.06 shows less, but still some asymmetry. These four lines are homologous.

Strontium 4832.23 is also peculiar, but here the violet component is shaded toward the red, while the red component is symmetrical. Moreover, the violet component is more intense than the red. Strontium 4876.35 shows the same difference in intensity but both components are symmetrical.

[1] Reese, Astrophysical Journal, **12**, 128, 1900.

TABLE XIII.

$H = 26{,}460$.

| Metal | λ[1] | Intensity and character | $\Delta\lambda$ | $\frac{\Delta\lambda}{\lambda^2}\times 10^{10}$ |
|---|---|---|---|---|
| *Zn* | 4680.38 | 10 r | 0.959 | 0.438 |
| | 4722.26 | 10 r | .859 | .394 |
| | 4810.71 | 10 r | .544 | .235 |
| | 3282.42 | 8 r | .146 | .135 |
| | 3303.03 | 8 r | .245 | .225 |
| | 3345.62 | 8 r | .337 | .201 |
| *Cd* | 4678.37 | 10 r | .947 | .433 |
| | 4800.09 | 10 r | .890 | .386 |
| | 5086.06 | 10 r | .619 | .239 |
| | 3252.63 | 8 bv | .308 | .291 |
| | 3403.74 | 10 r | .152 | .131 |
| | 3467.76 | 8 r | .275 | .228 |
| | 3613.04 | 8 r | .391 | .299 |
| *Hg* | 4046.78 | 6 r | .726 | .442 |
| | 4358.56 | 10 r | .738 | .388 |
| | 5460.97 | 10 r | .683 | .229 |
| *Mg* | 3838.44 | 10 r | .432 | .293 |
| | 5167.55 | 8 r | 1.140 | .427 |
| | 5172.87 | 10 r | 1.058 | .374 |
| | 5183.84 | 10 r | 0.643 | .239 |
| *Ca* | 4425.61 | 10 r | .246 | .125 |
| | 4435.86 | 8 r | .425 | .216 |
| | 4456.08 | 8 r | .594 | .298 |
| *Sr* | 4832.23 | 10 r | .792 | .339 |
| | 4876.35 | 8 r | .785 | .330 |

Also *approximately*[2] (*i. e.*, to 10 per cent.).

| Metal | λ[1] | Intensity and character | $\Delta\lambda$ | $\frac{\Delta\lambda}{\lambda^2}\times 10^{10}$ |
|---|---|---|---|---|
| *Ca* | 6122.46 | 10 r | 1.50 | 0.400 |
| | 6162.46 | 10 r | 1.05 | .274 |

1. Kayser and Runge's tables (arc spectrum). Wave-lengths, Rowland. r means "easily reversed;" bv stands for "band with sharp edge toward the violet."

2. Lines faint, and so only approximate measurements were made.

Magnesium 3838.44 is unsymmetrical in the same way as mercury 5460.97, but is probably correct to 0.01.

The three calcium lines 6102.99, 6122.46, 6126.46, all strong lines visually, could not be obtained by the method used with other lines, although an "Erythro" plate of the International Color-Photo Company was exposed to the radiation for four

hours. However, by throwing upon the upper terminal of the spark gap a continuous and rather large jet of a saturated solution of calcium chloride 6122.46 and 6162.46 were obtained in measurable form after an exposure of about 90 minutes.

The lines of the spectroscopic series which do not appear in Table XIII either could not be obtained because of (1) low intensity or (2) their position in the spectrum (below $\lambda$ 3300 or above $\lambda$ 5800), or could not be measured because of their diffuse character. Self-induction in the discharge circuit improved the condition of some lines and removed the air spectrum, but did not sufficiently narrow diffuse lines in all cases.

The values of $\frac{\Delta\lambda}{\lambda^2}$ in Table XIII are correct in general to 0.01 Å. U. Probably, however, $\Delta\lambda$ for zinc 3282.42 (also cadmium 3403.74 and calcium 4425.61) is not so accurate. The separation is small and the measurement of such a line is made with difficulty.

Table XV explains itself. Table XVI is taken from the mean values given in Table XV. Reese's values of $\Delta\lambda$ were obtained from the slope of the curves, and therefore no corrections are necessary in $\frac{\Delta\lambda}{\lambda^2 H}$. Correcting my values for $n = 3$, in the second subordinate series, by adding 3, 3 and 34 per cent. respectively; and treating the values for the three sets of lines for which $n = 4$ in the first subordinate series, and the one set for which $n = 4$ in the second subordinate series in a similar manner, because they give $H$–$\Delta\lambda$ curves homologous to those given by the $n = 3$, second subordinate sets;[1] also, raising by 16 per cent. Preston's mean value for the third set in $n = 3$, second subordinate series (see curve Fig. 2c) we obtain the results given in Table XVI. This table shows:

1. That Preston's law, that $\frac{\Delta\lambda}{\lambda^2 H}$ is a constant for homologous lines, established by him for the homologous lines of zinc, cadmium, and magnesium given by $n = 3$ in the second subordinate series, appears to hold for the homo logous lines in mercury and cadmium.

[1]This fact I have verified by experiment upon cadmium 3613.04 and cadmium 4425.61.

TABLE XIVa.

Lines of spectroscopic series investigated.

| Metal | First subordinate series | | | Second subordinate series | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $n=4$ | | | $n=3$ | | | $n=4$ | | |
| *Zn* | 3282.42 | 3303.03 | 3345.62 | 4680.38 | 4722.26 | 4810.71 | ...... | ...... | ...... |
| *Cd* | 3403.74 | 3467.76 | 3613.04 | 4678.37 | 4800.09 | 5086.06 | ...... | ...... | 3252.63 |
| *Hg* | ...... | ...... | ...... | 4046.78 | 4358.56 | 5460.97 | ...... | ...... | ...... |
| *Mg* | ...... | ...... | 3838.44 | 5167.55 | 5172.87 | 5783.84 | ...... | ...... | ...... |
| *Ca* | ...... | ...... | ...... | ...... | 6122.46 | 6162.46 | 4425.61 | 4435.86 | 4456.08 |
| *Sr* | 4832.23 | 4876.35 | ...... | ...... | ...... | ...... | ...... | ...... | ...... |

TABLE XIVb.

$\frac{\Delta\lambda}{\lambda^2}\times 10^{10}$ or lines given in homologous positions in Table XIVa.

| Metal | First subordinate series | | | Second subordinate series | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $n=4$ | | | $n=3$ | | | $n=4$ | | |
| *Zn* | 135 | 225 | 301 | 438 | 394 | 235 | ... | ... | ... |
| *Cd* | 131 | 228 | 299 | 433 | 386 | 239 | ... | ... | 291 |
| *Hg* | ... | ... | ... | 442 | 388 | 229 | ... | ... | ... |
| *Mg* | ... | ... | 293 | 427 | 374 | 239 | ... | ... | ... |
| *Ca* | ... | ... | ... | ... | 400 | 274 | 125 | 216 | 298 |
| *Sr* | 339 | 330 | ... | ... | ... | ... | ... | ... | ... |

2. That, within the limits of accuracy of measurement, $\frac{\lambda}{\lambda^2 H}$ is the same for homologous lines given by $n=4$ in both the subordinate series for zinc, cadmium, mercury, magnesium, and calcium. This assumes that the lines not investigated show values of $\frac{\Delta\lambda}{\lambda^2 H}$ which are the same as those investigated in each set—an assumption which is certainly not unjustifiable.

3. That the average value of $\frac{\Delta\lambda}{\lambda^2 H}$ —obtained from Preston's, Reese's, and my determinations—for the third set in $n=3$, or $\frac{1}{3}(12+10.7+11.1)=11.27$, is so related to the average value of $n=4$ (as given in the same table, XVI) for the homologous sets in both subordinate series, or $\frac{1}{2}(15.1+14.9)=15.00$, that $\left(\frac{\Delta\lambda}{\lambda^2 H}\right)_{n=3} : \left(\frac{\Delta\lambda}{\lambda^2 H}\right)_{n=4} :: 3 : 4$; as $\frac{1}{3}(11.27)=3.76$, while $\frac{1}{4}(15.00)=3.75$. Thus, if we assume $\frac{\Delta\lambda}{\lambda^2 H}$ proportional to $\frac{e}{m}$,

where "*e*" is the amount of electricity carried by the particle of mass "*m*", we may say that the ratio of the charge to the mass of the particle varies directly with "*n*" for the third set of lines in the second subordinate series where "*n*" has either the value 3 or 4.

TABLE XV[1].

$\frac{\Delta\lambda}{\lambda^2 H}\times 10^5$

| Metal | First subordinate | | | Second subordinate | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n=4$ | | | $n=3$ | | | | | | | | | $n=4$[4] | | |
| | | | | Calculated from Table XIVb | | | Reese[2] | | | Preston[3] | | | | | |
| *Zn* | 5.1 | 8.5 | 11.4 | 16.5 | 14.9 | 8.9 | 17.0 | 15.3 | 11.3 | 17. | 14.8 | 9.53 | .... | .... | .... |
| *Cd* | 5.0 | 8.6 | 11.3 | 16.4 | 14.6 | 9.1 | 17.0 | 15.5 | 10.5 | .... | .... | .... | .... | .... | 11.0 |
| *Hg* | .... | .. | .... | 16.7 | 14.7 | 8.7 | 16.7 | .... | 10.3 | .... | .... | .... | .... | .... | .... |
| *Mg* | .... | .... | 11.1 | 16.1 | 14.1 | 9.0 | 16.7 | 14.9 | 10.5 | .... | .... | .... | .... | .... | .... |
| *Ca*[5] | .... | .... | .... | .... | 15.1 | 10.4 | .... | .... | .... | .... | .... | .... | 4.8 | 8.2 | 11.3 |
| *Sr*[5] | 12.8 | 12.5 | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... |
| Mean | 5.1 | 8.6 | 11.3 | 16.4 | 14.6 | 8.9 | 16.9 | 15.2 | 10.7 | 17. | 14.8 | 9.5 | 4.8 | 8.2 | 11.2 |

1. Field 26,460.

2. Reese's values calculated from slope of curve on *H* diagrams.

3. Approximate mean value given by Preston for the homologous lines of *Zn*, *Cd*, and *Mg*. $H = 20{,}000$.

4. Calculated from data given by Reese. See his article before mentioned, ASTROPHYSICAL JOURNAL, **12**, 129, Sept. 1900.

5. Excluding strontium, as it is irregular, and calcium, $n=3$, the measurements of which are but approximate. Note that the calcium lines agree quite well with the homologous lines in *Zn*, *Cd*, *Hg*, and *Mg*.

TABLE XVI.

$\frac{\Delta\lambda}{\lambda^2 H}\times 10^5$, corrected values.

| | First subordinate | | | Second subordinate | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n=4$ | | | $n=3$ | | | | | | | | | $n=4$ | | |
| | | | | Table XIVb | | | Reese | | | Preston | | | | | |
| | 5.2 | 8.8 | 15.1 | 16.9 | 15.0 | 12.0 | 16.9 | 15.2 | 10.7 | 17. | 14.8 | 11.1 | 4.8 | 8.4 | 14.9 |
| Set | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |

(The first subordinate series shows no lines for $n=3$. The value 15.1 was averaged with 14.9 because of the probability of obtaining thus a more correct mean).

4. That there appears to be no relation between sets 1 and 2, $n=4$ and 1 and 2, $n=3$, or, taking total mean values, between

16.9, 15.0 for $n=3$ and
5.0, 8.6 $n=4$.

However, inasmuch as the wave-lengths of the three lines forming the spectroscopic triplet are so related to each other that, given that of one of them, the wave-lengths of the other two can be calculated, thus forming a connection between the members of the triplet, we would not expect that here again a relation would appear between the first, second, and third lines of the triplet in any one series, unless, indeed, it were a relation equivalent to that just mentioned, namely, the possibility of the calculation of the wave-lengths of the two lines of the triplet given that of any one.

## GENERAL SUMMARY.

The most important results obtained in this investigation are briefly as follows:

1. That for iron and nickel as well as zinc the Zeeman effect is not proportional to the field strength for high values of the latter.

2. That the divisions of large and small pressure shift in the case of iron are not absolutely the same as those of large and small Zeeman separation. Of 34 lines investigated, 26, or about 76 per cent., show both large pressure shift and large separation, or small pressure shift and small separation; while 8, or 24 per cent., show either small pressure shift and large separation, or large pressure shift and small separation.

3. That there is apparently no simple law connecting the separation of the various lines in either iron, nickel, or cobalt.

4. (*a*) That Preston's law, $\frac{\Delta\lambda}{\lambda^2 H}$ = constant, holds for the homologous lines given by $n=3$ for mercury and calcium as well as zinc, cadmium, and magnesium. (*b*) That Preston's law also

holds for the homologous lines given by $n=4$ in *both* the subordinate series of zinc, cadmium, magnesium, and calcium. ($c$) That the ratio of the charge carried to the mass of the particle carrying it varies directly as "$n$" for the third set of lines in the second subordinate series, when "$n$" has the value either 3 or 4. This, of course, assumes that the ratio of charge to mass varies directly as $\frac{\Delta\lambda}{\lambda^2 H}$.

In this investigation the author was assisted at various times by Messrs. J. H. Moore, J. E. Routh, R. E. Loving, G. W. Middlekauff, J. T. Barrett, and W. J. Crist.

In conclusion I wish to acknowledge the debt I owe to Professors H. A. Rowland and J. S. Ames for their valuable advice and encouragement.

PHYSICAL LABORATORY,
JOHNS HOPKINS UNIVERSITY,
June 1901.

# ON THE MOTION OF *α PERSEI* IN THE LINE OF SIGHT.[1]

By H. C. VOGEL.

IN *Monthly Notices*, Vol. LXI, No. 1, Mr. Newall, of Cambridge, England, calls attention to the fact that the velocity of *α Persei* in the line of sight, as deduced from observations on eleven evenings in September and October 1900, and three evenings in October 1899, varies from $-4$ km to $+8$ km, and he suspects a period of variability of 4.2 or 16.8 days. Since the Cambridge weather conditions in winter do not admit of continuous observations, Mr. Newall requests that the motion of the star be observed elsewhere. His observations were made with a new four-prism spectrograph of powerful dispersion attached to his large refractor of 63 cm aperture.

According to the data given on p. 12 of the above mentioned article, a difference in wave-length of 0.6 $\mu\mu$ on the spectrograms obtained last year corresponds to a linear distance of 1 mm, and a displacement of this amount to a velocity of 400 km. Nothing more definite is said as to the line to which these values refer; it is, however, clear from the above that the scale of the plates is very considerable, and about double that of the Mills spectrograph of the Lick Observatory or of the new spectrograph for the great Potsdam refractor.[2]

In view of our unfavorable conditions at Potsdam as regards steadiness of air, which of late years have been quite unusual, I had early arrived at the conclusion that spectrographic work with the large refractor would make but slow progress under further continuance of such state of air. Accordingly, toward the close of the year 1899 I designed a spectrograph for our excellent photographic short-focus refractor of 32 cm aperture and 3.44 m focal length, which was constructed by Toepfer of Potsdam and completed in the spring of 1900. In the course of

[1] Translated from author's proofs from *Sitzungsberichte der k. Akad. zu Berlin*. Session of Jan. 17, 1901.

[2] See my summary in ASTROPHYSICAL JOURNAL, 11, 399, 1900.

the summer it was subjected to an exceedingly thorough investigation and carefully corrected by Dr. Eberhard according to the method given by Dr. Hartmann.[1]

The triple collimator objective has a focal length of 30 cm; the camera objective, which is also triple, a focal length of 35 cm. The three prisms give a spectrum which is measurable and uniformly sharp between λ 4120 and λ 4420, the extent of this section of spectrum being 20 mm. At the center (λ 4250) a linear displacement of 0.25 mm (1 rev. of the screw of the measuring machine) corresponds to a motion of 261 km; at $H\gamma$ (λ 4340) the same displacement corresponds to 291 km. The instrument gives only about two fifths of the linear extent of Newall's apparatus. The spectrograph is enclosed in a case which is provided with means for maintaining the temperature constant to within 0°.1 C.

With this instrument Dr. Eberhard made photographs of the spectrum of *α Persei* on six nights (1900, Nov. 3, 5, 6, 8, 9, and 15), and four of these plates (Nov. 3, 5, 6, and 9) I measured in order to test the utility and efficiency of the apparatus for its proper purpose, as it had previously been investigated almost wholly in the laboratory. The spectra were good, and the measures of the four plates gave no deviations which would have allowed us to infer variations of motion of more than 2 km. The observations were, however, affected with one error which had its cause in a slight mechanical imperfection in the instrument that could not be observed in the laboratory investigations. After the correction of this defect and a further test of the apparatus by Dr. Eberhard, I regarded the instrument suitable to continue the observations by Newall, which had been published in the meantime.

The first two photographs in the list given below were made by Dr. Eberhard; the others by Dr. Ludendorff. The measurement of the spectrograms I undertook myself; it consists solely in the determination of the differences between the lines of the *Fe* spectrum and the corresponding lines of the star spectrum,

[1] *Zeitschrift für Instrumentenkunde*, 1900; ASTROPHYSICAL JOURNAL, 11, 400; 12, 30, 1900.

and the number of lines compared in the several spectra varies from 14 to 21. Although the spectrum of the star is to be classed among those with few lines, since, strictly speaking, it does not belong to spectral class IIa, but forms the transition from class $Ia_3$ to IIa, still 140 to 150 lines upon the better photographs are to be counted between $\lambda 4119$ and $\lambda 4415$; most of these lines, moreover, are extraordinarily sharp, owing to the fact that the slit width in the photographs of *a Persei* amounted to but 0.015 mm. Consequently, the spectra admits of a far more rigorous treatment than I at first undertook at this point. The final results, therefore, may still receive slight changes, which, indeed, would arise on measuring the displacement of the same lines in reference to the comparison spectrum in the reversed position of the spectrogram under the microscope. My measurements have been in but one direction, that in which increasing readings upon the screw of the measuring instrument correspond to greater wave-lengths. The changes mentioned have, however, no significance as regards the proof of a possible variation in the velocity, of the amount mentioned at the beginning of this article, and the observations are accordingly to be regarded as provisional only so far as the absolute amount of the star's motion in the line of sight is concerned.

OBSERVATIONS OF *a PERSEI.*

| Date 1900–1901 | Potsdam Mean Time | Temperature Centigrade | Number of plate | Velocity in ref. to Earth | Reduction to ⊙ | Velocity in ref. to ⊙ |
|---|---|---|---|---|---|---|
| Dec. 13 | $7^h\ 56^m$ | +5°4 | 416 | +7.8 km | −9.4 km | −1.6 km |
| 14 | 7 11 | +5.7 | 417 | +6.2 | −9.8 | −3.6 |
| 18 | 5 7 | +5.1 | 418 | +8.8 | −11.4 | −2.6 |
| 20 | 5 5 | +3.8 | 419 | +10.6 | −12.2 | −1.6 |
| 21 | 9 17 | +5.0 | 420 | +9.3 | −12.7 | −3.4 |
| 22 | 5 21 | +4.7 | 421 | +10.2 | −13.0 | −2.8 |
| Jan. 1 | 5 5 | −5.9 | 422 | +12.4 | −16.8 | −4.4 |
| 2 | 5 1 | −9.3 | 424 | +13.4 | −17.2 | −3.8 |
| 3 | 5 10 | −10.0 | 426 | +12.6 | −17.5 | −4.9 |
| 4 | 5 1 | −9.4 | 427 | +14.4 | −17.8 | −3.4 |
| 5 | 4 55 | −9.2 | 429 | +13.8 | −18.2 | −4.4 |
| 9 | 5 18 | −3.3 | 432 | +16.6 | −19.4 | −2.8 |
| 9 | 6 32 | −2.8 | 433 | +16.9 | −19.5 | −2.6 |
| | | | | | | −3.22 |

The spectrograms are almost without exception to be described as very good. In plate 417, however, the star spectrum is rather weak, and hence the measures are slightly less accurate. Morever, in plate 420 the *Fe* spectrum, and in plate 422 both the star and the *Fe* spectra are somewhat weak. In plate 426 the star lines are rather broad.

The preceding observations furnish no confirmation of Newall's results, since the largest deviations of the values obtained for the several evenings from the mean value are but −1.6 km and +1.7 km — deviations which may permissibly occur with the degree of accuracy attained in these observations. The probable error of a determination of the displacement between a line in the star and a line in the comparison spectrum varies between ±1.2 km and ±2.2 km for the different plates. Consequently the probable error of the mean value obtained from the measures of a plate would amount to from ±0.3 km to ±0.6 km. Experience shows it to be somewhat larger, and in the above case, as derived from the deviations of the values of the several plates, it amounts to ±0.69 km.

As I have already said, my results for the absolute value of the velocity of *α Persei* in the line of sight are as yet not to be considered definitive. Their close agreement with Campbell's observations, viz.,

| | | | |
|---|---|---|---|
| 1896 | Nov. | 11 | −2.0 km |
| | | 12 | −1.8 |
| 1897 | Jan. | 19 | −3.5 |
| 1898 | July | 12 | −2.1 |
| | | | −2.4 km |

is, however, worthy of note, and may well vouch for the invariability of the star's motion within narrow limits.

# THE SPECTROSCOPIC BINARY *MIZAR*.[1]

By H. C. VOGEL.

SHORTLY before 1890 photographic plates of the spectrum of *Mizar* (*a Ursae Majoris*) obtained at the Harvard College Observatory showed the brighter component of this well-known double star to be itself a binary, and from the extensive observational data[2] it was also inferred that both of these components are bright, and give spectra belonging to Class I. The motion of the components is shown by an occasional brief doubling of the spectral lines which occurs with fair regularity at intervals of fifty-two days, and the displacements of these lines give a maximum relative velocity of the two bodies of about 100 miles (160 km). According to Pickering[3] the assumption of a strongly eccentric orbit, with a major axis nearly perpendicular to the line of sight, agrees well with the observations. Consequently, only at the time of periastron, once in every 104 days, would the components of motion in the line of sight attain a sufficient value to allow of the separation of the lines of the composite spectrum formed by the overlying spectra of the two bodies. At apastron, in consequence of the slight orbital velocity, the lines would merely appear diffuse or considerably broadened. The Cambridge observations, however, show various irregularities to exist, so that up to the present time the relations in the system are to be regarded as not yet entirely explained.

The Potsdam observations of 1889 and 1890 are too few in number and too far apart in time to contribute toward the settlement of the question. Such is far from being the case, however, with the very excellent plates of this interesting double star

[1] Translated, at the author's request, from *Sitzungsberichte der k. Akad. zu Berlin*. Session of May 2.

[2] One hundred and thirteen photographs, eighty evenings of observation.

[3] *Monthly Notices*, R. A. S, 50, 297.

secured in March and April of the present year by Dr. Eberhard and Dr. Ludendorff with spectrograph IV of the 33 cm refractor. I have undertaken the measurement of these plates myself, and make the following preliminary announcements of the results, which are in complete contradiction to the previous views as to the system.

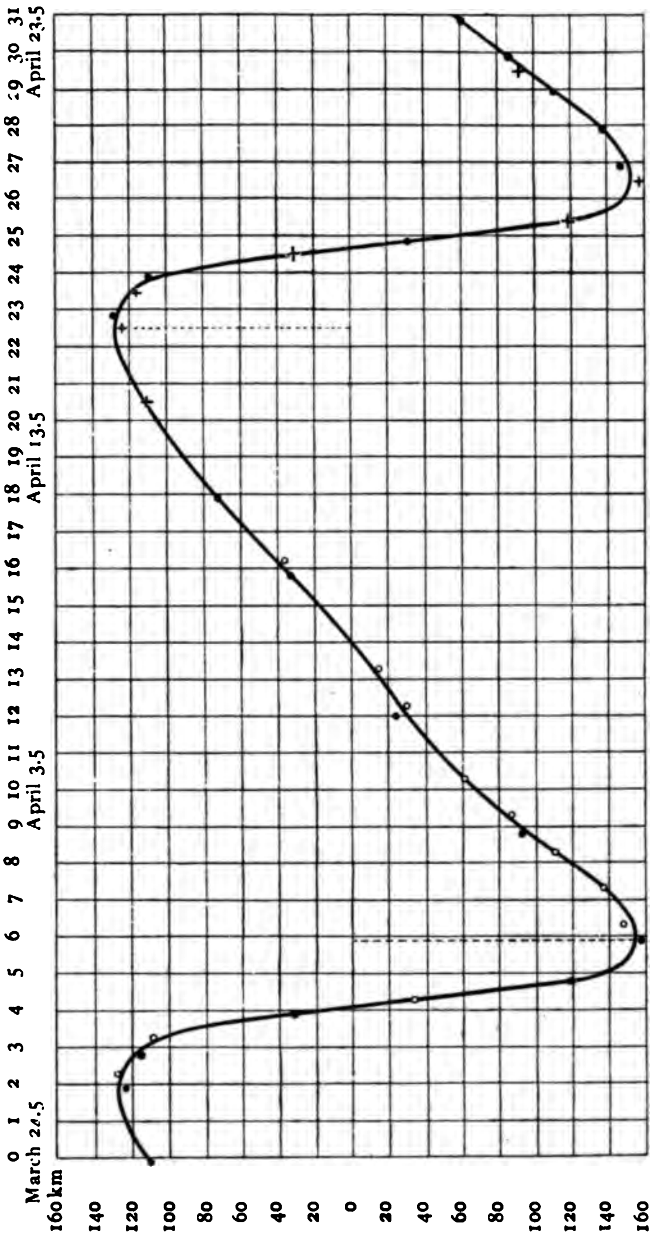
It is to be noted of the spectra themselves that they have few lines (Class Ia 2); at the time when the spectral lines of the two components nearly coincide, however, several of the strongest lines of the *Fe* spectrum and of a few other elements (as *Si*) in the part of the spectrum investigated ($\lambda$4120 to $\lambda$4500) appear as delicate lines, in addition to the broad $H\gamma$ and the strong *Mg* line at $\lambda$4481 which is always present. Thus on plate 602, obtained April 27, sixty-five lines are easily recognizable. When the spectra are more strongly displaced with reference to one another, most of the lines, which then appear double, become so weak that the measurement of their separation is rendered very difficult. On some plates, in fact, it was found possible to measure only the *Mg* line $\lambda$4481; especially good plates admitted of the measurement of some *Fe* lines as well, and, in case of exceptionally wide displacement, of $H\gamma$ also. It is to be noted that the fine lines of the spectra of Class Ia 2 are only obtained when the exposure time is exactly correct, and the plate is carefully developed. In general, the measures are to be classed as difficult, either on account of the excessive fineness or, in case of the *Mg* and *H* lines, on account of the too great breadth and diffuseness of the lines.

On several plates the *Mg* lines appear of unequal breadth, and I have endeavored to see whether I could observe a change in their behavior after a coincidence[1]—as I succeeded in doing in the case of $\beta$ *Aurigae*, in which, as is well known, a periodic doubling of the spectral lines occurs, but up to the present time I have arrived at no accordant results. I do not, however, consider it as impossible that a larger number of observations would give certainty on this point.

[1] *Publicationen des Astrophysikalischen Observatorium*, Bd. VII, p. 143.

| Date | | Potsdam Mean Time | Number of plate | Relative motion |
|---|---|---|---|---|
| 1901 March | 24 | 9^h 37^m | 537 | 111 km |
| | 26 | 9 52 | 542 | 124 |
| | 27 | 8 6 | 546 | 117: |
| | 28 | 10 22 | 549 | 31 |
| | 29 | 7 19 | 550 | 119 |
| | 30 | 8 33 | 554 | 158 |
| April | 2 | 8 8 | 556 | 93 |
| | 5 | 8 32 | 559 | 19 |
| | 5 | 11 12 | 561 | 23 |
| | 5 | 14 20 | 563 | 31 |
| | 5 | 16 15 | 565 | 21 |
| | 9 | 8 24 | 570 | 33 |
| | 11 | 9 7 | 573 | 73: |
| | 16 | 8 25 | 575 | 128 |
| | 17 | 8 30 | 577 | 109 |
| | 18 | 8 24 | 578 | 32 |
| | 20 | 8 22 | 580 | 148 |
| | 21 | 8 37 | 586 | 138 |
| | 22 | 8 20 | 589 | 111 |
| | 23 | 8 22 | 594 | 87 |
| | 24 | 8 49 | 597 | 61 |
| | 26 | 9 21 | 600 | 30 |
| | 27 | 9 24 | 602 | 15 |
| | 30 | 10 7 | 604 | 36 |
| May | 1 | 11 28 | 605 | 63 |

Measures of the motions of the system in the line of sight have also been secured upon plates in which the lines cannot be recognized as double. These, however, are of no great accuracy, since the distances between the individual lines of the star spectrum and the corresponding lines of the *Fe* spectrum exhibit greater variations than is to be expected upon plates made with this excellent apparatus. The reason for this may lie in the fact that with incomplete coincidence of the spectra the components of the different pairs of lines do not possess the same differences of intensity: thus, if in one of the apparently single lines the component lying toward the red were the stronger, and in another that lying toward the violet, a different estimate of the center would be formed in the two cases. Finally, in order to be able to draw some conclusion as to the ratio of the masses of the two bodies, I have tried to see whether a displacement of the centers of the pairs of lines in reference to the corresponding lines of the comparison spectrum occurs at the different phases when allowance




THE SPECTROSCOPIC BINARY *MIZAR*

is made for the motion of the system in the line of sight. The material so far secured has, however, proved insufficient for this investigation. The observation of the star will be continued with a view to detecting relations of this sort, but especially to determining the period accurately enough to establish a connection with the earlier observations.

The motion of the system according to my measures amounts to $-16$ km per second.

I have drawn in the figure the curve which agrees best with the results of the measures, the period used being 20.6 days. The observations of the first period are represented by points; those of the second period carried back to the first by small circles; while the points carried over from the first period to the section of the second period included in the drawing are denoted by crosses. The following provisional elements have been computed from the curve by Dr. Eberhard according to the method of Lehmann-Filhés,[1] on the assumption: $P = 20.6$ days, $A =$ 128 km, $B = 156$ km, $A$ and $B$ being the maximum relative intensities in the line of sight.

$T_v = 1901$ March 28.60 (Relative motion in line of sight=0)
$T = 1901$ March 28.88
$\omega = 101°3$
$e = 0.502$
$\log \mu = 9.4843$
$\mu = 17°476$
$a \sin i = 35$ million kilometers

$$m + m_1 = \frac{4\odot}{\sin^3 i}.$$

A curve computed from these elements agrees well with that drawn directly from the observations.

[1] *A. N.*, 136, 17, 1894.

# STANDARD LINES IN THE ARC SPECTRUM OF IRON.[1]

By H. KAYSER.

THE only really reliable method of determining wave-lengths is by producing in the spectrum under investigation a number of lines whose wave-lengths are accurately known. If we were reducing a photograph taken with a concave grating—hence a normal spectrum—a linear interpolation between two known lines at the end of the portion of spectrum would furnish approximate wave-lengths of the unknown lines intervening. But the spectrum is neither perfectly normal, nor is the measuring apparatus free from errors; and, moreover, there are errors in the standards themselves and in the settings made upon them. Hence a much greater accuracy is secured if we have at our disposal a greater number of standards between the terminal lines. The differences are then taken between the known wave-lengths of the standards and the wave-lengths computed from the terminal lines, and a curve is obtained by the method of least squares, or better graphically, which, instead of a straight line, best fits the measurements of the known lines. The measurements of the unknown lines are then corrected accordingly.

It is evidently desirable to have a large number of standards.

If the spectra are to be produced in the arc, iron lines are most convenient as standards, since the carbons contain so much iron that a large number of the principal lines will appear of themselves, and, if desired, some salt of iron, or the metal itself, may be introduced into the arc.

The basis of all determinations of wave-lengths for a long time to come will doubtless be Rowland's table,[2] which was obtained by the method of coincidences. It also includes many lines of the arc-spectrum of iron, but these are unfortunately not

[1] *Annalen der Physik* (4), 3, 1900.

[2] *Phil. Mag.* (5), 36, 49–75, 1893.

sufficient in number in all parts of the spectrum. The table gives the following number of such lines per hundred units from λ 2300 on, in the direction of greater wave-lengths; 8, 14, 13, 3, 17, 8, 17, 23, 3, 3, 3, 12, 7, 18, 22, 7, 4. Beyond λ 4000 the number becomes still less, and there are almost none beyond λ 4500.

Hence there is a pressing need of further measurements of the iron spectrum having an accuracy equal to that of Rowland's standards.

In beginning our investigations on the arc-spectra of the elements[1] Professor Runge and I accordingly first determined the iron spectrum. As Rowland's table had not then been published we had insufficient auxiliaries; and as they were our first measures, for which we thought we could be content with an accuracy of 0.1 tenth-meter, they did not turn out particularly well, and today they are entirely inadequate. Beside this, they were based upon a different value for the D lines than that later adopted by Rowland. Hence Runge and I later published a short list[2] of new measures on which our subsequent publications were based.

The accuracy of all measurements has meanwhile increased very considerably, and a limit of error of only a few thousandths of a tenth-meter is well attainable for sharp lines. I therefore undertook a new measurement of the iron spectrum for my determination of the spectra of the platinum group.[3] I have now made still another set of measures and I believe I have reached the limit of accuracy attainable with Rowland gratings, viz., for all lines a mean error of at most 0.003 tenth-meter.

My measures depend exclusively upon Rowland's standards, but in addition to his iron standards those of other elements have been employed, *Ni*, *Co*, *Mn*, *Ti*, *Mg*, *Ca*, *Sr*, *Zn*, *In*, *Ba*, etc., being introduced into the iron arc. It was thus possible to obtain a sufficient accuracy in those portions, as from λ 3200 to

[1] *Sitzungsberichte der k. Akad. d. Wissenschaft zu Berlin*, 1888.

[2] *Ibid.*, 1890. Also *Wied. Ann.*, 41, 302, 1890.

[3] *Ibid.*, 1897.

λ 3500, where Rowland's iron standards are absolutely inadequate. The standards are exclusively from the arc-spectrum, however, and never from the solar spectrum, for reasons presently to be mentioned.

It was Rowland's opinion that the error of none of his standards would exceed 0.01 tenth-meter. I believe, however, that it is larger in a very few instances, and such cases will be found on comparing my list with Rowland's. On the whole I think my values are more accurate than his, since the errors will balance each other in the large number of measures. Every wave-length in the following table is the mean of from six to fifteen determinations, on photographs made in different years and with three different gratings in different orders. The mean error lies between 0.001 and 0.003 tenth-meter.

This work might be thought superfluous, as Rowland has also published the wave-lengths of all the iron lines in his list of Fraunhofer lines. We must not forget, however, that with the accuracy aimed at here the wave-lengths of the solar lines are by no means to be regarded as identical with those of the same lines in the arc. A glance at Rowland's table shows what differences may occur for the two cases, even exceeding 0.2 tenth-meter. It appears, moreover, from the early observations of Lockyer on the varied displacements and distortions of the lines of the same element in Sun-spots, and from Jewell's observations, as if the different lines of iron, for instance, originated in different layers of the solar atmosphere where different conditions of pressure prevail. It is then entirely unpermissible to employ the wave-lengths of the solar spectrum for terrestrial spectra. For this reason I have not taken any solar lines from Rowland's table of standards.

The region near λ 3400 offers special difficulties. Rowland has two standards near λ 3306, then follow one at 3389, two at 3406, one at 3427, and several from 3440 on. In my opinion the standards at λ 3389, 3406, and 3427 are all given a value too large by 0.02 to 0.03 tenth-meter. My correction curves depending on these lines always showed a quite impossible bend at

$\lambda$ 3400, so that I was finally compelled to omit these standards. That I hit upon the right thing in so doing is rendered probable by the fact that the values I obtained for these lines agree well with Rowland's determinations in the solar spectrum.

The following table gives a number of iron lines entirely sufficient for the purpose of interpolation in the region of spectrum between $\lambda$ 2300 and 4500 photographable with ordinary plates. Those lines have been chosen which appear most readily and at the same time are as sharp as possible, hence the most easily reversible lines. The intensities depend upon quite rough estimates, and range from 1 for the faintest to 10 for the strongest lines. *r* indicates that the line easily shows a self-reversal, *u* that its edges are not sharp. Rowland's standards in the arc-spectrum, and in some few cases in the solar spectrum, are added for comparison.

In the later photographs and computations I have been assisted by Dr. H. Konen.

| Wave-length | I. | Rowland | Wave-length | I. | Rowland | Wave length | I. | Rowland |
|---|---|---|---|---|---|---|---|---|
| 2327.468 | 3 | ....... | 2413.393 | 4*r* | ....... | 2493.331 | 7*r* | ....... |
| 31.384 | 3 | ....... | 24.231 | 3 | ....... | 2496.625 | 4*r* | ....... |
| 32.869 | 3 | ....... | 31.126 | 2 | ....... | 2501.228 | 8*r* | 2501.223 |
| 43.567 | 3 | 2343.571 | 38.274 | 2 | ....... | 07.991 | 4*r* | ....... |
| 48.196 | 2 | ....... | 39.834 | 4*r* | ....... | 10.927 | 8*r* | 2510.934 |
| 48.380 | 2 | 2348.385 | 40.201 | 4*r* | ....... | 11.857 | 3 | ....... |
| 54.969 | 2 | ....... | 42.658 | 4*r* | ....... | 17.754 | 2 | ....... |
| 59.187 | 3 | ....... | 47.808 | 4*r* | 2447.785 | 18.198 | 8*r* | 2518.188 |
| 64.904 | 2 | 2364.897 | 53.568 | 2 | ....... | 22.950 | 20*r* | 2522.948 |
| 68.670 | 2 | ....... | 57.686 | 5*r* | 2457.680 | 23.754 | 4*r* | ....... |
| 73.813 | 3*r* | ....... | 62.279 | 4*r* | ....... | 24.393 | 5*r* | ....... |
| 75.273 | 3 | ....... | 62.740 | 10*r* | 2462.743 | 27.525 | 10*r* | 2527.530 |
| 79.355 | 3 | ....... | 65.244 | 5*r* | ....... | 29.223 | 8*r* | ....... |
| 80.840 | 4 | ....... | 68.974 | 4*r* | ....... | 29.928 | 6*r* | ....... |
| 82.114 | 7*r* | 2382.122 | 72.436 | 4*r* | ....... | 33.911 | 4 | ....... |
| 83.324 | 3 | ....... | 72.976 | 10*r* | 2472.974 | 35.699 | 6*r* | 2535.699 |
| 84.473 | 3 | ....... | 74.906 | 4*r* | ....... | 37.263 | 4*r* | ....... |
| 88.711 | 2 | 2388.710 | 79.872 | 10*r* | 2479.871 | 41.064 | 8*r* | 2541.058 |
| 90.058 | 2 | ....... | 83.361 | 20*r* | 2483.359 | 42.192 | 5*r* | ....... |
| 95.709 | 5*r* | 2395.715 | 83.618 | 3*r* | ....... | 44.016 | 4*r* | ....... |
| 2399.322 | 5*r* | 2399.328 | 84.280 | 8*r* | 2484.283 | 46.072 | 10*r* | 2546.068 |
| 2404.519 | 3 | ....... | 87.155 | 2*r* | ....... | 49.708 | 8*r* | 2549.704 |
| 04.969 | 5*r* | 2404.971 | 88.232 | 10*r* | 2488.238 | 51.192 | 3 | ....... |
| 06.742 | 5*r* | 2406.743 | 89.844 | 8*r* | 2489.838 | 56.963 | 2 | ....... |
| 10.601 | 5*r* | 2410.604 | 90.737 | 10*r* | 2490.723 | 62.619 | 5 | ....... |
| 2411.152 | 4*r* | ....... | 2491.249 | 10*r* | 2491.244 | 2567.001 | 4 | ....... |

| Wave-length | I. | Rowland | Wave-length | I. | Rowland | Wave-length | I. | Rowland |
|---|---|---|---|---|---|---|---|---|
| 2574.462 | 2 | ....... | 2747.080 | 5r | ....... | 2973.254 | 8r | 2973.255 |
| 75.845 | 3 | ....... | 50.238 | 10r | 2750.237 | 73.366 | 5r | 2973.358 |
| 78.012 | 3 | ....... | 55.834 | 5r | 2755.837 | 76.253 | 3 | ....... |
| 82.408 | 2 | ....... | 56.412 | 4r | 2756.427 | 81.565 | 7r | 2981.575 |
| 84.623 | 5r | 2584.629 | 57.413 | 4r | ....... | 83.690 | 10r | ....... |
| 85.964 | 3 | 2585.963 | 61.883 | 5r | 2761.876 | 87.410 | 4 | 2987.410 |
| 88.102 | 5r | ....... | 62.125 | 5r | 2762.110 | 90.511 | 4 | ....... |
| 98.456 | 5r | 2598.460 | 68.621 | 5r | 2768.630 | 94.554 | 10r | 2994.457 |
| 99.483 | 5r | 2599.494 | 72.205 | 8r | 2772.206 | 2999.630 | 8r | 2999.632 |
| 2599.663 | 4r | ....... | 78.327 | 6r | 2778.340 | 3001.068 | 10r | 3001.070 |
| 2606.920 | 3r | ....... | 81.936 | 3 | 2781.945 | 07.262 | 2 | 3007.260 |
| 07.155 | 3r | ....... | 88.207 | 10r | 2788.201 | 07.409 | 2r | 3007.408 |
| 11.963 | 5r | 2611.965 | 91.989 | 3 | ....... | 08.254 | 8r | 3008.255 |
| 13.914 | 4r | ....... | 2797.877 | 2 | ....... | 09.690 | 4r | 3009.696 |
| 17.706 | 4r | ....... | 2804.622 | 5r | ....... | 16.305 | 3 | 3016.296 |
| 18.108 | 2r | ....... | 07.088 | 5r | ....... | 17.747 | 8r | 3017.747 |
| 20.499 | 3 | ....... | 13.391 | 8r | 2813.388 | 19.105 | 4 | 3019.109 |
| 23.627 | 5r | ....... | 17.612 | 3 | ....... | 20.619 | 4r | 3020.611 |
| 25.754 | 5r | ....... | 23.382 | 5r | 2823.389 | 20.764 | 10r | 3020.759 |
| 28.383 | 5r | ....... | 25.660 | 6r | 2825.667 | 21.194 | 10r | 3021.191 |
| 31.139 | 5r | 2631.125 | 25.803 | 4r | ....... | 24.153 | 3r | 3024.154 |
| 35.899 | 3r | ....... | 32.543 | 8r | 2832.545 | 25.960 | 8r | 3025.958 |
| 44.085 | 3r | ....... | 35.562 | 4r | ....... | 31.753 | 4r | ....... |
| 47.649 | 3 | ....... | 38.231 | 3r | 2838.226 | 37.505 | 10r | 3037.505 |
| 51.800 | 2 | ....... | 43.742 | 3r | 2843.744 | 41.753 | 3 | ....... |
| 56.232 | 3 | ....... | 44.083 | 8r | 2844.085 | 41.860 | 3 | ....... |
| 66.897 | 3r | ....... | 48.828 | 3 | ....... | 47.719 | 10r | 3047.720 |
| 69.581 | 2 | ....... | 51.910 | 5r | 2851.904 | 51.179 | 3 | 3051.173 |
| 73.315 | 2 | ....... | 59.007 | 3 | ....... | 57.562 | 8r | 3057.557 |
| 79.148 | 8r | 2679.148 | 63.973 | 3 | ....... | 59.202 | 10r | 3059.200 |
| 80.544 | 3 | ....... | 69.418 | 5r | ....... | 64.042 | 2 | ....... |
| 89.302 | .. | ....... | 74.284 | 5r | ....... | 67.363 | 8r | 3067.363 |
| 90.153 | 2 | ....... | 77.414 | 3 | ....... | 68.286 | 3 | ....... |
| 2699 193 | 3 | ....... | 80.867 | 2 | ....... | 75.830 | 6r | 3075.849 |
| 2706.672 | 4r | 2706.684 | 87.920 | 3 | ....... | 83.853 | 5r | 3083.849 |
| 08.663 | 2 | ....... | 90.000 | 3r | ....... | 91.687 | 3 | ....... |
| 14.503 | 3 | ....... | 94.617 | 3 | ....... | 3095.013 | 2 | 3095.003 |
| 18.530 | 4r | ....... | 2899.531 | 3 | ....... | 3100.057 | 4r | 3100.064 |
| 19.121 | 10r | 2719.119 | 2901.496 | 3 | ....... | 00.418 | 4r | 3100.415 |
| 20.997 | 10r | 2720.989 | 07.630 | 3 | ....... | 00.778 | 4r | 3100.779 |
| 23.671 | 8r | 2723.668 | 12.273 | 8r | 2912.275 | 12.183 | 2 | ....... |
| 25.024 | 4r | ....... | 18.144 | 3 | ....... | 16.747 | 3 | ....... |
| 28.914 | 3 | ....... | 23.409 | 5 | ....... | 19.609 | 3 | ....... |
| 30.832 | 3 | ....... | 26.699 | 3 | ....... | 25.770 | 3 | ....... |
| 33.978 | 8r | 2733.973 | 29.119 | 8r | 2929.127 | 32.627 | 5r | ....... |
| 35.566 | 8r | ....... | 37.030 | 10r | 2937.020 | 40.503 | 3u | ....... |
| 37.407 | 10r | 2737.405 | 41.462 | 8r | ....... | 42.565 | 3u | ....... |
| 39.639 | 8r | ....... | 47.996 | 9r | 2947.993 | 44.096 | 3u | ....... |
| 42.349 | 5r | ....... | 48.557 | 4 | ....... | 51.460 | 3u | ....... |
| 42.506 | 10r | 2742.485 | 54.061 | 9r | 2954.058 | 57.157 | 4 | ....... |
| 44.163 | 8r | ....... | 57.484 | 9r | 2957.485 | 60.764 | 3 | ....... |
| 44.624 | 4r | ....... | 65.379 | 9r | 2965.381 | 62.064 | 3 | ....... |
| 45.177 | 5r | ....... | 67.019 | 10r | 2967.016 | 65.129 | 3 | ....... |
| 2746.580 | 4r | ....... | 2970.227 | 10r | 2970.223 | 3166.551 | 3 | ....... |

| Wave-length | I. | Rowland | Wave length | I. | Rowland | Wave-length | I. | Rowland |
|---|---|---|---|---|---|---|---|---|
| 3171.473 | 3 | ....... | 3397.117 | 3 | ....... | 3609.011 | 2 | 3609.015 |
| 75.556 | 7 | ....... | 3399.468 | 7 | ....... | 12.242 | 2 | 3612.237 |
| 78.122 | 5 | ....... | 3402.392 | 4 | ....... | 17.474 | 1 | ....... |
| 85.015 | 3 | ....... | | | 3406.602, | 17.944 | 5u | 3617.939 |
| 88.947 | 5 | ....... | 06.578 | 2 | in Sun | 18.918 | 7r | 3618.922 |
| 91.778 | 5 | ....... | | | 3406.572 | 22.158 | 5 | 3622.161 |
| 92.921 | 8 | ....... | | | 3406.965, | 30.506 | 3 | ....... |
| 93.423 | 8 | ....... | 06.938 | 4 | in Sun | 31.617 | 6r | 3631.616 |
| 3199.638 | 7 | ....... | | | 3406.943 | 32.195 | 5 | ....... |
| 3200.595 | 7 | ....... | 13.275 | 5 | ....... | 40.541 | 5 | 3640.545 |
| 05.513 | 8 | ....... | 18.649 | 5 | ....... | 47.997 | 7r | 3647.995 |
| 10.953 | 5 | ....... | 24.430 | 5r | ....... | 50.429 | 3 | ....... |
| 12.112 | 8 | ....... | | | 3427.282, | 51.615 | 5 | ....... |
| 14.158 | 10 | 3214.152 | 27.263 | 5 | in Sun | 55.625 | 3 | ....... |
| 16.057 | 5 | ....... | | | 3427.263 | 59.673 | 5 | ....... |
| 22.187 | 10r | 3222.197 | 40.762 | 9r | 3440.756 | 69.674 | 5 | ....... |
| 25.905 | 10r | 3225.907 | 41.138 | 8r | 3440.135 | 76.461 | 3 | ....... |
| 28.379 | 3 | ....... | 44.025 | 7r | 3444.024 | 80.062 | 4r | 3680.064 |
| 31.091 | 8 | ....... | 45.301 | 5 | ....... | 83.205 | 3 | 3683.209 |
| 39.564 | 8 | ....... | 50.484 | 4 | ....... | 87.609 | 4r | 3687.609 |
| 44.308 | 5 | ....... | 58.454 | 3 | ....... | 3695.202 | 3 | 3695.208 |
| 46.617 | 3 | ....... | 60.067 | 4 | ....... | 3702.180 | 2 | ....... |
| 48.332 | 5 | ....... | 66.006 | 5r | 3466.010 | 05.714 | 4r | 3705.715 |
| 53.043 | 3 | ....... | 71.413 | 3 | ....... | 07.199 | 3 | 3707.201 |
| 57.724 | 3 | ....... | 71.497 | 3 | ....... | 09.395 | 5r | 3709.395 |
| 65.746 | 8 | ....... | 75.600 | 6r | 3475.602 | 20.083 | 10r | 3720.082 |
| 71.129 | 5 | ....... | 76.850 | 6r | 3476.848 | 22.710 | 6r | 3722.712 |
| 80.386 | 5 | ....... | 83.159 | 3 | ....... | 24.527 | 5 | ....... |
| 84.720 | 3 | ....... | 85.490 | 3 | ....... | 27.769 | 5r | 3727.768 |
| 86.884 | 7 | ....... | 90.721 | 6r | 3490.724 | 31.102 | 2 | ....... |
| 92.721 | 5 | ....... | 3497.989 | 5r | 3497.991 | 32.541 | 5 | 3732.549 |
| 3298.263 | 5 | ....... | 3500.716 | 3 | ....... | 33.470 | 5r | 3733.467 |
| 3306.106 | 7 | 3306.119 | 06.650 | 3 | ....... | 35.016 | 9r | 3735.012 |
| 06.479 | 7 | 3306.481 | 08.627 | 2 | ....... | 37.278 | 8r | 3737.280 |
| 14.868 | 5 | ....... | 08.663 | 2 | ....... | 43.510 | 6r | 3743.506 |
| 17.251 | 3 | ....... | 13.974 | 5 | 3513.981 | 45.710 | 7r | 3745.708 |
| 25.589 | 3 | ....... | 21.415 | 5r | 3521.409 | 48.409 | 7r | 3748.410 |
| 28.992 | 5 | ....... | 26.196 | 4r | .... .. | 49.634 | 8r | 3749.633 |
| 37.793 | 4 | ....... | 26.822 | 4 | ....... | 58.381 | 8r | 3758.380 |
| 42.034 | 3 | ....... | 29.960 | 3 | ....... | 63.940 | 8r | 3763.939 |
| 42.340 | 4 | ....... | 36.694 | 4 | ....... | 67.339 | 7r | 3767.342 |
| 48.056 | 4 | ....... | 40.287 | 2 | ....... | 70.452 | 2 | ....... |
| 51.882 | 3 | ....... | 45.793 | 4 | ....... | 76.606 | 3 | ....... |
| 55.355 | 4 | ....... | 53.898 | 3 | ....... | 78.670 | 2 | ....... |
| 66.917 | 3 | ....... | 58.672 | 5r | 3558.674 | 88.031 | 5 | 3788.029 |
| 66.993 | 3 | ....... | 65.535 | 8r | 3565.530 | 90.242 | 5 | ....... |
| 67.675 | 5 | ....... | 70.257 | 8r | 3570.253 | 95.149 | 8r | 3795.148 |
| 78.814 | 5r | ....... | 81.348 | 7r | 3581.344 | 98.658 | 6r | ....... |
| 80.242 | 4 | ....... | 85.478 | 4r | ....... | 3799.694 | 6r | ....... |
| 84.113 | 4 | ....... | 87.137 | 4r | ....... | 3801.822 | 3 | ....... |
| | | 3389.913, | 94.767 | 4u | ....... | 06.847 | 3 | ....... |
| 89.882 | 2 | in Sun | 3599.781 | 2 | ....... | 13.202 | 5 | ....... |
| | | 3389.884 | 3605.619 | 4 | 3605.621 | 15.987 | 8r | 3815.984 |
| 3394.721 | 3 | ....... | 3606.836 | 4 | 3606.836 | 20.573 | 9r | 3820.566 |

| Wave-length | I. | Rowland | Wave-length | I. | Rowland | Wave-length | I. | Rowland |
|---|---|---|---|---|---|---|---|---|
| 3824.591 | 6r | ....... | 4007.429 | 3 | ....... | 4247.604 | 5 | ....... |
| 26.028 | 8r | 3826.024 | 17.303 | 2 | ....... | 50.299 | 8 | 4250.300 |
| 27.967 | 7r | 3827.973 | 22.029 | 5 | ....... | 50.948 | 8 | 4250.949 |
| 33.463 | 3 | ....... | 30.670 | 3 | ....... | 60.656 | 9 | 4260.647 |
| 34.370 | 8r | ....... | 32.796 | 2 | ....... | 71.333 | 7 | ....... |
| 40.586 | 7r | 3840.589 | 44.776 | 2 | ....... | | | 4271.920, |
| 41.194 | 8r | ....... | 45.978 | 10r | 4045.975 | 71.933 | 10r | in Sun |
| 50.114 | 8r | ....... | 55.706 | 3 | ....... | | | 4271.934 |
| 56.515 | 6r | ....... | 62.605 | 5 | ....... | 82.567 | 7 | ....... |
| 60.054 | 10r | 3860.050 | 63.755 | 10r | 4063.755 | 85.614 | 4 | ....... |
| 65.670 | 6r | ....... | 68.138 | 5 | ....... | 91.631 | 3 | ....... |
| 72.640 | 6r | ....... | 71.901 | 8r | 4071.903 | 94.290 | 6r | ....... |
| 78.166 | 6r | ....... | 79.999 | 3 | ....... | 4299.420 | 6r | ....... |
| 78.722 | 4 | ....... | 84.666 | 5 | ....... | 4309.542 | 4 | ....... |
| 86.426 | 6r | 3886.421 | 96.135 | 5 | ....... | 15.255 | 6 | ....... |
| 87.193 | 5r | ....... | 4098.346 | 5 | ....... | | | 4325.932, |
| 93.538 | 3 | ....... | 4107.646 | 5 | ....... | 25.941 | 8 | in Sun |
| 95.801 | 5r | ....... | 14.608 | 4 | ....... | | | 4325.939 |
| 3899.853 | 5r | ....... | 18.709 | 8 | ....... | 37.219 | 6 | ....... |
| 3903.097 | 6r | ...... | 37.156 | 6 | ....... | 46.739 | 3 | ....... |
| 06.624 | 6 | ....... | 44.033 | 10u | ....... | 52.910 | 5 | 4352.908 |
| 09.980 | 3 | ....... | 54.662 | 4 | ....... | 58.689 | 3 | ....... |
| 13.784 | 3 | ....... | 71.069 | 4 | ....... | 67.759 | 5 | ....... |
| 16.880 | 4r | 3916.886 | 75.799 | 5 | ....... | 69.954 | 5 | 4369.948 |
| 18.467 | 3r | ....... | 81.918 | 5 | ....... | 76.104 | 6 | 4376.108 |
| 20.404 | 6r | ....... | 87.221 | 8 | ....... | 83.724 | 8r | 4383.721 |
| 23.059 | 3r | ....... | 91.611 | 8 | ....... | 4391.137 | 4 | ....... |
| 28.073 | 5r | 3928.060 | 4199.256 | 6 | 4199.257 | 4404.929 | 8 | 4404.928 |
| 35.966 | 4 | ....... | | | 4202.187, | 15.301 | 8 | 4415.298 |
| 41.032 | 4 | 3941.034 | 4202.195 | 8 | in Sun | 27.490 | 6 | ....... |
| 45.269 | 2 | ....... | | | 4202.195 | 30.801 | 5 | ....... |
| 48.927 | 4 | ....... | 10.521 | 5 | ....... | 42.522 | 6 | ....... |
| 56.610 | 3 | ....... | 19.523 | 5 | ....... | 47.907 | 6 | 4447.912 |
| 56.823 | 5 | ....... | | | 4222.396, | 54.572 | 4 | ....... |
| 66.219 | 3 | ........ | 22.387 | 5 | in Sun | 61.838 | 5 | ....... |
| 69.411 | 6r | ....... | | | 4222.382 | 66.737 | 6 | ....... |
| 77.892 | 6 | ....... | 27.606 | 6 | ....... | 69.566 | 6 | ....... |
| 84.112 | 4 | ....... | 33.771 | 7 | ....... | 76.207 | 6 | ....... |
| 86.330 | 4 | ....... | 36.118 | 8 | ....... | 84.420 | 5 | ....... |
| 96.147 | 3 | ....... | 38.980 | 6 | ....... | 89.929 | 4 | ....... |
| 3998.211 | 3 | ....... | 4245.423 | 5 | ....... | 4494.755 | 6 | 4494.756 |

# OBSERVATIONS OF THE BRIGHTNESS OF *NOVA PERSEI.*

By George C. Comstock and Joel Stebbins.

The following comparisons of the brightness of *Nova Persei* with surrounding stars have been made by the method of Argelander, and in their reduction the magnitudes of the comparison stars have been taken from the column heading "H. P." in Hagen's First Chart and Catalogue for Observing *Nova Persei.* An opera glass has been used in making the larger part of the comparisons, but a few of the earlier ones were made with the naked eye.

Each observer has made his estimates quite independently of the other, but the observations have been continuously compared one with another, so that there has probably been produced in the later observations some tendency toward an artificial agreement in the estimates. Each observer has determined his light scale, value of one grade, from all of his own observations suitable for that purpose, excluding comparison stars where the estimated difference of brightness was less than three grades. We find for C, 1 grade = 0.12 magnitude and for S, 1 grade = 0.10 magnitude. From an examination of the residuals furnished by simultaneous comparisons of the *Nova* with different stars, the probable error of a single comparison is found to be aproximately 0.1 magnitude.

In the following table the *Nova* is represented by the letter *N*, and, save in the case of *Aldebaran*, all comparison stars are to be supposed to have the word "*Persei*" printed after the letter or number by which they are designated. The letters "SS" in the last column indicate that the estimate by S was made, not in grades, but in fractional parts of the total interval between the comparison stars. For example, $\kappa$, 1, *N*, 2, $\sigma$ . . . . SS, means than in respect of brightness *N* was one third of the way from $\kappa$ to $\sigma$. The hour at which the observation was made is expressed in Central Standard time, *i. e.*, six hours slower than Greenwich mean time.

| 1901 | Hour | Comparison | Mag. | Obs. | 1901 | Hour | Comparison | Mag. | Obs. |
|---|---|---|---|---|---|---|---|---|---|
| Feb. 24 | 11.0 | *N Aldebaran* | 1.1 | C | April 2 | 8.0 | *N*, 36 | 5.3 | S |
| 26 | 7.0 | *Aldebaran*, 6 *N* | 1.8 | C | 2 | 8.0 | 30, 1, *N* | 5.5 | S |
| 26 | 7.0 | *N*, 3, α | 1.5 | C | 2 | 8.0 | κ, 2, l, 1, *N* | 5.6 | SS |
| 27 | — | *N*, α | 1.9 | C | 3 | 7.3 | 30, 2, *N* | 5.6 | C |
| 27 | — | *N*, 3, β | 2.0 | C | 3 | 9.0 | *N*, 36 | 5.3 | S |
| 28 | — | *N*, 1, α | 1.8 | C | 3 | 9.0 | 30, 1, *N* | 5.5 | S |
| Mar. 3 | 9.0 | *N* β | 2.4 | C | 8 | 9.0 | (κ σ) *N* | 4.3 | C |
| 3 | 9.0 | *N* (α ε) | 2.4 | C | 8 | 9.0 | ν, 2, *N* | 4.1 | C |
| 4 | 7.0 | β, 2, *N* | 2.6 | C | 9 | 8.0 | *N*, 2, σ | 4.3 | C |
| 4 | 7.0 | *N*, 4, δ | 2.6 | C | 9 | 8.0 | *N* ψ | 4.2 | C |
| 6 | 7.3 | δ, 2, *N* | 3.3 | C | 10 | 8.0 | 30, 1, *N* | 5.5 | C |
| 6 | 7.3 | *N*, 4, ν | 3.4 | C | 14 | 9.0 | *N*, 36 | 5.3 | S |
| 11 | 6.7 | *N*, κ | 4.1 | C | 14 | 9.0 | 30, 1, *N* | 5.5 | S |
| 15 | 7.0 | κ = *N* = ν | 4.0 | C | 14 | 9.0 | *N*,1,B.D.+45°,811 | 5.5 | S |
| 15 | 8.7 | (ν κ), 1, *N* | 4.1 | C | 14 | 10.0 | 30, 2, *N* | 5.6 | C |
| 16 | — | *N*, 2, (ν κ) | 3.8 | C | 14 | 10.0 | *N*, 1, 36 | 5.2 | C |
| 17 | — | *N*, 2, (ν κ) | 3.8 | C | 15 | 8.2 | 30, 2, *N* | 5.6 | S |
| 20 | 9.0 | *N*, 3, ν | 3.5 | C | 15 | 8.2 | *N*,1,B.D.+45°,811 | 5.5 | S |
| 20 | 9.0 | *N*, δ | 3.1 | C | 15 | 8.2 | 36, 1, *N* | 5.4 | S |
| 21 | 8.2 | *N*, l | 5.1 | S | 15 | 8.5 | 30, 2, *N* | 5.6 | C |
| 21 | 9.2 | κ, 5, *N* | 4.7 | C | 15 | 8.5 | *N*, 36 | 5.3 | C |
| 21 | 9.2 | *N*, 2, σ | 4.3 | C | 18 | 9.0 | ν, 1, *N*, 2, l | 4.3 | SS |
| 21 | 9.2 | *N*, 2, l | 4.5 | C | 19 | 8.8 | *N*, (36, 30) | 5.4 | S |
| 22 | 7.2 | *N*, 1, σ | 4.4 | C | 20 | 8.7 | 36, 1, *N* | 5.4 | C |
| 22 | 7.2 | *N*, 3, l | 4.7 | C | 20 | 8.7 | 30, 2, *N* | 5.6 | C |
| 22 | 7.5 | *N*, σ | 4.5 | S | 20 | — | 36, 1, *N* | 5.4 | S |
| 22 | 7.5 | *N*, (κ l) | 4.6 | S | 20 | — | *N*,1,B.D.+45°,811 | 5.5 | S |
| 22 | 9.0 | *N*, (ν l) | 4.5 | S | 20 | — | 30, 3, *N* | 5.7 | S |
| 22 | 9.0 | κ, 1, *N*, 2 σ | 4.2 | SS | 22 | 10.0 | *N* (ν l)* | 4.5 | S |
| 22 | 9.0 | *N*, 1, ι | 4.1 | S | 23 | 8.2 | κ, 2, *N** | 4.3 | C |
| 22 | 9.2 | κ, 2, *N* | 4.3 | C | 23 | 8.2 | *N*, 1, ψ* | 4.1 | C |
| 27 | 7.3 | *N*, ξ | 4.1 | S | 23 | 9.0 | ν, 2, *N** | 4.1 | S |
| 27 | 7.3 | *N*, ι | 4.2 | S | 24 | 8.2 | *N*, 30 | 5.4 | C |
| 27 | 7.3 | (ν κ), 1, *N*, 4, σ | 4.1 | SS | 24 | 8.2 | 36, 2, *N* | 5.5 | C |
| 27 | 8.5 | *N*, 1, σ | 4.4 | C | 24 | 8.5 | *N*,1,B.D.+45°,811 | 5.5 | S |
| 27 | 8.5 | ν, 5, *N* | 4.5 | C | 24 | 8.5 | 36, 1, *N* | 5.4 | S |
| 27 | 8.5 | *N*, 2, l | 4.9 | C | 24 | 8.5 | 30, 2, *N* | 5.6 | S |
| 28 | 7.2 | σ, 1, *N* | 4.6 | S | 25 | 8.0 | 30, 1, *N* | 5.5 | C |
| 28 | 7.2 | ν, 1, *N*, 2, l | 4.7 | SS | 25 | 8.0 | 36, 2, *N* | 5.5 | C |
| 28 | 7.2 | *N*, 1, l, 1, 30 | 4.8 | SS | 25 | 9.8 | *N*,1,B.D.+45°,811 | 5.5 | S |
| 31 | 8.0 | κ, 4, *N* | 4.6 | C | 25 | 9.8 | 36, 1, *N* | 5.4 | S |
| 31 | 8.0 | *N*, 2, σ | 4.3 | C | 25 | 9.8 | 30, 2, *N* | 5.6 | S |
| 31 | 8.0 | ν, 3, *N* | 4.3 | C | 26 | 8.0 | 30, 2, *N* | 5.6 | C |
| 31 | 8.0 | *N*, 2, ω | 4.7 | C | 26 | 8.0 | 36, 3, *N* | 5.7 | C |
| 31 | 8.0 | (ν κ), 1, *N* | 4.1 | S | 26 | 8.0 | *N*, B.D.+45°, 811 | 5.6 | S |
| 31 | 8.0 | *N*, ι | 4.2 | S | 26 | 8.0 | 36, 2, *N* | 5.5 | S |
| 31 | 8.0 | *N* ξ | 4.1 | S | 26 | 8.0 | 30, 3, *N* | 5.7 | S |
| 31 | 8.0 | *N*, (ν σ) | 4.2 | S | May 1 | 8.5 | l, 2, *N* | 5.3 | S |
| April 2 | 7.8 | l, 2, *N* | 5.3 | C | 1 | 8.5 | *N*, 2, 36 | 5.1 | S |
| 2 | 7.8 | *N*, 1, 30 | 5.3 | C | 12 | 10.0 | *N* (l, 36)* | 5.2 | S |
| 2 | 7.8 | 36, 1, *N* | 5.4 | C | | | | | |

*** Through clouds.**

**WASHBURN OBSERVATORY,**
**Madison, May 1901.**

# ON THE DENSITY OF THE SOLAR NEBULA.

By ANNE SEWELL YOUNG.

IT would not seem unreasonable to suppose that a possible law of density of the original solar nebula might be determined from considerations based upon the distribution of the moment of momentum of the solar system.[1]

In this paper I will outline the method by which a law of density has been determined for a nebula such that the distribution of its moment of momentum is not inconsistent with that of the solar system. The test of the correctness of this law, or rather of the hypotheses upon which it depends, will be the comparison between the distribution of mass in the supposed nebula and in the present system. The results may have some bearing upon the nebular hypothesis, inasmuch as the assumptions made do not differ widely from those made in regard to the development of the solar nebula.

Briefly stated, this was the method of procedure. It was assumed that the original solar nebula was spherical in form, extending at least to the limits of the orbit of the planet *Neptune*; also that its mass, if not homogeneous, was arranged in homogeneous concentric layers such that the law of its density might be expressed as the sum of a series of $i + 1$ terms of the form $\frac{a_i}{r^i}$, in which the $a_i$ are constants to be determined from the investigation, and $r$ is the distance from the center of the nebula. This expression for density was substituted in the integral for the moment of momentum of a sphere, which could then be integrated, its value being expressed in terms of $a_i$ and $r$. Values of $r$ corresponding to the mean distances of the various planets were substituted in this expression; the results were placed equal

[1] For this suggestion I am indebted to Dr. F. R. Moulton, of The University of Chicago.

to the numerical values of the moment of momentum of respective systems consisting of the Sun and the planets, exclusive of those whose orbits lie beyond the orbit of the one whose distance from the Sun was used as the value of $r$. This gave a set of equations which would of necessity be simultaneous if the conditions upon which they were based were actual, and whose unknowns were the constants, $a_i$, of the assumed law of density.

It was also assumed that the planets were formed from the outer limits of the nebula in the order of their distances from the Sun; that when the nebula extended to the limits of the orbit of any planet, the angular rotational velocity of the entire mass was the same as the present angular velocity of the planet; and that the moment of momentum of the system has been constant. This last assumption must be true unless the system has been acted upon by some external force since the time under consideration.

It will be shown first that the nebula could not have been homogeneous.

Let $I\omega$ represent the moment of momentum of a sphere of radius $r$, rotating with the angular velocity $\omega$, and let $\sigma$ represent the density of the sphere. Then

$$I\omega = \omega \int_0^{\pi} \int_0^{2\pi} \int_0^{r} \sigma r^4 \sin^3 \phi \, d\phi \, d\theta \, dr \,. \quad (a) \,.$$

If $\sigma$ is a constant (the condition for homogeneity),

$$I\omega = \frac{8}{15} \pi \sigma r^5 \omega = \frac{2}{5} M r^2 \omega \,,$$

where $M$ is the whole mass of the system. If we neglect the diminution of mass of the original nebula and the consequent loss of moment of momentum due to the abandonment of the planets in succession, $r^2 \omega$ should be a constant. The logarithmic values for this product, computed for the mean distance of each planet from the Sun in terms of the Earth's distance, are as follows:

| | | | | | |
|---|---|---|---|---|---|
| *Neptune* | 0.732561 | *Jupiter* | 0.350505 | *Venus* | 9.922976 |
| *Uranus* | 0.635268 | *Mars* | 0.084746 | *Mercury* | 9.787242 |
| *Saturn* | 0.482886 | Earth | 9.993712 | Sun | 5.755774 |

A certain part of this variation could be accounted for by the separation of the planets from the parent mass. The masses of the planets are small as compared with the entire mass, hence the mass factor would change but little. That the variations in the values of $r^2\omega$ are entirely out of proportion to the loss of moment of momentum occasioned in this way will be made evident by reference to the table of orbital momenta given later. It will be seen that a very great change would be expected between the orbits of *Saturn* and *Mars*, because of the great orbital momentum of *Jupiter*, but that only small changes should take place between *Mars* and the Sun. We find, however, that $r^2\omega$ was reduced to about *two-fifths* of its former value in the first case, and to less than *one ten-thousandth* in the latter. It is, therefore, evident that the nebula could not have been homogeneous.

Let us now consider the assumption that the mass was arranged in homogeneous concentric layers, the density being a function of the radius. Let us suppose that the density may be represented by a series of the form

$$\frac{a_0}{r^0}+\frac{a_1}{r}+\frac{a_2}{r^2}+\frac{a_3}{r^3}+\ldots\ldots,$$

in which $r$ represents the distance from the center to any given point, and $a_0$, $a_1$, $a_2$, etc., are constants to be determined. Any one of them may equal unity or reduce to zero. In order to avoid the difficulties arising from terms which become infinite in the expression for the moment of momentum, I limited the series to five terms. Substituting in $(a)$ the value of $\sigma$, which was, therefore,

$$a_0+\frac{a_1}{r}+\frac{a_2}{r^2}+\frac{a_3}{r^3}+\frac{a_4}{r^4}\,,$$

we obtain

$$I\omega=\frac{8\pi}{3}\left[\frac{1}{5}a_0r^5+\frac{1}{4}a_1r^4+\frac{1}{3}a_2r^3+\frac{1}{2}a_3r^2+a_4r\right]\omega\,.$$

In the formation of the equations, the following data were used, the unit of time being the mean solar day, the unit of distance the mean distance of the Earth from the Sun:

| | Sid. Period[1] | Mean Distance[2] | Orbital Momentum[2] |
|---|---|---|---|
| *Neptune* | 60193.2$^d$ | 30.05660 | 1.806 |
| *Uranus* | 30681 | 19.18239 | 1.323 |
| *Saturn* | 10774.9 | 9.538786 | 5.456 |
| *Jupiter* | 4346.5 | 5.202776 | 13.469 |
| *Mars* | 687 | 1.523692 | 0.00253 |
| Earth | 365.25 | 1.000000 | 0.01720 |
| *Venus* | 224.7 | 0.723332 | 0.01309 |
| *Mercury* | 88 | 0.387098 | 0.00079 |
| Sun | 25.35 | 0.004664 | 0.444 |

Darwin has shown that the rotational moments of the planets are insignificant in comparison with their orbital moments,[3] and I have therefore disregarded them entirely. The maximum rotational moment is that of the planet *Jupiter*, and is only about $\frac{1}{6000}$ of the orbital moment of momentum of that planet.

The second members of the equations were determined by considering the original total momentum as unity, and deducting for each successive equation that fraction of the whole represented by the orbital momentum of the planet supposed to have been abandoned; that is, the second member would be unity in the equation whose $r$ was the radius of *Neptune's* orbit; in the equation for *Uranus*, the second member would be one diminished by the fraction representing the orbital momentum of *Neptune*, etc.

In this way the following set of equations was formed:[4]

$$(1)\left\{\begin{array}{lll} 29341.620a_0 & +1220.2680a_1 & +54.13189a_2 \\ +2.701498a_3 & +\ \ 0.1797608a_4 & =0.1193663 \end{array}\right. .$$

$$(2)\left\{\begin{array}{lll} 6095.019a_0 & +\ 397.1767a_1 & +27.60704a_2 \\ +2.158780a_3 & +\ \ 0.2250794a_4 & =0.1097987 \end{array}\right. .$$

$$(3)\left\{\begin{array}{lll} 527.69680a_0 & +\ \ 69.15147a_1 & +\ 9.666016a_2 \\ +1.520006a_3 & +\ \ 0.318700a_4 & =0.1027897 \end{array}\right. .$$

$$(4)\left\{\begin{array}{lll} 63.14962a_0 & +\ \ 15.17210a_1 & +\ 3.888206a_2 \\ +1.121000a_3 & +\ \ 0.43092237a_4 & =0.0738853 \end{array}\right. .$$

[1] C. A. YOUNG, *General Astronomy*, Art. 489.

[2] G. H. DARWIN, *Philosophical Transactions of the Royal Society*, Part II, 1881, pp. 516, 517.

[3] *Ibid.*, p. 523.

[4] In these equations both members have been divided by $\frac{8\pi}{3}$.

$$(5)\left\{\begin{array}{lll} 0.8607198a_0 + & 0.7061136a_1 & + 0.6178970a_2 \\ +0.6082893a_3 + & 0.7984413a_4 & = 0.0025303 \end{array}\right. .$$

$$(6)\left\{\begin{array}{lll} 0.1971253a_0 + & 0.2464056a_1 & + 0.3285420a_2 \\ +0.4928132a_3 + & 0.9847189a_4 & = 0.0025168 \end{array}\right. .$$

$$(7)\left\{\begin{array}{lll} 0.0634480a_0 + & 0.1096454a_1 & + 0.2021116a_2 \\ +0.4191262a_3 + & 1.1588760a_4 & = 0.0024257 \end{array}\right. .$$

$$(8)\left\{\begin{array}{lll} 0.0071114a_0 + & 0.0229638a_1 & + 0.0790973a_2 \\ +0.3065009a_3 + & 1.5835830a_4 & = 0.0023564 \end{array}\right. .$$

$$(9)\left\{\begin{array}{lll} 0.0000000a_0 + & 0.00000002a_1 & + 0.00000048a_2 \\ +0.00015444a_3 + & 0.0662310a_4 & = 0.0023522 \end{array}\right. .$$

I first attempted to effect a solution by using the four equations derived by combining (1) and (2), (3) and (4), etc., together with the solar equation. The values of $a_0$, $a_1$, $a_2$, etc., determined in this way failed to satisfy any of the original equations of the planets, a large negative residual in equation (1) being balanced by an equal positive residual in equation (2); the same result was found for the other three pairs.

It could hardly be expected that the solar equation would be found consistent with the others, as the present condition of the Sun must be entirely unlike what it was when it extended even to the limits of *Mercury's* orbit. It was decided, therefore, to confine the solution to the first five equations, formed from the five planets which lie beyond the orbit of the Earth. There is no evident reason why the distribution of mass should change radically between the limits of the orbits of *Mars* and *Neptune*, and, because of the rarity of the gases and the slow rotation of the entire mass, the assumption of sphericity of form would not be a violent one. The results obtained gave the following expression for the density of the solar nebula:

$$\sigma = 0.00000767 - \frac{0.00019634}{r} - \frac{0.0025714}{r^2} + \frac{0.10607164}{r^3} - \frac{0.07754859}{r^4}$$

These values of $a_0$, $a_1$, $a_2$, etc., satisfied the five equations in every case to the sixth decimal place, and in all but one to the seventh; but they failed entirely to satisfy the conditions expressed by the equations for the Earth, *Venus*, *Mercury* and Sun.

Having found a law of density consistent with the moment of momentum of the superior planets, the next step was the

computation of the amount of material within a spherical shell contained between the limits of the orbits of *Mars* and *Saturn*. This mass should be at least equal to the mass of the planet *Jupiter*, as it is altogether improbable that the nebula should have expanded instead of contracted during the process of development. This was done by computing the value of the integral

$$M = 4\pi \int_{r\,(Mars)}^{r\,(Saturn)} \left(a_0 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \frac{a_4}{r^4}\right) r^2 dr \ .$$

It was found that the entire mass lying between these limits would be only about *one fifth* of the Earth's mass, whereas the mass of *Jupiter* is more than *three hundred* times that of the Earth. These results show that the assumed law of density is an impossible one, and yet the method of attack seems legitimate. Because of the great discrepancies in the figures, it would seem that the law of density cannot be represented by a series of this general type such that the distribution of both the moment of momentum and the mass of the nebula will be satisfied. While one should not attach great importance to conclusions based upon such a large number of assumptions, several of which may be incorrect, from the preceding discussion it seems probable that the density of the solar nebula was irregular.

I may add that in an earlier attempt to find an expression for density which should be consistent with all nine equations, I assumed the form

$$\sigma = \frac{a_0}{(1+r)^0} + \frac{a_1}{1+r} + \frac{a_2}{(1+r)^2} + \frac{a_3}{(1+r)^3} + \dots ,$$

using nine terms of the series. This particular form removes all difficulty arising from terms which become infinity in the expression for the moment of momentum when $r$ is zero. The equations were formed and solved, but I do not consider the results of much value, because, as has been suggested, the solar equation probably could not be consistent with the others. The mass tests in this case also show that, with such a distribution of mass, the formation of a planet like *Jupiter* would have been impossible.

MT. HOLYOKE COLLEGE,
March 1901.

# *REVIEWS*

*Ueber die Ursache der Nordlichter.* SVANTE ARRHENIUS. *Öfversigt af Konigl. Vetanskaps-Akademiens Förhandlingar, 1900.* Pp. 545–580. Reprinted in the *Physikalische Zeitschrift*, Nov. 10 and 17, 1900.

THE manner in which two domains of science, apparently unrelated, are sometimes united and simplified by a keen observation is beautifully illustrated in the paper whose title has just been given. The brilliant Swedish chemist, S. Arrhenius, here applies Maxwell's electromagnetic theory to the explanation of solar repulsion on comets and to the explanation of the Aurora Borealis. Following is an abstract of the highly plausible result which he obtains:

Solar repulsion of the tails of comets, and the apparent ejection of matter from the Sun to form the corona and solar projections, have long puzzled scientists as seeming exceptions to the law of gravitation. Many theories in explanation of these phenomena have been proposed, electrical repulsion being perhaps the one most generally given.

Kepler[1] attempted the first explanation, basing his hypothesis on the emission theory of light, supposing that the matter might be repelled by the impact of the corpuscles. Newton accounted for the phenomena by supposing such a difference in the density of the surrounding medium as causes the ascension of hot air and smoke.

Euler[2] in the 18th century held that light waves, which he supposed to consist of longitudinal vibrations in the ether, were competent to produce repulsion. This view was so severely criticised that it was soon abandoned. Nevertheless, if Maxwell's electro-magnetic theory of light be accepted, it appears that Euler was, in the main, right. Maxwell[3] proves that in a medium in which electro-magnetic or light waves are propagated, a pressure is produced in the direction of propagation which, at any point, is numerically equal to the total energy per unit volume.

[1] KEPLER, *Principia Mathematica*, I, III, Prop. 41.

[2] EULER, *Mémoires de l'Academie de Berlin*, 1746, 2, 121, 135.

[3] MAXWELL, *Electricity and Magnetism*, 1873. Art. 792.

The amount of solar energy per square centimeter per second at the distance of the Earth is about 0.0417 calories; or 1775 ($42600 \times .0417 = 1775$) gram-centimeters per second per square centimeter. Since the velocity of the Sun's radiations is about $3 \times 10^{10}$ centimeters per second, the amount of solar energy per cubic centimeter $= 1775 \div 3 \times 10^{10} = 592 \times 10^{-10}$ gram-centimeters. This pressure acts only on the side of bodies toward the Sun, hence bodies are urged away from the Sun in the direction of the beam of light. Though the repulsion produced by the Sun's rays in the vicinity of the Earth is too small to be detected, near the Sun it is vastly greater. The average distance of the Earth from the Sun is about 215.7 times the Sun's radius; at the surface of the Sun the repulsion will then be $(215.7)^2 \times 592 \times 10^{-10} = 2.75 \times 10^{-3}$ grams per square centimeter. The weight of a body at the Sun is 27.47 times that at the Earth. Then a cubical body, 1 centimeter on an edge, of unit density, suspended so that its lower surface were perpendicular to the Sun's rays, would lose about one ten-thousandth part of its weight. If the body were more or less transparent, a deduction would have to be made for the light transmitted; but if the body were a perfect reflector the effect would be doubled, so perhaps computations based on the assumption that all the radiations are absorbed will be near the truth.

If now a cube of the same density $10^{-4}$ centimeters in diameter were taken, its weight would be $10^{-12}$ and its area $10^{-8}$ times that of the first; such a body would lose all its weight when subjected to the Sun's radiations. According to Bredichin[4] the matter composing the tails of comets is, at perihelion, repelled from the Sun with a force 1.5 to 18.5 times its weight. Assuming that the tails of comets are composed of gaseous hydrocarbons whose density could hardly exceed 0.8, the computed diameter of the particles to be thus repelled would lie between $0.1\mu$ and $1.25\mu$. Such particles would be much larger than simple molecules. Micro-organisms of a diameter not greater than $0.3\mu$ have been observed. When it is considered that these organisms are composed of many complicated organic molecules, it is evident that inorganic particles may be vastly smaller. Indeed liquid films have been produced as thin as $5\mu\mu$ ($0.005\mu$). Particles of this diameter would be 20 times smaller than is necessary to account for the maximum observed cometary repulsion.

[4] BREDICHIN, *Revision des valeurs numeriques de la force répulsive.* Leipsic, Voss, 1885.

As a comet approaches the Sun there is developed on the side toward the Sun an extension of the coma. This is accounted for by supposing that the head of the comet is composed of solid or liquid hydrocarbons of relatively high boiling point, which are vaporized under the intense heat of the Sun; while the particles are comparatively large they fall toward the Sun, but with their further dissipation they will be repelled and form a part of the tail. If the nucleus is heterogeneous, particles of many sizes may be formed, which, by their varying degrees of repulsion, may give rise to several distinct tails, as in the comet of 1744.

The apparent force of repulsion of the tail is not always proportional to the inverse square of its distance from the Sun. This is easily accounted for on the supposition that the size of the particles, and hence their force of repulsion, varies with the distance. It has been observed that comets are more numerous and brighter in years when Sun-spots are plentiful. Measurements made by Savélieff[1] in the summers of 1890, 1891, and 1892 when the number of Sun-spots were in the ratios of 7, 47, and 86, gave for the solar energy values of 29.8, 34.2, and 36 calories per square centimeter per hour. Thus Sun-spots accompany intense solar radiation; this means increased repulsion and the carrying away of a large amount of "cosmic dust," into colder space where the particles may aggregate till they again fall toward the Sun in the form of comets.

A particle having half the critical diameter, and projected from the surface of the Sun, would, in traveling a distance equal to the Sun's radius, acquire a velocity of 430 km per second. Such a particle would traverse a distance equal to the Sun's diameter in less than an hour. A particle with $\frac{1}{18}$ the critical diameter would travel that distance in four minutes. This "cosmic dust" thus shot off from the Sun may account for many of the phenomena of the solar corona, and the zodiacal light.

That these particles thus ejected from the Sun would be strongly electrified is almost certain, when we consider the violent electrical disturbances which always accompany volcanic eruptions. Cathode and Roentgen rays would be developed if the electrification were sufficiently intense, by which the surrounding gases would be ionized. The negative ions, as has been shown by Wilson,[2] would form centers

[1] SAVÉLIEFF, *Comptes Rendus*, 118, 62, 1894.

[2] WILSON, *Phil. Mag.*, 193, 289–308, 1899.

of condensation for the "cosmic dust" and consequently the particles finally leaving the neighborhood of the Sun, would be negatively charged, while the positive ions would remain behind. The side of the Earth turned toward the Sun would receive a constant stream of these negatively charged particles, which would, for the most part, remain in the upper strata of the air. Particles of a diameter of $1\,\mu$ would probably remain as high as 200 km. The atmosphere would be most strongly charged in the direct line between the Earth and the Sun, and in this region cathode rays might be developed. Under the action of the ultra-violet light, which would render the air conducting, the charges would be gradually conducted toward the less illuminated regions to the north and south. The normal circulation of the air would also work toward the same end.

Dr. Paulsen[1] found such a remarkable agreement in essential characteristics between the Aurora Borealis and the cathode rays that he declared the first to be a special form of the second. The great obstacle to the acceptance of his conclusions has been the difficulty of imagining any way in which the cathode rays could be produced; a difficulty which the present theory aims to overcome.

Since cathode rays tend to follow the lines of force in a magnetic field, the rays will, near the equator, where the lines of force are parallel to the Earth's surface, remain in the upper air, never penetrating deep enough to produce any visible illumination. Still aurora lines are found even in equatorial regions in the diffused light after sunset. The nearer the poles the greater the angle which the lines of force make with the Earth's surface, and the deeper the cathode rays will penetrate, till they reach strata sufficiently dense to produce a very considerable illumination, thus causing the aurora.

Practically all the known facts concerning the aurora harmonize with the theory that the light is produced by cathode rays which arise from negatively electrified particles repelled from the Sun. The remarkable identity of the 11-year periods of the aurora and Sun-spots; the annual, monthly, and daily variations in the number and intensity of the aurora following closely the variations in the position of the Sun and the intensity of its light may all be much more satisfactorily explained by this theory than perhaps by any other.

A. W. AUGUR.

[1] PAULSEN, "Sur la nature et l'origine de l'aurore boreale." *Bull. d. l'Ac. Roy. d. Sc. de Copenhague*, 1894.

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